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ORIGINAL PAPER

Application of meta-heuristic optimization algorithm to determine the optimal cutoff grade of open pit mines

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Abstract

The optimal cutoff grade leads to maximizing the profit or the net present value (NPV). One of the most important issues in open pit mining is cutoff grade optimization with regard to NPV maximization over the mine life. The most famous method that applied for cutoff grade optimization is the Lane methodology. Mining capacity, concentration plant capacity, and refining plant capacity are the three major parts of the abovementioned methodology and further, more time value of money and ore grade distribution are considered in Lane method. Repeated and cyclic calculations are the most disadvantages of lane methodology and therefore, in this research, the imperialist competitive algorithm (ICA) as a robust meta-heuristic optimization method has been applied to calculate the optimal cutoff grade. The accuracy of calculation was set to 0.001% and also the production of each unit and the NPV have been determined by ICA method. The optimum cutoff grades have been obtained from 0.232 to 0.135% in mine life time and NPV was calculated 37,750 billion Iranian Rials but the range of cutoff grades that calculated by Lane method was from 0.233 to 0.155% and NPV was determined 37,434 billion Iranian Rials. The results of this study showed that the determination of the optimum cutoff grade by the ICA has high accuracy and more speed of calculation in comparison with traditional lane method.

Keywords Optimum cutoff grade · Net present value · ICA · Sarcheshmeh copper mine

Introduction

The destinations of mine materials are determined by cutoff grade criterion (King 1999, 2001; Wooler, 2001). With consideration of several technical and economic parameters on the extent of the limit, cutoff grade determination in different periods of mine life is a fundamental problem in mine planning and furthermore is one of the most difficult issues faced by engineers (Rendu 2014). Head-to-head ratio of cutoff grade is defined as the grade that the income of ore covers the cost of mining, processing, and refining stages without consideration of the waste dump costs. This grade is used to determine the initial mine limit (Azimi et al. 2013; Osanloo et al. 2008). Unfortunately, several important factors are not included, e.g., capacity ofmining, in the abovementioned cutoff grade and in conclusion, material extraction with regard to the head-to-head ratio of cutoff grade will not lead to optimum results. There are several methodologies for the optimal cutoff grade determination, but most recent studies showed that the consideration of NPV maximization as the objective function leads to more realistic results (Bascetin and Nieto 2007; Wang et al. 2010). In early life of mining the annual NPV increases because of extration of higher grade materials (Baird and Satchwell 2001; Tatiya 1996). Mining, processing plant, and refinery are the three main stages that proposed by Lane method. With consideration of these stages, three economic and balancing cutoff grade for each pair of stage should be obtained. After determinantion these values, via a graphical approach, one of them is determined as the optimal cutoff grade (Ataei and Osanloo 2004; Lane 1988; Tatiya 1996). Therefore, the process of cutoff grade calculation finality has been time-

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consuming by using Lane's method. After presenting Lane's theory, no new independent method or algorithm has been presented yet, and the work done by other researchers has focused on using other methods of optimization based on this method. The following works are examples of such approaches: using knock-on optimization cutoff grade (Ataei 2003; Osanloo and Ataei 2003), the dynamic price metal-adjusted cost optimization cutoff grade (Asad and Dessureault 2005), set the cutoff grade to optimize NPV agent-based optimization algorithm GRG (Bascetin and Nieto 2007), minimizing acid mine drainage (Rashidinejad et al. 2008), considering the costs of the accumulation of waste in the Lane algorithm (Gholamnejad 2008), using a nerve-genetic cutoff grade optimization (He et al. 2009), the combination of stockpile with low grade mineral and the economic parameters in cutoff grade optimization (Asad and Topal 2011), optimization of dynamic random cutoff grade (Barr 2012), the impact of changes in the copper price on the cutoff grade optimization problem (Jafarnejad 2012), cutoff grade optimization by multi-stage stochastic programming (Li and Yang 2012), modification of Lane algorithm by consideration of the dynamic unit capacity (mining, processing plant, and refinery) during mine life (Abdollahisharif et al. 2012), and uncertainty analysis by the scoring system with MCDM methods (Azimi et al. 2012). Recently, several researches have been done in the area of cutoff grade optimization by derivative-free algorithms, which are known as meta-heuristic methods and other optimization methods for nonlinear programming problems, for example, using genetic algorithm (GA), particle swarm optimization (PSO), heuristic approach, stochastic methods, grid search, and dynamic programming and nonlinear programming (Asad and Dimitrakopoulos 2013; Cetin and Dowd 2013; Meagher et al. 2014; Yasrebi et al. 2015; Rahimi et al. 2015a; Rahimi et al. 2015b; Rahimi and Ghasemzadeh 2015; Rahimi and Akbari 2016; Cetin and Dowd 2016; Ahmadi 2018; King and Newman 2018). Lane model in a metal mine that producing and selling several types of products could not be applied (Ahmadi and Shahabi 2018; Maleki et al. 2016; Mohammadi et al. 2015). According to this viewpoint, in the present research, by modeling the operational process of the mine and then the economic parameters such as cost, income and profit are calculated, and based on these parameters, the objective function (NPV maximization) of the optimization problem is defined. To optimize this objective function, the ICA algorithm was applied as a meta-heuristic optimization method. ICA is a derivative-free method which avoids any local optimum trapping. Accordingly, it should be noted that high speed of calculation is the one the main advantages of ICA application in this problem. To this end, for coding the MATLAB R2016a software, a colonial competition algorithm is used to process the answer and using it, the values of optimal cutoff grades, various mine units production, operating profit, and NPV of biggest copper mine of Iran (Sarcheshmeh copper mine) were calculated. In this research, the optimum cutoff grade determination with the goal of the NPV maximization by ICA method and then the proficiency of the ICA approach and Lane's model were compared.

