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Cutoff grades optimization in open pit mines using meta-heuristic algorithms

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ABSTRACT

To have a sound production planning one of the main factors that should be considered is the cutoff grade. The cutoff grade is used as a criterion to identify waste of minerals in a mining reserve. The cutoff grade is one of the most sensitive parameters that can have a significant impact on net present value (NPV) and cash flow of projects. Since the cutoff grade has a significant impact on the operation, the choice of the correct level of this grade is of considerable importance. Choosing the optimal cutoff grade maximizes the NPV and the total profit of the mining operation and the project. The optimization of the cutoff grades considering the maximum achievable NPV over the life of the mine is one of the key issues in the mining of open pits. In this paper, two different meta-heuristic optimization algorithms are employed to determine the optimal cutoff grade. For this purpose, taking into account the precision of 0.001%, the optimum cutoff grades, the production amount of each unit and the NPV are calculated. Accordingly, the optimum cutoff grades of iron mine No. 1 Golgohar was obtained using the PSO algorithm is 49.11–40.6%, and using the imperialist competitive algorithm, the optimum cutoff grades of iron mine No. 1 Golgohar was obtained from 48.56% to 40.5%. The results show that the determination of the cutoff grade by using these two methods has high accuracy and speed. According to the results, the ICA algorithm has a higher accuracy than the PSO algorithm.

1. Introduction

The cutoff grade is used to detect minerals of a deposit from waste. (The higher the amount of minerals, the mine more economical). The cutoff grade is determined by various factors such as geological characteristics, practical restrictions of processes, and various economic parameters (Asad and Dimitrakopoulos, 2013; Azimi and Osanloo, 2011). Considering the effect of several practical and economic factors on the cutoff grade, determining it in different periods of mine life is a key issue in the planning of mining production and the most difficult and sensitive issues facing mining engineers (Gholinejad and Moosavi, 2016; Muttaqin et al., 2017; Rahimi and Ghasemzadeh, 2015). The cutoff grade calculated on the basis of the break-even analysis is called the break-even cutoff grade. This grade is evident in which the mining, processing, and refining costs of each of the minerals with the proceeds are equal. In other words, the income proceeds from each ton of minerals cover mining costs, processing, and refining of each ton of minerals without taking into account the cost of the waste. The break-even cutoff grade is employed to figure out the initial range of the mine; this value is useful for finding the mine floor and mine wall (Minnitt, 2004; Osanloo and Ataei, 2003; Osanloo et al., 2008). In the computation of the break-even cutoff grade, the time value of money, the distribution of ore deposits and mining capacities, processing plant, refining plant, and market are not considered, therefore extraction according to the break-even cutoff grade will not result in optimization of operations. There are several theories in determining the optimum cut-off grade; however, according to the recent works, calculation of cut-off grade based on maximizing net present value (NPV) gives more reliable results compared to other approaches (Ahmadi, 2018; Bascetin and Nieto, 2007; Wang et al., 2010). Therefore, during the first years of the mine's life, minerals will be extracted with high-grade minerals and the NPV is increased (Baird and Satchwell, 2001; Tatiya, 1996). There are numerous works have been done to figure out the optimum value of cutoff using different approaches, for instance, use of knockout methods to optimization the cutoff grade (Ataei and Osanloo, 2003), Determining the cutoff grade for optimizing the NPV based on the optimization factor and using the GRG algorithm (Bascetin and Nieto, 2007), Creation of a model with environmental considerations to optimization the cutoff grade with consideration of environmental impact (Rashidinejad et al., 2008), Taking into account the cost of tailings disposal in Lane's algorithm (Gholamnejad, 2008), Use of the neurological method - PSO to optimization the cutoff grade (He et al., 2009), Taking into account

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the combined mineral deposit stock, economic parameters and adjustments in the optimization of the cutoff grade (Asad and Topal, 2011), Dynamic Randomized cutoff grade Optimization (Barr, 2012), The effect of mineral price changes on the optimal cutoff grade (Jafarnejad, 2012), Optimization of the cutoff grade using multi-stage random planning (Li and Yang, 2012), Correcting the lane method by considering the variable capacities of units over time (Abdollahisharif et al., 2012), Optimization of the cutoff grade under uncertain prices using the Multi-criteria Decision Scoring System (Azimi et al., 2012). To optimize the cutoff grade, the Lane modeled the operational process of a mine, which is the only refined product to be sold, and accordingly, defines the objective function; As a result, his model in the mines of a metal that has the ability to produce and sell several types of products can not be used (Mohammadi et al., 2017). Therefore, in this research, the operational route of the mine is modeled and using it, the cost, income, and profit relationships are obtained and based on that, the objective function of the problem is determined for maximizing the NPV. To optimize this objective function, we use the metamorphic algorithms for particle swarm optimization (PSO) and imperialist competitive, including ultra-intelligent methods. The important point is that, in intelligent methods, it is not necessary to derive an objective function. Given this point, the answer to the problem is easily and quickly achieved with this method. So to get the answer, in the MATLAB R2016a software environment, for PSO algorithms and imperialist competitive, coding takes place. Using it, optimal cutoff grades values, the production of different mine units, the operating profit and the NPV of the hypothetical deposit are calculated. In this paper, the following sections present how to find the optimum value of the cut-off grade considering the limitations of mining, processing, and refining using PSO and ICA algorithms.

2. Cutoff grade optimization based on Lane's method

In order to determine the optimum cutoff grade, the mineralization operation consists of three steps of extraction, processing, melting, and refining based on the lane Theory. Each of these steps has a cost and a limited capacity. In addition, fixed costs are also included. With regard to costs and incomes in this operation, operating profit can be determined using below expression (Hustrulid et al., 2013):

$$P = (s - r) \quad Q_r - mQ_m - cQ_c - fT \tag{1}$$

In the above equation, T stands for the period of the production, Q_m denotes the amount of the required ore, Q_c represents the amount of the delivered mineral to the processing plant, Q_r stands for the value of the final product, f denotes the fixed costs per unit time, S represents the sale price of the final product, m is the cost of mining per ton ore, c stands for the cost of processing per ton of minerals and r denotes the cost of melting and refining each unit of the final product. The change of NPV of the reserves at $t = 0$ and $t = T$ after the mining operation is calculated from the following equation (Hustrulid et al., 2013):

$$V = (s - r) \quad Q_r - mQ_m - cQ_c \quad (f + Vd)T \tag{2}$$

In this relation, d stands for the discount rate, V represents the present value of NPV, which is calculated using the repeat process. The amount of refined ore (Q_r) depends on the amount of mineral delivered to the processing plant (Q_c). The amount of refined ore (Q_r) can be written in the following form:

$$Q_r = \bar{g}.\, y.\, Q_c \tag{3}$$

$$V = [(s - r) \quad g \quad \bar{} y - c] \quad Q_c - mQ_m - (f + Vd)T \tag{4}$$

In order to maximize NPV, the value of V must be maximized. If the capacity of the mine (M) is a decisive limitation, T is equal to Q_m/M, and if the capacity of the processing plant (C) is a decisive limitation, T is equal to Q_c/C, and if the refining capacity (R) is a decisive limitation, T is equal to Q_r/R will be. Then for each of these limitations, a value V is obtained (Hustrulid et al., 2013):

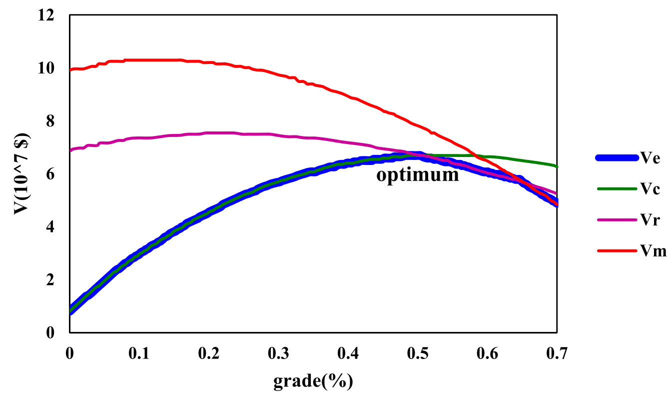

Fig. 1. Curves, V_m, V_c, V_r, V_e (Ahmadi and Shahabi, 2018).