Lane Model

Three stages of extraction, concentrate production, melting, and refining are the main steps in Lane's methodology for cutoff grade optimization. Capacity and operation cost should be considered in this method. It should be noted that fixed cost are also included in it. Considering economic parameters in this operation, the annual profit can be evaluated using the formula below (Hustrulid et al. 2013):

$$P = (s-r)\ Q_{\mathrm{r}} - mQ_m - cQ_c - fT \tag{1}$$

In this equation, T is the production period, Q_{m} stands for the targeted mining of materials, Q_{c} represents the delivered materials to the concentration plant, Q_{r} denotes the final product, f stands for the fixed costs per unit time, S represents the final selling price, m denotes the mining cost in terms of ton material, c is the cost of concentration per ton, and r resembles the cost of melting and purifying of the final product per unit. If d is the discount rate, the difference between the NPV of the remaining reserves at $t = 0$ and $t = T$ after the mining operation is calculated by Eq. (2) (Hustrulid et al. 2013; Ahmadi 2018).

$$v = (s-r)\ Q_r - mQ_m - cQ_c\ (f + Vd)T \tag{2}$$

In this equation, V is the NPV of the remaining reserve at the starting point (time $t = 0$), which is calculated by the process given in Table 1. The amount of refined matter (Q_r) cahnges with the delivered minerals to the processing plant (Q_c). Regarding the average amount of minerals mined to the concentration plant ($\overline{g}$) and recovery percentage (y), the amount of refined matter (Q_r) is equal to (Ahmadi 2018):

$$Q_r = \overline{g}.y.Q_c \tag{3}$$

$$v = \left[(s-r)\ \overline{g}\ y - c\right]\ Q_c - mQ_m - (f + Vd) \tag{4}$$

Table 1 Effective parameters in optimization cutoff grade (Ahmadi, 2018)

Symbol	Definition	Unit
Q_m	Material mined	Ton
Q_c	Ore processed	Ton
Q_r	Ore produced	Ton
M	Mining capacity	ton/year
C	Processing plant capacity	ton/year
R	Units purification capacity	ton/year
$\boldsymbol{S}$	Final selling price	Rial/ton
$\boldsymbol{m}$	Mining cost	Rial/ton
$\boldsymbol{c}$	Processing cost	Rial/ton
$\boldsymbol{r}$	Refining cost	Rial/ton
F	Fixed cost	Rial/year
T	Years of production	Year
y_c	Recovery of processing	%
D	Discount rate	%

The value of v should be optimized for NPV maximization and in this process, the capacity of each mining unit, concentration plant and refinery plant may be acted as a limiting factor. Each of these capacity may be acted as limiting factor and by consideration of which of these capacities is limiting the operation, the value of T will be changed (Eq. (4)).

Therefore, in each case, the value of v can be evaluated by using Eqs. (5–7) (Ahmadi and Shahabi 2018).

$$v_m = \left[(s-r)\,\overline{g}y-c\right]Q_c-[m+(f+Vd)/M]Q_m \tag{5}$$

$$v_c = \left[(s-r)\,\overline{g}\,y-(c+(f+Vd)/C)\right]Q_c-m\,Q_m \tag{6}$$

$$v_r = \left[(s-r-(f+Vd)/R)\,\overline{g}\,y-c\right]Q_c-mQ_m \tag{7}$$

v_m, v_c, and v_r can be plotted as a function of the cutoff grade. As showed in Fig. 1, the curve convexity is upward for these equations. As mentioned above, for finding the optimum cutoff grade, the intersection of these plots should be considered. Therefore, in order to find the optimal cutoff grade, the first part of the three curves must be obtained (v_e). Secondly, the magnitude that maximizes this curve. The goal is to obtain an intelligence that satisfies the function below.

$$\max\,(\mathrm{v_e}) = \max\,[\min\,(v_m, v_c, v_r)] \tag{8}$$

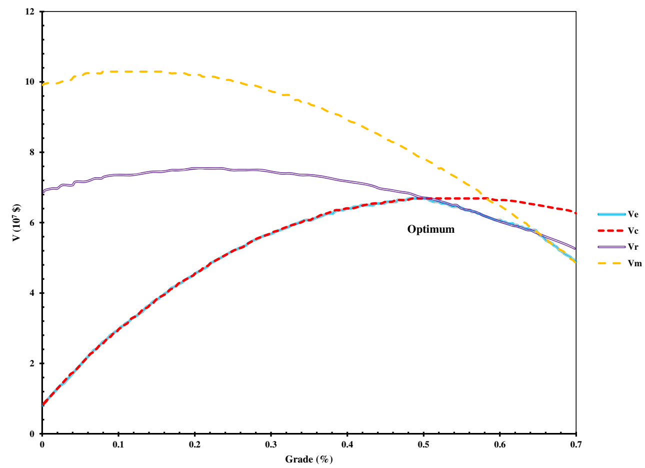

Fig. 1 Curves V_m, V_c, V_r, V_e (Ahmadi and Bazzazi 2019)

Maximizing the objective function

There numerous approaches can be implemented to maximize the given function. These techniques comprise heuristic and meta heuristic, derivative based approach, and numerical algorithms. Each of these methods has pros and cons. The user should decide which one works best for the problem. The main drawback of derivative based approach is the high probablity of being trapped in the local extermum. However, the prime advantage of meta-heuristic ones is easy-to-apply without being trapped in local extremum.

Imperialist competitive algorithm

The imperialist competitive algorithm (ICA) is a method in evolutionary computation that tries to find an optimal answer for different optimization problems. This method presents an algorithm for solving the optimization problems based on the nature of political relations. The process of ICA algorithm is that in the first step, it begins with a number of primary populations and then the value of the objective function is calculated for each populations that named country in the ICA algorithm. By comparing the values of the objective function of all countries, some of the best ones are selected and colonialist countries are chosen. On the other hand, the rest of the countries called colonies in this technique. It should be noted that, under the same conditions, several levels of the objective function are selected randomly for the colonialist countries. Now, colonies countries are assigned to colonialist countries and therefore, these collections (colonialist countries and colonies) are named empires. It should be mentioned that the greater the power of a colonialist country (it means that objective function is better) resulted in the more colonies countries are allocated to it. (Atashpaz Gargari and Lucas 2007; Ahmadi 2011; Ahmadi et al. 2013).

Strategy of ICA

In optimization problems, taking into account the function f (x), try to find the x argument in such a way that the corresponding NPV, each country's NPV (cutoff grade) by examining the function f for parameters (P_1, P_2, P_3,…,P_{var}), so:

$$NPV_i = f(country_i) = f(P_{i1}, P_{i2}, P_{i3}, \dots, P_{ivar}) \tag{9}$$

In the ICA algorithm to start, the initial country $N_{country}$ number (initial cutoff grade) is created and N_{imp} denotes the cutoff grades with the largest NPV as Colonialist. Colonial countries (the cutoff grades with the highest amount of NPV), by applying the policy of absorbing (assimilating) along optimization different axes, carry the colonies countries (the cutoff grades with the least amount of NPV). According to the power they have, colonialists are pulling these colonies. The total power of each empire is determined by calculating the power of both its constitutive parts, namely, the colonial country power (the cutoff grades with the highest amount of NPV), together with a percentage of the colonies power average (the cutoff grades with the least amount of NPV) (Atashpaz Gargari et al. 2008):

$$T.C_n = NPV(imperialist_n) + (\varepsilon mean\{NPV(colonies\ of\ empire_n)\}) \tag{10}$$

The colonial country moves to size x units in the direction of the colonialist and leads to a new position. In Fig. 2, the distance between the colonialist (the cutoff grades with the highest amount of NPV) and the colony (the cutoff grades with the least amount of NPV) is shown with d and x are random numbers. The following is for x (Atashpaz Gargari and Lucas 2007):

$$x = U(0, \beta \times d) \tag{11}$$

where $1<\beta\leq 2$ and the motion angle is considered uniformly distributed.

$$\theta \approx U(-\gamma, \gamma) \tag{12}$$

In the ICA algorithm, it is possible that the colony moves attraction path to colonialist with a possible deviation. The deviation represented by the angle θ, which is selected randomly from uniform distribution span (Fig. 3).