$$V_m = [(s - r) \quad g \quad \bar{} y - c] \quad Q_c - [m + (f + Vd)/M]Q_m \tag{5}$$

$$V_c = [(s - r) \quad \bar{g}y - (c + (f + Vd)/C)] \quad Q_c - mQ_m \tag{6}$$

$$V_r = [(s - r - (f + Vd)/R) \quad \bar{g}y - c] \quad Q_c - mQ_m \tag{7}$$

In the top three cases, V_m, V_c, V_r can be plotted as a function of the grade. Convex is all upward. This is shown in Fig. 1. As stated, the goal of obtaining the optimal cutoff grade is to be maximized for V. The main goal is to obtain a grade that fulfilled the following equation.

$$maxV_e = max\,[min\,(V_m, \quad V_c, \quad V_r)] \tag{8}$$

3. Maximization of the objective function

As explained in the previous sections, three objective functions are employed to find the optimal cutoff grade of an ore deposit. For such an optimization problem with different objective functions, there are several options including numerical methods, mathematical techniques, and artificial intelligence based algorithms. For the mathematical techniques, we should take a derivative from the objective functions and then solve a system of ordinary differential equations. The main drawback of such methods is trapping in a local optimum, time consumption and limited to a linear system of equations. For numerical methods, there are two main approaches such as elimination and interpolation techniques. The main drawback of those methods is a simplification of the nonlinear system into linear ones; however, in some cases, it might be work. Simplifying the nonlinear systems into linear ones can mislead into the wrong solution. Metaheuristic or intelligent approaches inspired from the different aspects of nature and using those concepts occur in nature. The main advantages of those methods are derivative-free and easy-to-use.(Fan et al., 2008).

3.1. Particle Swarm Optimization (PSO) Algorithm

The PSO optimization method was introduced in 1995 by James Kennedy and Russell Eberhart. They initially intended to create a kind of computational intelligence using social models and existing social relationships that did not require individual abilities. Their work led to the creation of a robust algorithm for optimization, called the Particle Swarm Optimization or PSO algorithm. This method has been adapted from the collective function of animal groups, such as birds and fish (Shi and Eberhart, 1998). PSO particle swarm optimization algorithm has memory, so that knowledge of good solutions is maintained by all particles. In other words, in the particle swarm optimization algorithm, each particle benefits from its past information, while such behavior and features do not exist in other evolutionary algorithms. In the PSO particle swarm optimization algorithm, each member of the community changes its position according to personal experiences and the experiences of the entire community. In the particle swarm optimization algorithm, members of the population are interconnected and through the exchange of information, they are able to solve the problem and have a high convergence rate. The collective motion of the particle is an optimization technique, in which each particle tries to move towards the best of personal and collective experience in those parts. The PSO algorithm is easy to implement and is used to solve many discrete and nonlinear continuous optimization problems. This algorithm uses only basic mathematical operators and provides good results. Due to the benefits of a simple concept, easy implementation and rapid convergence compared to other optimization algorithms, today particle swarm optimization algorithm has many applications in different fields. According to these features, the PSO algorithm is used to optimize the cutoff grade for this research. In the PSO algorithm, there are a number of organisms that call them particles and are spun in the search space. Each particle calculates the value of the objective function in a position of space in which it is located. Then, using the combination of its current location data and the best place previously provided, as well as the information of one or more particles of the best particles in the aggregate, select a direction to move. After a collective move, one step of the algorithm is completed. These steps are repeated several times until the desired answer is obtained (Shi, 2001). The steps to implement the PSO algorithm are as follows (Shi, 2001):

1. Creating a Primary Population and Assessing It
2. Determine the best personal experiences and best collective experience
3. Update speed and position and evaluate new responses
4. If the conditions are not fulfilled, the stop will go to step 2.
5. End

The flowchart structure of the PSO algorithm is shown in Fig. 2:

3.1.1. PSO algorithm strategy

In the PSO algorithm, the particle speed in each step consists of two parts, the first part of which is the current particle size, and the second part is to follow the best personal experience and best band experience. Without the second part of the algorithm, the global search mode will be blind, and without the first part, the algorithm will become a local search near the best particle, which will be incapable of reaching large parts of the search space. By combining these two parts, the PSO algorithm tries to create a balance between global and local searches.

Eq. (9) is used to update the speed as following as (Shi, 2001; Zhang et al., 2004):

$$Velocity[t+1] = W + Velocity[t] + c1*rand(0, 1)*(pbest[t] - Position[t]) + c2*rand(0, 1)*(gbest - Position[t]) \tag{9}$$

W: Inertia weight. $Velocity[t]$: The velocity of the particle is at the instant t. $Position[t]$: The current position of the particle is at the instant t. $c2andc1$: Learning factors. $rand(0,1)$: A random number is in the range (0,1). $pbest$: The amount is allocated to the best position of each particle during the preceding steps with attention to a target function to that particle. $gbest$: The best value is for the position of all population particles during the previous steps.

Inertia Weight (W): The inertia weight W in the above relationship is used to ensure convergence in the PSO. Inertia weight is used to control the effect of previous speed records on current speeds. The proper value of W creates a balance between the ability of the algorithm to search for the general and the local. An appropriate amount of inertia weight usually creates the equilibrium between the inclusive and local exploration capabilities of the group. By choosing the right amount of inertial weight, the amount of repetition decreases to find the optimal response. The constant weight of the inertia is larger than one, although it causes the search algorithm to be faster, but the algorithm becomes unstable because it increases the speed of the previous one.

The $c1$ and $c2$ parameters are not very critical for the PSO convergence. The appropriate amount $c1$ and $c2$ may converge the answer sooner and prevent the possibility of being placed in the local committee. At first, the value of $c1 = c2 = 2$ was suggested, but various experiments showed that $c1 = c2 = 1$ could be more useful in achieving the better answer. In the general case, $c1$ and $c2$ can be selected differently depending on the case and with trial and error, but for the better and more precise search space of the answer, the condition $c1 + c2 \leq 4$ must always be observed.

The rand parameter is used to preserve the variety and diversity of the group; the appropriate value of the rand parameter is randomly selected in the interval between zero and one. These values allow particles to move in ranges between gbest and pbest in random steps (Shi, 2001; Zhang et al., 2004).

Eq. (10) is used to update the particle position (present particle position):

$$Position_i(t) + Velocity_i(t) \rightarrow Position_i(t+1) \tag{10}$$

In the upper equation, $Position_i(t)$ is the position of the particle at time t, and $Velocity_i(t)$ is the velocity of the particle i in time t. The particle update in the two-dimensional space is shown in Fig. 3(Shi, 2001; Zhang et al., 2004).

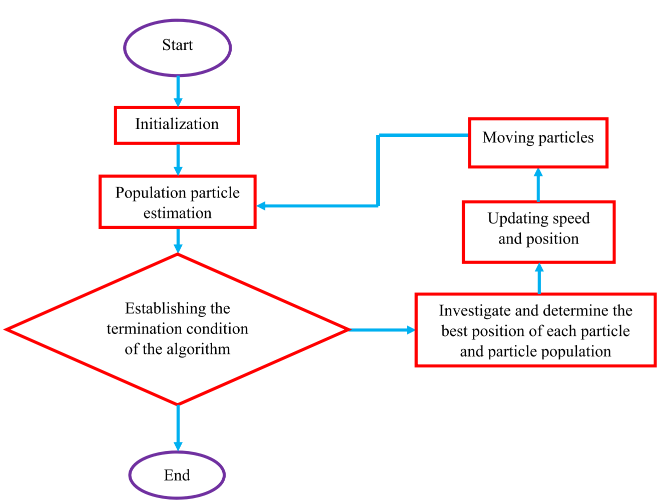

Fig. 2. Flowchart of the PSO algorithm (Shi, 2001).

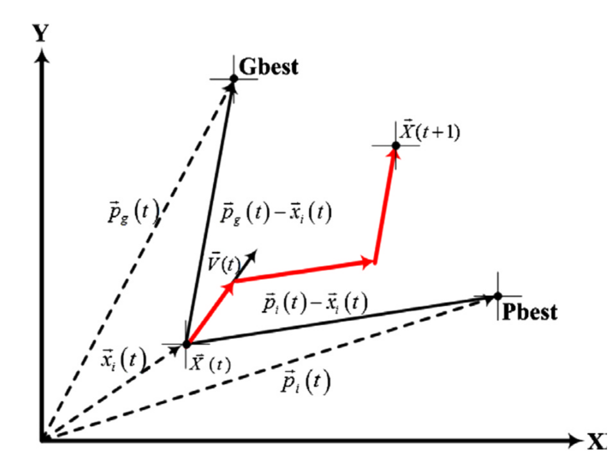

Fig. 3. Updating particles in a two-dimensional space (Shi, 2001; Zhu et al., 2011).

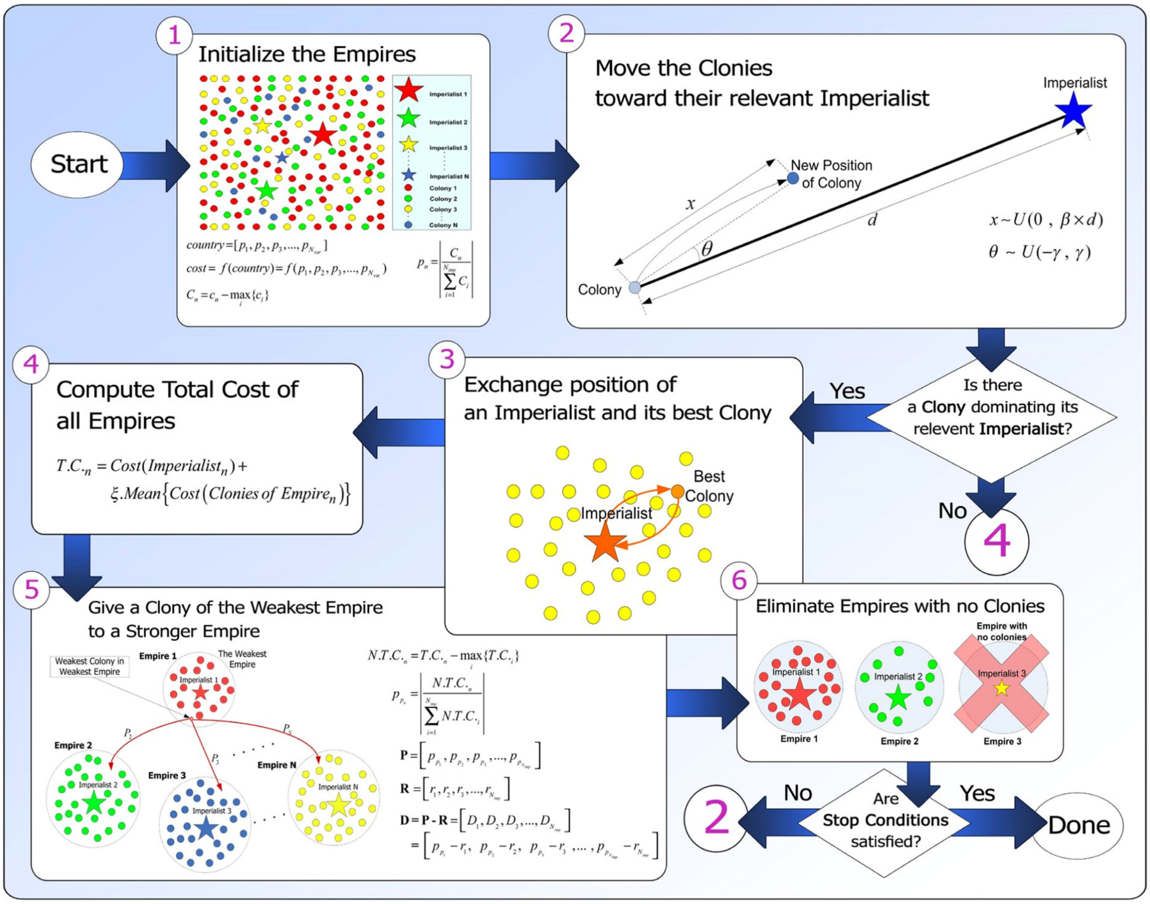

Fig. 4. The general idea of the ICA (Atashpaz Gargari et al., 2008).

3.2. Imperialist Competitive Algorithm (ICA)

The ICA was developed by Atashpaz and Lucas in 2007. This algorithm, in the first place, with a completely new perspective on optimization, establishes a new link between the humanities and the social sciences on the one hand and the technical and mathematical sciences on the other. This algorithm looks at the optimization problems in the form of countries and tries to improve these responses during the repetitive process and ultimately to the optimal solution to the problem. Shortly after introducing this algorithm, it has been used to solve many issues in the area of optimization. In order to evaluate the efficiency of this algorithm, the problems that have been solved with other evolutionary algorithms have also been solved with this algorithm, with better results both in terms of time and in terms of optimal response (Atashpaz Gargari et al., 2008). The speed of finding the optimal answer to this algorithm has been used to optimize the extent of this research. The elements inside the ICA are country and imperialist. Imperialist defined as the best countries and in our case the imperialists are the cu-off grade values. To have a better understanding how ICA works in optimization problems, the general scheme of this algorithm is shown in Fig. 4 (Atashpaz Gargari et al., 2008).

As shown in Fig. 4. The developed algorithm, like other evolutionary optimization methods, starts with a number of initial population (Part 1 of Fig. 4). Solutions gained from ICA are split into colonizers and colonies. Each colonizer, depending on its power, controls a number of colonial countries. Here, the stronger colonial country, represented by the larger star, has the largest number of colonial countries with colored circles. And the colonial country of all the weaker that appears with the smaller star has the fewest influential countries. The next stage is the colonial capture and competition policy that forms the core of this algorithm (Part 2 of Fig. 4). In the next stage, the costs are calculated, and if the cost of the colony falls less than that of the colonizer, the function of the revolutionary function changes the colonial and colonial position (Part 3 of Fig. 4). This action will make the colonial country move in line with the colonial country. In other words, each colonial country approaches the colonial country using a certain procedure, respectively. At this stage, according to the natural routine, some colonial states may revolutionize and capture the power of the empire (the operator of the revolution). After the absorption and revolutionary functions are performed, in the next step, the objective function is calculated for the total cost of the empires (Part 4 of Fig. 4). Now, for each of the empires, if the best function of the colonial countries of the colonial country was better than the objective function, then the two countries would change together. Meanwhile, after doing this for all the empires, the best answer and the amount of all colonial countries are stored as the best answer and the current amount in this algorithm's repetition. In the next stage, the colonies are separated from the weak empires and become stronger empires (Part 5 of Fig. 4). Finally, the weak empires are eliminated, and with the continuation of the algorithm, only one empire remains, which is the "optimal answer" (part 6 of Fig. 4).