In Eq. (12), γ stands for the favorable factor, which increases the searches around the imperialists (the cutoff grade with the highest amount of NPV) and its decline also shrinks the distance between colony and colonialist. Given the radian unit for θ, a number close to π/4 has been a good selection for most implementations (Atashpaz Gargari and Lucas 2007). As the colonies (the cutoff grades with the least amount of NPV) move towards a colonialist country (the cutoff grade with the highest amount of NPV), compared to the colonialist, some of these colonies may attain a better position. In this case, the colonialist and the colony change their position. To take into account such a competition, first, the possibility of seizing colonies by each empire, using the empire total cost, is calculated as follows (Atashpaz Gargari and Lucas 2007; Ahmadi 2011; Ahmadi et al. 2013).

$$N.T.C_n = T.C_n - min_i\{T.C_i\} \tag{13}$$

Where empire's total NPV represented by $T.\ C_n$ and N.T.C_n stands for the normalized total NPV of the given empire; this parameter demonstrates the total power of the empire. The highest power among all empires belong to the empire with the lighest NPV value. Considering the concept explained above, Eq. (14) calculates the power of the empire in the world (Ahmadi and Chen 2019):

$$P_{pn} = \left| \frac{N.T.C_n}{\sum_{i=1}^{N_{imp}} N.T.C_i} \right| \tag{14}$$

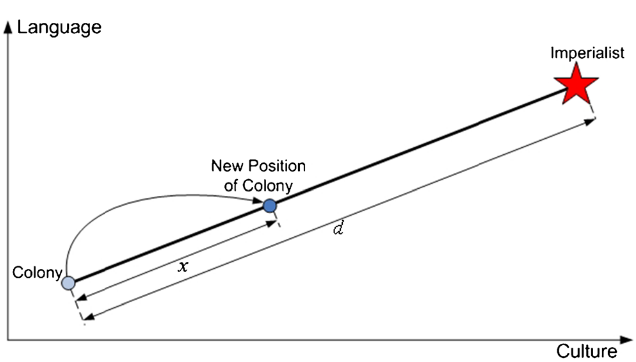

Fig. 2 Movement of the colonies towards the imperialists in the culture and language axes direction (attraction policy)(Atashpaz Gargari et al. 2008)

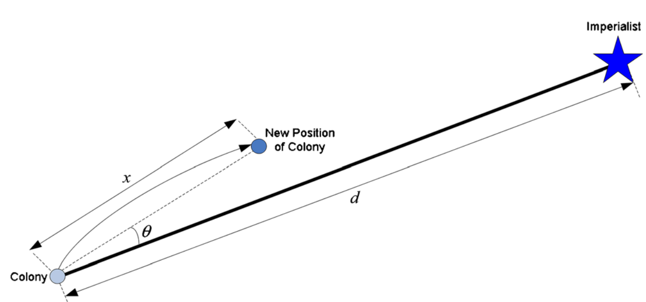

Fig. 3 The actual colony movement to the imperialist (Atashpaz Gargari et al., 2008)

According the power calculated via Eq. (14) for each empire, all colonies are randomly distributed in the world, and the vector P created as shown in Eq. (15) (Ahmadi and Chen 2019; Ahmadi et al. 2013):

$$P = \left[P_{P_1}, P_{P_2}, P_{P_3}, \dots, P_{P_{N\,imp}}\right] \tag{15}$$

The P vector has $1 \times N_{imp}$ arrays and comprised by the empires relative power. We consider another vector with the same size as P vector; however, all arrays are arbitrary values between zero and one.

$$R = \left[r_1, r_2, r_3, \dots, r_{N_{imp}}\right] \tag{16}$$

$$r_1, r_2, r_3, \dots, r_{N_{imp}} \sim U(0,1) \tag{17}$$

Then, the vector D can be defined as Eq. (18) (Ahmadi and Chen 2019):

$$D = P - R = \left[D_1, D_2, D_3, \dots, D_{N_{imp}}\right] = \left[P_{P_1} - r_1, P_{P_2} - r_2, P_{P_3} - r_3, \dots, P_{P_{N_{imp}}} - r_{N_{imp}}\right] \tag{18}$$

Via vector D, colonies can be assigned to the empire with the highest array inside this vector (Atashpaz-Gargari and Lucas 2007). In Fig. 4, the process of representing this approach in the cutoff grade optimization is shown.