3.2.1. Strategy of the imperialist competitive algorithm (ICA)

In optimization, the goal is to find an optimal answer based on the variables of the problem. Here it is called a country (cutoff grade). In the next optimization problem N_{var}, a country is an array of $1 \times N_{var}$. This array is defined as follows.

$$\text{Countrry} = [P_1, P_2, P_3, \ldots, P_{N_{var}}] \tag{11}$$

The values of variables in a country are represented as decimal numbers. In fact, in solving an optimization problem by the proposed algorithm, it is looking for the best country with social-political features (Which is looking for the best cutoff grade in this research). Finding this country (cut off grade) is, in fact, equivalent to finding the best parameters of the problem that produces the highest amount of the net present value function. To start the algorithm, there must be a number of these countries (the number of countries in the initial algorithm). Therefore, the matrix of all countries is formed randomly.

$$\text{COUNTRY} = \begin{bmatrix} country_1 \\ country_2 \\ country_3 \\ \vdots \\ country_{N_{country}} \end{bmatrix} \tag{12}$$

The NPV of a country is found by evaluating the function f in variables ($P_1, P_2, P_3, \ldots, P_{N_{var}}$) so:

$$NPV_i = f(country_i) = f(P_1, P_2, P_3, \ldots, P_{N_{var}}) \tag{13}$$

The algorithm presented in this paper, by producing a basic set of answers and categorizing them in the form of empires (the countries with the highest NPV), and applying the policy of absorbing colonialists to the colonies, as well as creating a colonial competition between the empires, searches for the best country (cutoff grade).

To start the algorithm, we create the $N_{country}$ as the number of the primary country (primary cutoff grade). N_{imp} selects the best members of this population (the solutions with the highest amount of the NPV function) as an imperialist. The remainder of N_{col} is composed of countries (cutoff grades), colonies (solutions with the lowest amount of the NPV function),each of which belongs to an empire. For the division of the initial colonies between the imperialists, each imperialist gives a number of colonies, which are in proportion to their strength. To do this, with the NPV of all imperialists, they consider their normalized NPV as follows (Atashpaz-Gargari and Lucas, 2007).

$$C_n = c_n - min\{c_i\} \tag{14}$$

Where C_n, is the NPV of the imperialist n, $min\{c_i\}$ the lowest NPV among imperialists, and c_n is the normalized NPV of this imperialist. Any imperialist who has NPV is less (imperialist weaker) will have less normalized NPV. With the normalization NPV, the normalized normal power of each imperialist is calculated as follows and based on that colonial countries are divided between imperialists.

$$P_n = \left| \frac{c_n}{\sum_{i=1}^{N_{imp}} c_i} \right| \tag{15}$$

From another point of view, the normalized power of an imperialist is the colonial proportions that the imperialist administration is managing. Therefore, the initial number of imperialist colonies will be equal to:

$$N.\,C_n = round\{p_n.(N_{col})\} \tag{16}$$

Where $N.C_n$ is the initial number of colonies in an empire and N_{col} is also the total number of colonial states in the population of the primary countries. Round is also a function that assigns the nearest integer to a decimal number. Given $N.C_n$ for each empire, it selects this number of colonial countries randomly and gives it to the imperialist n. With the initial state of all empires, the colonial competition algorithm begins. The evolutionary process is in a loop that continues until a condition for a moratorium is fulfilled. Considering the country's way of solving the optimization problem, in fact, the central government sought to bring the colonial country closer to different social-political dimensions through its policy of attraction. This part of the colonial process is modeled in the optimization algorithm, as the colonies move towards the imperialist state. Fig. 5 shows the general schema of this move (Atashpaz-Gargari and Lucas, 2007).

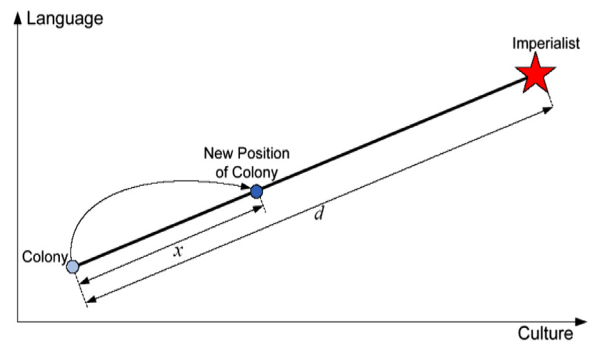

Fig. 5. The general concept of movement of the colonies towards the imperialist (Atashpaz Gargari et al., 2008).

As shown in this figure, the colonial country moves to a colonial position as large as x in the direction of the colonial colony's line, leading to a new position. In this figure, the distance between the colonizer and the colony is indicated by d. x is also a random number with uniform distribution(or any other suitable distribution). That means for x (Atashpaz-Gargari and Lucas, 2007).

$$x \sim U(0, \beta \times d) \tag{17}$$

Where β is a number greater than one and close to 2. A suitable choice can be $\beta = 2$. The presence of the coefficient $\beta > 1$ makes the colonial country closer to the colonial country as it moves in different directions (Atashpaz-Gargari and Lucas, 2007).

With the historical analysis of the phenomenon of assimilation, one obvious fact is that, despite the fact that colonial countries were consistently pursuing a policy of attraction, the events were not fully in line with their policies, and there were deviations as a result of work. In the proposed algorithm, this probable deviation is done by adding a random angle to the colonial absorption path. To this end, moving the colonies towards colonialism adds a bit of a random angle to the colonial movement. Fig. 6 shows this state. For this purpose, instead of moving x to the colonial country, this movement continues to move in the direction of the vector of the colony's colony to the same extent but with a deviation of θ on the path. θ is considered randomly and distributed uniformly (but any other suitable distribution may also be used). So:

$$\theta \sim U(-\gamma, \gamma) \tag{18}$$

In this regard, γ is a desirable parameter, which increases its search for imperialist surroundings and reduces it to allow the colonies to move as far as possible to colonial colonialism. Considering the radian unit for θ, the number is nearly $\frac{\pi}{4}$, in most implementations, it was a good choice.

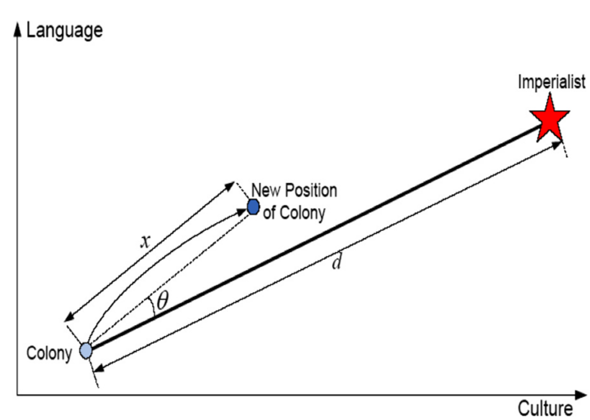

Fig. 6. Real movement of the colonies to the imperialist (Atashpaz Gargari et al., 2008).

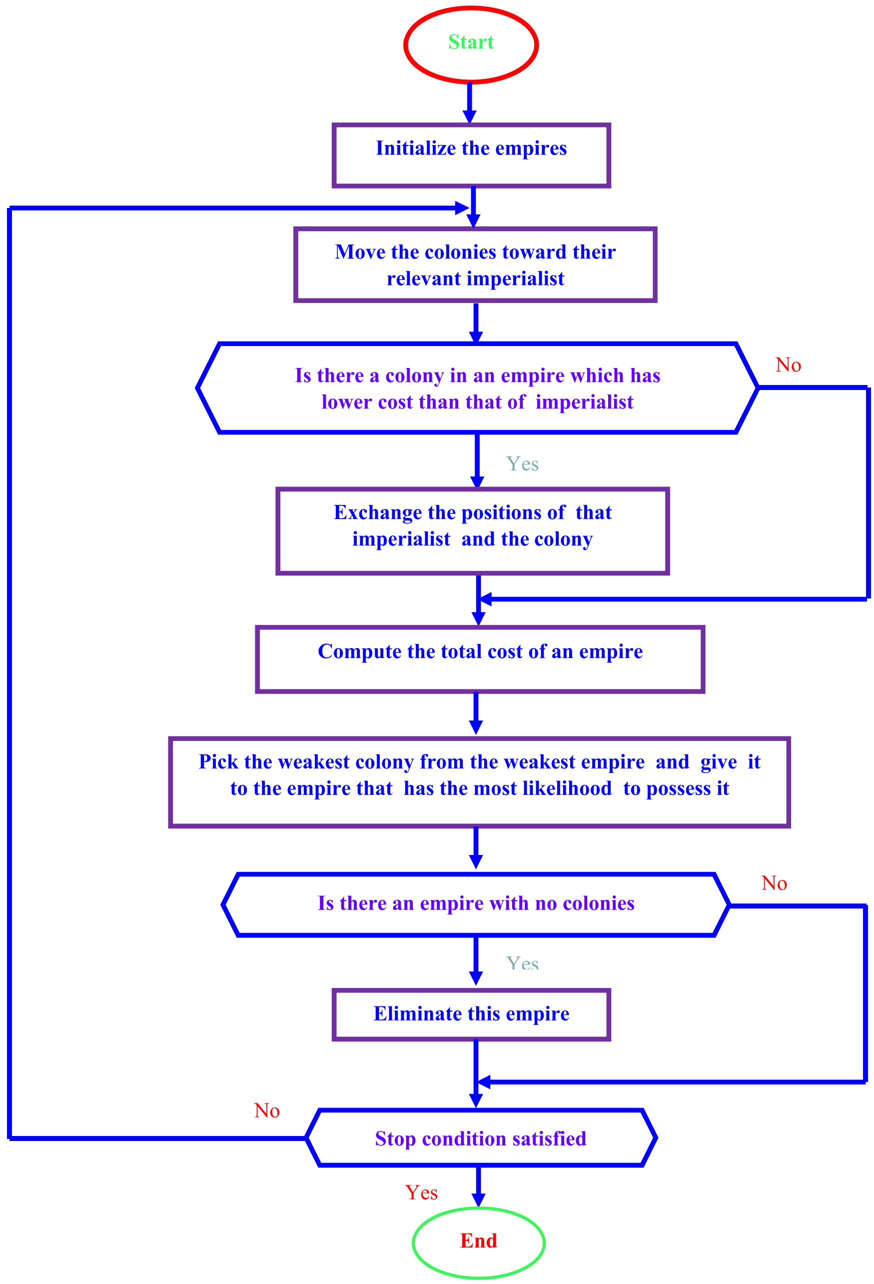

Fig. 7. : The trend of the ICA (Atashpaz-Gargari and Lucas, 2007).

In the modeling of this historical event, in the above-mentioned algorithm, it has been shown that during the movement of the colonies to the colonial country, it is possible that some of these colonies reach a better position than the imperialist (To points in the NPV function that produces more NPV than the amount of the NPV function in the imperial position). In this case, the colonial country and the colonial country have replaced each other and the algorithm with the colonial country has continued in a new position, and this time it is a new imperialist country that begins to apply a policy of assimilation on its colonies. The power of an empire equals the power of the colonial country, plus a percentage of the total power of its colonies, thus, for the total NPV of an empire (Atashpaz-Gargari and Lucas, 2007):

$$T.\ C_n = NPV\,(imperalist_n) + \xi mean\{NPV\,(colonies\ of\ empire_n)\} \tag{19}$$

Where $T.C_n$ is the total NPV of the empire n and ξ is a positive number, which is usually considered between zero and one and close to zero. The small consideration of ξ causes the total NPV of an empire to be approximately equal to the NPV of its central government (the imperial country), and the increase in ξ also increases the effect of the NPV of the empire's colonies in determining its total NPV. In a typical case, $\xi = 0.05$ has led to satisfactory answers in most implementations. As previously stated, any empire that can not increase its strength and lose its competitive power will be eliminated during imperialist rivalries. This deletion is made gradually. It means that, in the meantime, the weak empires of our colonies will be lost, and they will seize the stronger empires of these colonies and add to their power. To model this fact, they assume that the empire is removing the weakest empire. In this way, in repeating the algorithm, one or more of the weakest colonies of the weakest empire is taken and, in order to seize these colonies, create a competition among all the empires. The colonies will not necessarily be seized by the strongest empire, but stronger empires will be more likely to be seized. For this purpose, the total NPV of the empire is determined by its the NPV of the normalized total (Atashpaz-Gargari and Lucas, 2007):

$$N.\ T.\ C_n = T.\ C_n - min_i\{T.\ C_i\} \tag{20}$$

In this regard, $T.C_n$ is the total NPV of the empire n and $N.\ T.C_n$, as well, the total normalized NPV of that empire. Each Empire with less $T.C_n$ will have less $N.\ T.C_n$. In fact, $T.C_n$ is equivalent to the total NPV of an empire, and $N.\ T.C_n$ is equivalent to its total power. Empire with the highest NPV has the highest power. With the normalized NPV, the probability (power) of colonial conquest is calculated by each empire as follows:

$$P_{P_n} = \left| \frac{N.\ T.\ C_n}{\sum_{i=1}^{N_{imp}} N.\ T.\ C_i} \right| \tag{21}$$

With the possibility of seizing each empire so that the colonies are randomly divided, with the probability that each empire is properly seized, between empires, the vector P is formed from the above probability values as follows:

$$P = [P_{P_1}, P_{P_2}, P_{P_3}, ..., P_{PNimp}] \tag{22}$$

The P vector has a size of $1 \times N_{imp}$ and is composed of probabilities for the capture of empires. Then the random vector R is the same size as the vector P. Arrays of this vector are random numbers with uniform distribution in the interval [0,1].

$$R = [r_1, r_2, r_3, ..., r_{Nimp}] \tag{23}$$

$$r_1, r_2, r_3, \quad ... \quad, r_{Nimp} \sim \quad U(0, 1) \tag{24}$$

Then the vector D is formed as follows.

$$D = P - R = [D_1, D_2, D_3, ..., D_{Nimp}] = [P_{P_1} - r_1, \ P_{P_2} - r_2, P_{P_3} - r_3, ..., P_{P_{Nimp}} - r_{Nimp}] \tag{25}$$

With the vector D, they give the colonies an empire whose index of the vector D is larger than the rest. The empire that has the most probability of taking possession is likely to have the highest index in D in its index. With the colonial takeover by one of the empires, the operation of this phase of the algorithm ends (Atashpaz-Gargari and Lucas, 2007). In Fig. 7, the trend of the representation of this algorithm is shown.

3.3. Implementing meta-heuristic algorithms for determining the optimal cutoff grade

In order to investigate the meta-heuristic methods, the optimum cutoff grade of the hypothetical deposit is calculated (Lane, 1964, 1988). In the final range of 100 million tons of minerals with the distribution of grade listed in Table 1. Mining capacity (ore extraction and waste disposal) is 20 million tons per year, the capacity of the processing plant is 10 million tons per year and the refining unit capacity is 90 thousand tons per year, the mining cost is $ 0.5 per ton, the processing cost is $ 0.6 per ton, the melting and refining cost is $ 50 per ton, the fixed cost of $ 4 million per year, the sales price of the mineral is $ 550 per ton, recovery of 0.9% and a discount rate of 15%.

Table 1
Distribution of grade deposit.