Determining the optimum cutoff grade using ICA approach

In order to investigate the method of the ICA, the optimum cutoff grade of Sarcheshmeh copper mine of Iran is calculated. This model is based on base metal ores such as

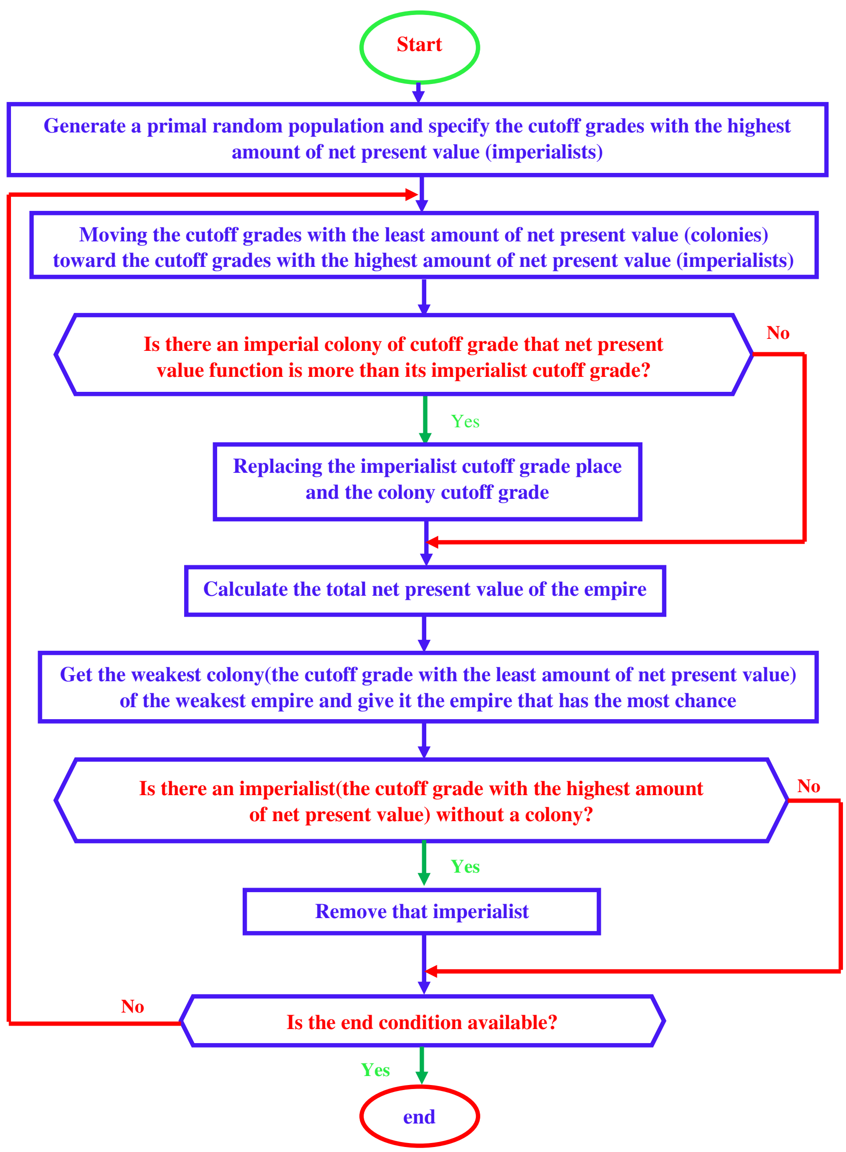

Fig. 4 The process of the ICA model in the cutoff grade optimization (Atashpaz-Gargari and Lucas 2007; Ahmadi et al. 2013; Ahmadi and Chen 2019)

Table 2 Distribution of cathode in ore deposit (Maleki et al. 2016)

Grade classes		Quantity (tons)	Grade classes		Quantity (tons)
From	Until		From	Until	
0.0	0.10	573,343,900	1.60	1.70	2,530,700
0.10	0.20	138,415,800	1.70	1.80	1,673,000
0.20	0.30	99,533,800	1.80	1.90	2,458,800
0.30	0.40	62,322,200	1.90	2.00	713,300
0.40	0.50	46,943,100	2.00	2.10	281,800
0.50	0.60	61,191,300	2.10	2.20	108,500
0.60	0.70	76,426,300	2.20	2.30	108,700
0.70	0.80	66,685,700	2.30	2.40	67,400
0.80	0.90	50,973,000	2.40	2.50	51,500
0.90	1.00	34,095,400	2.50	2.60	35,300
1.00	1.10	24,165,800	2.60	2.70	25,000
1.10	1.20	17,055,800	2.70	2.80	10,900
1.20	1.30	13,004,900	2.80	2.90	11,000
1.30	1.40	8,993,100	2.90	3.00	20,800
1.40	1.50	5,372,700	3.00	3.10	5000
1.50	1.60	3,615,500	3.10	3.20	5000
Total tonnage			1,290,245,000		