Grade classes		Quantity (tons)	Grade classes		Quantity (tons)
From	Until		From	Until	
0.0	0.15	14,400,000	0.45	0.5	3,800,000
0.15	0.2	4,600,000	0.5	0.55	3,700,000
0.2	0.25	4,400,000	0.55	0.6	3,600,000
0.25	0.3	4,300,000	0.6	0.65	3,400,000
0.3	0.35	4,200,000	0.65	0.7	3,300,000
0.35	0.4	4,100,000	0.7	1.5	42,300,000
0.4	0.45	3,900,000	Total tonnage		100,000,000

3.3.1. Implementing the PSO algorithm

In solving this problem, it is intended to find a combination of cutoff grade, mining capacity, processing plant and refining unit that will bring the most profit and NPV. To accomplish this, the PSO algorithm is coded in the MATLAB R2016a software environment. The advantage of this program is that it is able to find the cutoff grade, which is at the same time optimized and based on maximizing the NPV. The first step in implementing the PSO algorithm and objective function and optimizing the cutoff grade of the mine is that all the required parameters are defined. In PSO, the number of parameters needed to solve the problem is 28 intervals. The number of these parameters is equal to the sum of the number of the cutoff grade, the amount of minerals, the amount of the processing plant and the amount of the refining per year. Determining the number of particles of a problem is one of the parameters whose correct adjustment is important in reaching the optimal answer at the right time. To solve this problem, at first, the 300 primitive particles used for optimization has been selected randomly. The inertia weight is also considered 0.9. Also, to generate new answers with several times the implementation of the program and review the results, the C_1 and C_2 parameters that are not critical for the PSO convergence, but their appropriate size converges the answer earlier, which showed several times with the implementation of the program that $C_1 = C_2 = 1$ It can be more helpful to get a better answer. In this research, the condition for executing the program in the PSO algorithm is determined based on the maximum number of repetitions, and the maximum repetition for the completion condition is 300 repetitions. Now, implementation of the objective function in MATLAB software is addressed in a way that is consistent with the PSO algorithm and the correct result is achieved at this stage, which took place in two steps. In the first step, the operational limitations that include the mining limitation, the processing plant limitation, and the refining unit limitation, and should not be higher than the limit, apply to the implementation of the PSO algorithm in optimizing the cutoff grade. If any of these restrictions are violated, the value of the objective function for this field of specific solutions is a very low value, so the probability that this solution will be chosen is very low, and we are guaranteed to be using the penalty function when it comes to a final answer. In the second step, the constraints are applied to the main function, at which point the objective function achieved by the equation is implemented using the fining method, and each of the constraints is added separately to the main function. The values (β_1), (β_2), (β_3) are the factors of the penalty function. The amount of fines for each limitation is derived from the multiplication of the factors of the fine imposed on the amount of the violation. Relationships 26–28 are the penalty functions of the PSO algorithm.

$$X_1(n) = \beta_1 \times \left(\frac{Q_m(n)}{M} - \frac{Q_c(n)}{C} \right) \tag{26}$$

$$X_2(n) = \beta_2 \times \left(\frac{Q_m(n)}{M} - \frac{Q_r(n)}{R} \right) \tag{27}$$

$$X_3(n) = \beta_3 \times \left(\frac{Q_c(n)}{C} - \frac{Q_r(n)}{R} \right) \tag{28}$$

Where, $X_1(n)$ stands for the annual value of fines related to the restriction between the capacity of mining and the processing plant, $X_2(n)$ represents the annual value of fines related to the restriction between the capacity of mining and the refining section, $X_3(n)$ denotes the annual value of fines related to the constraints between the processing plant and the refining section, $Q_m(n)$ stands for the annual value of mining minerals ,$Q_c(n)$ represents the annual amount of the mineral sent to processing plant, $Q_r(n)$ denotes the annual value of refined matter, M is the capacity of Mining, C is the capacity of the processing plant, and R stands for the refining unit capacity.

PSO algorithm employs the Eq. 29 as an objective function to figure out the optimum cu-off grade value.

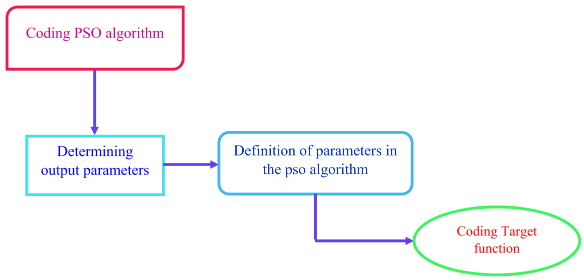

Fig. 8. PSO Algorithm Implementation Steps.

$$NPV(n) = \left(\left(\frac{I(n)}{(1+d)^n}\right) + X_1(n) + X_2(n) + X_3(n)\right) \tag{29}$$

Where, $NPV(n)$ stands for the annual NPV, $I(n)$ represents the annual profit, and d denotes the rate of discount. Fig. 8 shows the steps to implement the PSO algorithm.

By running more than 30 times the PSO algorithm, only the error rates of the cutoff grade and the NPV were reached zero, but there was no significant reduction in the objective function, the amount of mining capacity, the processing plant and the refining unit and the grade. Therefore, it can be concluded that the NPV of the hypothetical deposit after the optimization has been increased to 369,248,759 dollars, which is shown in Fig. 8 for the increase of the objective function. Therefore, it can be concluded that the NPV of the hypothetical deposit after the optimization has been increased to 369,248,759 dollars, which is shown in Fig. 9 for the increase of the objective function. Given that the objective of this research is the cutoff grade optimization based on maximizing the NPV, Fig. 8 shows the increase of the NPV for different repetitions. In this figure, the horizontal axis (Iter) represents the number of repetitions, and the vertical axis represents the NPV for each repetition. In this problem, the repeat rate for the PSO algorithm was tested with a try and error of 300 repetitions, and the condition for the completion of the program is applied.

3.3.2. Implementation of the ICA

At first, the 40 primary countries (cutoff grade) used for optimization were selected randomly. Six countries (cutoff grades) have been selected as imperialists (the cutoff grade with more NPV) and controlled by the rest of the 34 primary empires is formed. In the next stage, the policy of assimilation and absorption is achieved, and the colonies (the cutoff grades with less NPV) are attracted to empires according to the power of each empire (all calculations are done in MATLAB software). By continuing the algorithm of removing the weaker empires, and their colonies are divided among other empires.

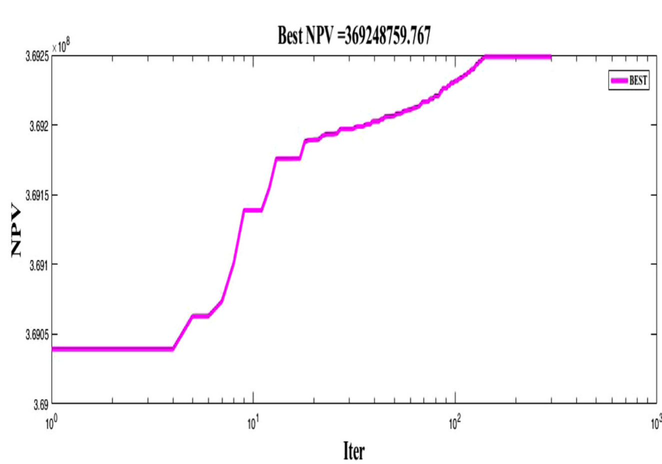

Fig. 9. Optimization of NPV using the PSO algorithm.

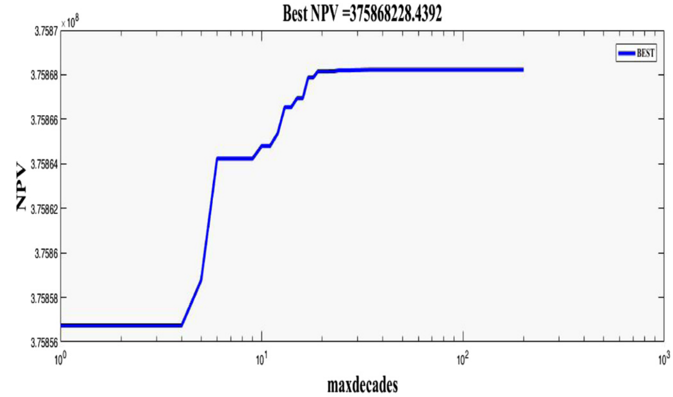

Fig. 10. Optimization NPV using the ICA.

During the rotation process, after the 5th generation, 2 empires have been removed and the colonies are divided according to the relationship 15 between the four remaining empires. As the algorithm continues, the process continues and the weak empires have lost their colonies and eliminated from the competition cycle. After 10 generations, only two empires have survived, and rivalry between these two empires continues to occupy more colonies. One of these two empires will be an optimal answer. With the continuation of the algorithm in the fifteen generation, all empires, with the exception of one, collapsed and become a monopolar world; the whole of which forms a single empire, and all the colonies and even the imperialists themselves have the same status. This Empire is the optimal answer to the problem. Using the imperialist competitive algorithm method, calculations determine the optimal cutoff grade for the desired deposit, the results are shown in Table 6. Therefore, the NPV of the hypothetical deposit after optimization has increased to 375,868,228 dollars, which is shown in Fig. 10 for the graph the increase of the objective function. Given that the aim is to optimize the cutoff grade maximization of the NPV, Fig. 10 shows the increase of the NPV for various repetitions. In this form, the horizontal axis (maxdecades) represents the number of repetitions, and the vertical axis represents the NPV for each repetition. In this problem, the repeat rate for the ICA was tested with a try and error of 200 repetitions.

4. Validation of models

To verify the validity of the proposed models, the optimum cutoff grade value of the hypothetical deposit was calculated using a method based on the Lane theory. In the proposed method, the optimal cutoff grade starts at 0.503% and at the end of the project lifetime to 0.22%. as well as the NPV obtained from the proposed method is $ 94,408,000. In the PSO algorithm, optimum cutoff grade at the start of the project of 0.512% and at the end of life reaching 0.225% and the NPV is $ 94,867,884. Also, in the imperialist competitive algorithm, the optimal cutoff grade at the start of the life of the mine of 0.507% and at the end of the life of the mine to 0.225%, and the NPV is $ 95,427,114. According to Figs. 11 and 12, respectively, the optimal cutoff grade and the NPV are close in each of the three methods. In Tables 2 and 3, the percentage of errors of the optimal cutoff grade is shown by the methods of the PSO and the imperialist competitive. The average error with the PSO algorithm model for an optimal cutoff grade is 2.73% and the average error with the ICA algorithm model for an optimal cutoff grade of 0.54%. As can be seen, the results of the proposed algorithms are in good agreement with the method based on the lane's theory. Among the two PSO and ICA algorithms, the imperialist competitive algorithm has less error.

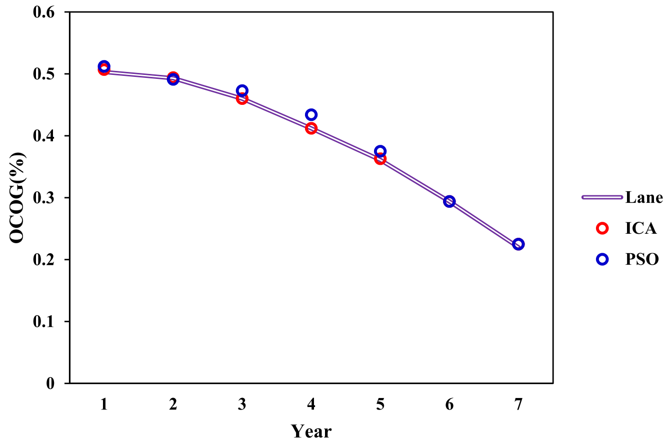

Fig. 11. Comparison of the optimal cutoff grades obtained from all three models.

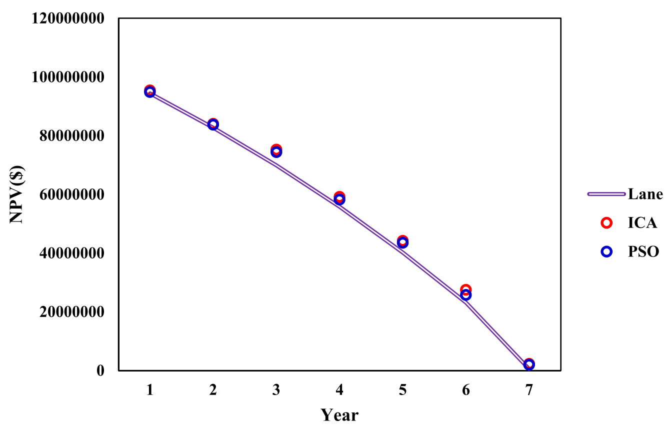

Fig. 12. Comparison of the NPV obtained each year in all three models.

Table 2
The error rate of the optimum cut-off grade of the PSO model.

Year	Lane Method (%)	PSO Algorithm (%)	Relative Error (%)
1	0.503	0.512	1.7
2	0.493	0.491	0.4
3	0.460	0.473	2.82
4	0.412	0.434	5.33
5	0.361	0.375	3.87
6	0.294	0.294	0.00
7	0.220	0.225	2.27
Average Error		2.73	

Table 3
The error rate of the optimum cut-off grade of the ICA model.

Year	Lane Method (%)	ICA Algorithm (%)	Relative Error (%)
1	0.503	0.507	0.80
2	0.493	0.494	0.20
3	0.460	0.460	0.00
4	0.412	0.412	0.00
5	0.361	0.363	0.55
6	0.294	0.294	0.00
7	0.220	0.225	2.27
Average Error		0.54	

5. Case study

Now, the optimization of the cutoff grade of iron mine Golgohar No. 1 using PSO and ICA algorithms is discussed. Mining capacity of 40 million tons per year, capacity of processing plant 12 million tons per year, refining capacity 42004,200,000 t per year, fixed cost $ 2 million per year, mining cost $ 0.2 per tonne, condensation cost $ 1.3 Per ton, the cost of melting and refining $ 28.1 per tonne, the sale price of the mineral $ 300 per tonne, recovery of 0.67 and the discount rate of 21% plan (Ahmadi, 2018). The distribution of the grade of Golgohar mineral deposit is shown in Table 4.

Table 4
Distribution of the grade of Golgohar mineral deposits in the 5-year plan (Ahmadi, 2018).

Grade(%)	Tonnage (ton)
40.5–45	61376,137,335,335
45–49.5	27,346,643
49.5–54	33,254,956
54–58.5	11,258,398
58.5–63	438,098
Total ore (ton)	78,435,430
Total waste (ton)	109,305,000
Total material (ton)	187,740,430

The optimum cutoff grade of the iron mine No. 1 of Golgohar was calculated using a method based on Lane's theory. The results obtained from this method are shown in Table 7. In the proposed method, the optimal cutoff grade starts at 48.72% and reaches 40.3% at the end of the project life. also, The NPV obtained from the proposed method is equal to 92,660 thousand dollars. In Table 5 presents the results of the PSO model, which at the beginning of the mine's lifetime in this 5-year plan, the mine is not in equilibrium and does not work at full capacity, but the processing plant and the refinery are in balance. Also, the optimal cutoff grade at the beginning of the life of the mine with 48/71% and at the end of the life of the mine reaches 40.6% and the NPV is 92,955,483 dollars. The results of the ICA model are also shown in Table 6. In the ICA model, the mine does not work at full capacity, but the processing plant and the refinery plant do work at full capacity. The optimum cutoff grade obtained from the ICA model initially starts with 48.56%, and at the end of its lifetime it will reach 40.5% and the NPV is 93,060,514 dollars. Also, in Figs. 13 and 14, accordingly,optimal cutoff grade variations, and the changes in the NPV of all three methods were graphically compared in years of mine life.

Also, by determining the amount of annual waste dump and mineral extraction amount, the value of the annual stripping ratio is determined. In Table 8, the total extraction, mineral content, waste and stripping ratios in the PSO algorithm are shown. Also, in Tables 9 and 10, the total extraction, mineral content, waste, and the stripping ratio were shown in ICA and lane's theory, respectively.