copper, where limitations are split into three phases of mining, processing, melting, and refining units. The deposit is assumed to be mono-metallic mine and its linear grade distribution is employed. The final product of the complex, cathode copper was considered, and the price of the product in the target function is considered constant. In the final range, there are 1290 million tons of minerals with grade distribution reported in Table 2. Mining capacity (ore and tailing extraction) of 55 million tons annually, the capacity of the processing plant is 23 million tons annually, and the unit capacity of refining is 150 thousand tons annually, the cost of the mining was assumed 38 thousand Rials per ton (capital and operational cost), the processing cost 83 thousand Rials per ton (capital and operational cost). It should be mentioned that the capital cost includes items such as the cost of manufacturing a factory, warehouses, material transportation lines (Conveyor), power transmission lines, purchase and installation costs of equipment, the water supply system, the cost of machinery. The operating cost includes items such as the cost of spare parts for the factory, the crusher and the milling cost, repairs and maintenance cost of the devices, the cost of supply and transfer of water and the recovery of water, etc.; melting and refining cost was assumed 26.4 million Rials per ton (The melting and refining cost includes the cost of constructing a melting plant, purchasing and installing equipment, spare parts, energy consumption, etc.), and finally, 428 billion Rials per ton was assumed for fixed cost (costs that are independent of mine production and also, as long as the mine is active, these costs exist). The commodity price is 121 million Rials per ton, the recovery and the discount rate was assumed 0.66% and 15%, respectively (Maleki et al. 2016).

For optimizing our function, the 120 countries (cutoff grades in this optimization problem) have been selected, randomly. Twenty-four countries (cutoff grades) are selected as imperialists (the cutoff grades with the highest amount of NPV) and the 96 countries are chosen as colonies. In the next stage, the colonies moved toward colonialist countries based on the power of them by application of MATLAB software. As depicted in Fig. 4, the process of empire elimination contineues upto monopolar world system. In the 25th

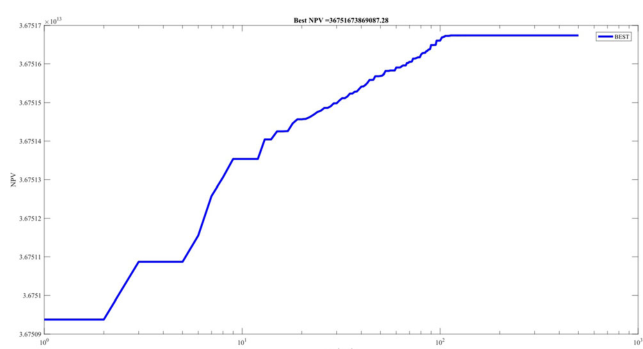

Fig. 5 Optimization of the NPV value using an imperialist competitive algorithm

generation, all empires have fallen apart from an empire and have become a monopolar world. A world in which it all forms a single empire, and all the colonies and even imperialist themselves (the cutoff grade with the highest amount of NPV) have the same status. The cutoff grade achieved at this stage stands for the optimum value. By application of the ICA algorithm, the optimum cutoff grade have been calculated for the Sarcheshmeh copper ore deposit, the results are shown in Table 5. Hence, 36,751 billion Rials is the highest NPV can be achieved via meta-heuristic optimization for Sarcheshmeh copper ore deposit, as illustrated in Fig. 5.

Validation of models

Sarcheshmeh copper deposit was employed as a real case study in this work to examine the effectiveness of two different approaches in optimization of cutoff grade. With regard to the range of the cutoff grades from 0 to 3.2% in this research, maximum available grade, the tonnage of the mineral, and the average grade of ore deposit can be calculated under any cutoff grade. These values are shown in Table 3. In this table, the tonnage under each cutoff grade (assumed that the cutoff grade is the same as the lower bounds of the grade ranges) is obtained from the cumulative aggregation of the tonnage of the grade classes that more than the cutoff grade, and it should

Table 3 Calculation of tonnage and average grade of deposit under different cutoff grades

Grade classes (%)		The tonnage of each class (ton)	Average grade of each class (%)	Cutoff grade (%)	Cumulative tonnage under any range of grade (ton)	Average grade (%)
From	Until					
0.00	0.10	573,343,900	0.01	0.00	1,290,245,000	0.31
0.10	0.20	138,415,800	0.15	0.10	716,901,100	0.55
0.20	0.30	99,533,800	0.24	0.20	578,485,300	0.65
0.30	0.40	62,322,200	0.34	0.30	478,951,500	0.74
0.40	0.50	46,943,100	0.45	0.40	416,629,300	0.79
0.50	0.60	61,191,300	0.55	0.50	369,686,200	0.84
0.60	0.70	76,426,300	0.65	0.60	308,494,900	0.90
0.70	0.80	66,685,700	0.74	0.70	232,068,600	0.98
0.80	0.90	50,973,000	0.85	0.80	165,382,900	1.07
0.90	1.00	34,095,400	0.95	0.90	144,409,900	1.17
1.00	1.10	24,165,800	1.05	1.00	80,314,500	1.26
1.10	1.20	17,055,800	1.14	1.10	56,148,700	1.36
1.20	1.30	13,004,900	1.25	1.20	39,092,900	1.45
1.30	1.40	8,993,100	1.34	1.30	26,088,000	1.55
1.40	1.50	5,372,700	1.45	1.40	17,094,900	1.66
1.50	1.60	3,615,500	1.55	1.50	11,722,200	1.76
1.60	1.70	2,530,700	1.64	1.60	8,106,700	1.85
1.70	1.80	1,673,000	1.75	1.70	5,576,000	1.95
1.80	1.90	2,458,800	1.99	1.80	3,903,000	2.04
1.90	2.00	713,300	1.95	1.90	1,444,200	2.12
2.00	2.10	281,800	2.04	2.00	730,900	2.29
2.10	2.20	108,500	2.38	2.10	449,100	2.45
2.20	2.30	108,700	2.26	2.20	340,600	2.47
2.30	2.40	67,400	2.35	2.30	231,900	2.56
2.40	2.50	51,500	2.46	2.40	164,500	2.65
2.50	2.60	35,300	2.54	2.50	113,000	2.74
2.60	2.70	25,000	2.65	2.60	77,700	2.83
2.70	2.80	10,900	2.76	2.70	52,700	2.92
2.80	2.90	11,000	2.84	2.80	41,800	2.95
2.90	3.00	20,800	2.93	2.90	30,800	2.99
3.00	3.10	5000	3.02	3.00	10,000	3.12
3.10	3.20	5000	3.21	3.10	5000	3.21

Table 4 Optimum cutoff grade, production of different units, annual profit, and NPV of project life by the ICA approach

Year	OCOG (%)	Average grade (%)	The mining (tons)	The concentrate (tons)	Filtration rate (tons)	Profit (billion Rials)	NPV (billion Rials)
1	0.232	0.678	55,000,000	20,415,249	106,214	5844	37,750
2	0.229	0.676	54,996,123	20,413,798	106,211	5844	37,420
3	0.228	0.675	54,998,192	20,415,409	106,210	5844	37,260
4	0.227	0.674	54,998,017	20,414,135	106,210	5836	36,970
5	0.227	0.674	54,994,086	20,415,464	106,211	5836	36,630
6	0.224	0.671	54,998,143	20,414,819	106,209	5836	36,340
7	0.222	0.669	54,999,729	20,413,782	106,208	5836	35,950
8	0.221	0.668	54,998,287	20,415,895	106,209	5836	35,580
9	0.218	0.666	54,995,874	20,413,938	106,213	5836	34,890
10	0.216	0.664	55,000,000	20,413,066	106,208	5836	34,250
11	0.214	0.662	55,000,000	20,415,031	106,209	5836	33,550
12	0.198	0.648	45,535,407	20,414,157	92,805	4956	28,940
13	0.197	0.647	45,535,614	20,414,892	92,803	4956	28,310
14	0.195	0.645	45,534,325	20,413,730	92,806	4956	27,650
15	0.193	0.643	45,534,168	20,415,215	92,804	4956	26,860
16	0.189	0.639	45,536,116	20,415,107	92,805	4956	25,970
17	0.187	0.637	45,534,673	20,416,294	92,806	4956	24,680
18	0.184	0.634	45,534,203	20,413,334	92,805	4956	23,500
19	0.180	0.630	45,534,371	20,414,871	92,805	4956	22,380
20	0.176	0.626	45,535,080	20,416,483	92,804	4956	20,670
21	0.171	0.621	45,534,253	20,417,152	92,803	4956	18,870
22	0.167	0.617	45,534,669	20,415,858	92,805	4956	16,660
23	0.162	0.612	45,535,994	20,413,906	92,806	4956	14,380
24	0.156	0.606	45,536,607	20,415,281	92,803	4956	11,370
25	0.149	0.599	45,534,793	20,416,974	92,802	4956	8050
26	0.143	0.593	45,536,712	20,413,583	92,806	4956	4360
27	0.135	0.585	2,175,759	975,550	4435	184	160