6. Conclusions

The optimum cutoff grade and the NPV help us to have an appropriate plan of production from an open pit mine. The present study has been used to determine the optimum cut-off grade of PSO metaheuristic algorithms and ICA, which is part of intelligent algorithms. The performance of the PSO algorithm and the ICA and Lane's theory in optimizing the cutoff grade on the basis of maximizing the NPV are compared. For this purpose, the hypothetical deposit was considered with different grades. In the PSO algorithm, the correct setting of the parameters is effective in achieving the optimal response, and the values of the parameters of the PSO algorithm are important in achieving the speed and accuracy of the results. Then, the ICA was implemented and considering the precision of 0.001%, the optimal cutoff grades, the amount of production of each unit and the NPV were calculated. In the ICA method to calculate the probability of seizing each empire, a new mechanism for implementing this process has been used which has much less computational cost than the roulette cycle in the genetic algorithm. In this mechanism, the relatively large operation associated

Table 5
Optimum cutoff grade, production of various units, profit and NPV in the different years of the mine life using PSO algorithm.

Year	Optimum cutoff grade (%)	Ore mined (ton)	Ore sent to processing (ton)	Refining rate (ton)	Profit ($)	NPV ($)
1	49.11	39,998,245	12,000,000	42004,200,000,000	25,882,985	93,955,239
2	48.26	39,983,627	12,000,000	42004,200,000,000	22,894,602	74,865,349
3	47.09	39,992,538	12,000,000	42004,200,000,000	18,134,955	54,635,619
4	45.82	39,990,764	12,000,000	42004,200,000,000	15,066,058	35,905,237
5	40.6	28,694,821	11,783,659	35383,538,124,124	2,328,131	2,188,443

Table 6
Optimum cutoff grade, production of various units, profit and NPV in the different years of the mine life using ICA algorithm.

Year	Optimum cutoff grade (%)	Ore mined (ton)	Ore sent to processing (ton)	Refining rate (ton)	Profit ($)	NPV ($)
1	48.56	39,992,025	12,000,000	42004,200,000,000	26,462,367	95,264,519
2	47.78	39,975,547	12,000,000	42004,200,000,000	23,137,999	75,892,637
3	46.81	39,991,158	12,000,000	42004,200,000,000	18,460,633	55,197,294
4	45.47	39,985,381	12,000,000	42004,200,000,000	15,639,627	37,065,916
5	40.5	27,795,319	11,904,539	34963,496,218,218	2,671,404	2,448,406

Table 7
Optimum cutoff grade, production of various units, profit and NPV in the different years of the mine life using lane method.

Year	Optimum cutoff grade (%)	Ore mined (ton)	Ore sent to processing (ton)	Refining rate (ton)	Profit ($)	NPV ($)
1	47.72	39,997,215	12,000,000	42004,200,000,000	25,596,000	92,660,000
2	47.14	39,970,427	12,000,000	42004,200,000,000	23,239,000	74,135,000
3	46.23	39,991,378	12,000,000	42004,200,000,000	17,836,000	53,840,000
4	45.28	39,987,153	12,000,000	42004,200,000,000	14,855,000	34,705,000
5	40.3	27,7943,257	11,426,729	37533,753,126,126	2,475,000	569,000

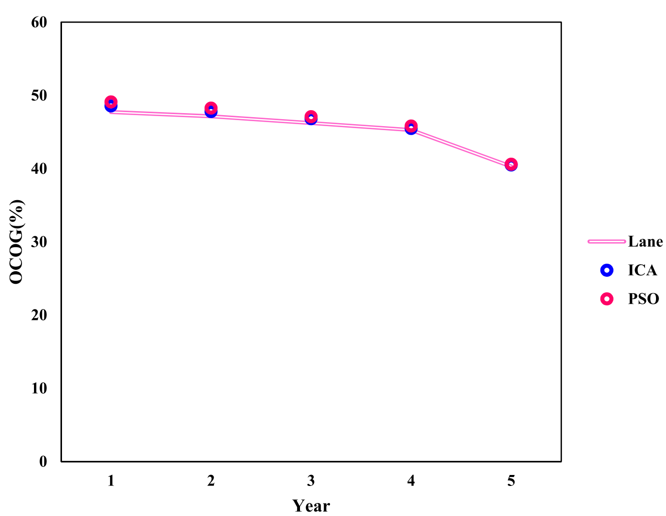

Fig. 13. Comparison of the optimum cutoff grades obtained from all three models for iron mine No. 1 Golgohar.

with calculating the probability cumulative distribution function (CDF) required in the roulette cycle is eliminated and only requires having a probability density function (PDF). Comparison of the economic results of the three methods shows that the NPV and the optimal cutoff grades obtained from PSO and ICA algorithms with the NPV and the optimal cutoff grades calculated by the Lane method have a reasonable agreement. Also, a real case study is used to determine the performance of those meta-heuristic optimization methods. In a real case, the economic results obtained from the PSO algorithm show that the optimal cutoff grade of the mine in the year of the start of this 5-year plan is 49.11% and in the final years reaches 40.6%, as well as the NPV in the initial year of the mine's life of 93955239 dollars. Also, the economic results of the ICA algorithm show that the NPV in the initial year of the mine's life is $ 95,264,519 and the optimum cutoff grade of the mine in the year of the start of this five year plan, 48.56%, and in the final years of the life of the mine reaches 40.5%. As a result, these meta-heuristic algorithms converge faster to the optimal answer.

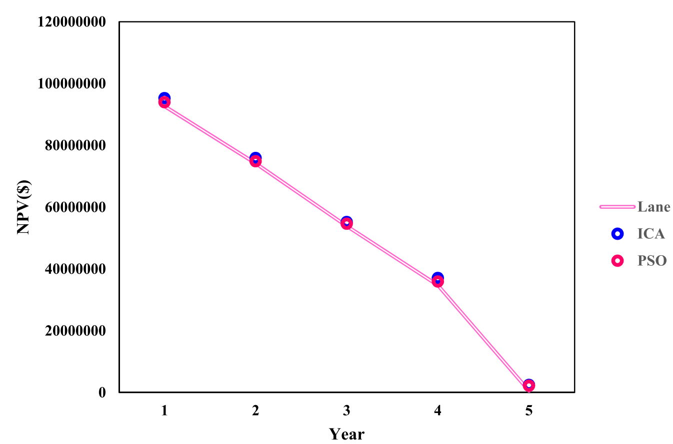

Fig. 14. Comparison of the NPV obtained each year in each three models for iron mine No. 1 Golgohar.

Table 8
Waste extraction and mineral extraction values and stripping ratio in the PSO algorithm.

Year	Total material (ton)	Waste (ton)	Ore (ton)	Stripping ratio
1	39,991,245	22,299,886	17,691,359	1.26
2	39,973,627	22,869,043	17,104,584	1.33
3	39,989,973	23,765,374	16,324,599	1.45
4	39,989,764	23,839,329	16,150,435	1.47
5	27,794,821	16,530,368	11,164,453	1.48

Table 9
Extraction values of waste and mineral matter and stripping ratio in the ICA.

Year	Total material (ton)	Waste (ton)	Ore (ton)	Stripping ratio
1	39,992,025	22,283,807	17,708,218	1.25
2	39,975,547	22,717,505	17,258,042	1.31
3	39,991,158	23,813,480	16,177,678	1.47
4	39,985,381	23,854,352	16,130,029	1.47
5	27,795,319	16,633,856	11,161,463	1.49

Table 10
Extraction amounts of waste and mineral matter and the stripping ratio in the lane's theory.

Year	Total material (ton)	Waste (ton)	Ore (ton)	Stripping ratio
1	39,997,215	22,544,100	17,453,115	1.29
2	39,970,427	22,871,679	17,098,748	1.33
3	39,991,378	23,443,443	16,547,935	1.35
4	39,987,153	23,868,115	16,119,038	1.48
5	27,793,257	16,576,663	11,216,594	1.48

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