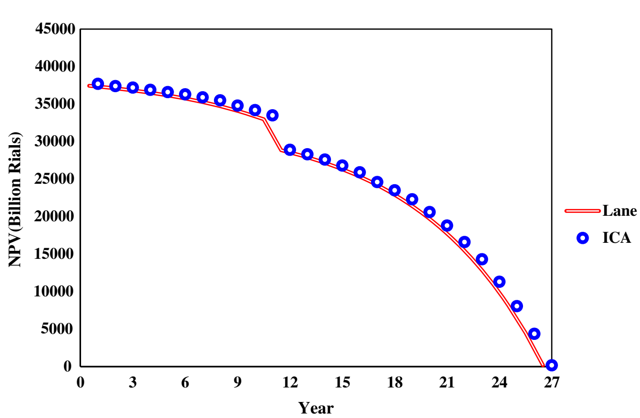

Fig. 6 Comparison of the annual NPV achieved from two models

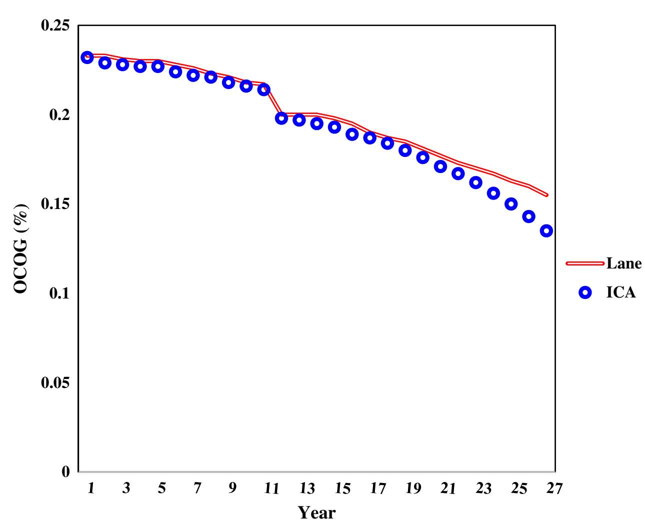

Fig. 7 Comparison of the optimal cutoff grades achieved from the two models

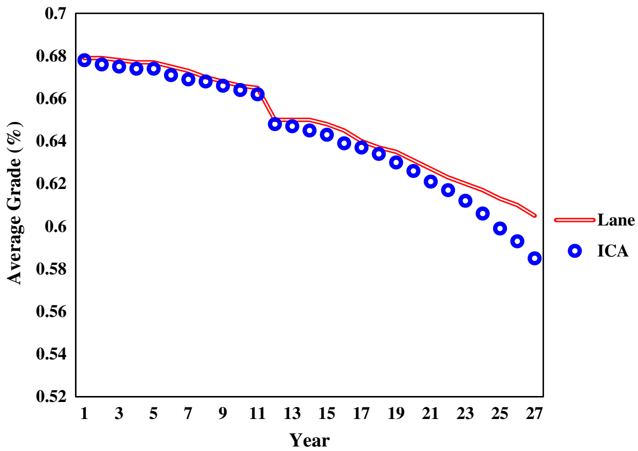

Fig. 8 Comparison of the average grades achieved from the two models

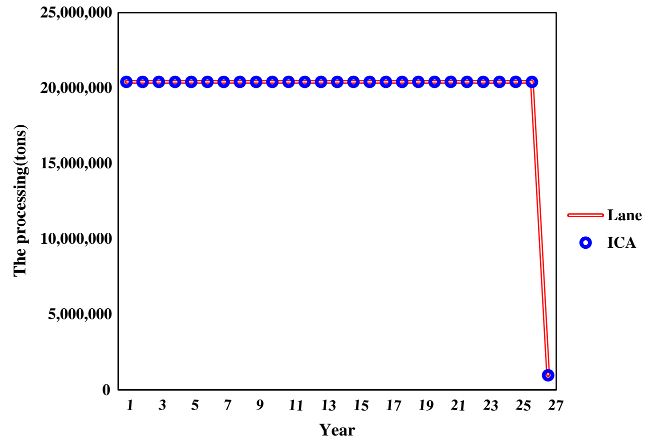

Fig. 10 Comparison of the capacity of the processing plant achieved from the two models

be noted that the average grade of the mineral is also calculated. The results that obtained from the Lane method are listed in Table 5. In the Lane method, the optimal cutoff grade begins at 0.233% and declines to 0.155% at the end of the mine life. In the early years of the mine's life, mine operation works at full capacity; however, the full capacity of both processing and refinary plants are not used. The NPV of the Lane method in the first year of the mine is 37,434 billion Rials. On the other hand, the ICA results are shown in Table 4, which shows that the optimum cutoff grade at the start of the project is 0.232% and at the end of its life reaches 0.135%, and the NPV in the primary year is 37,750 billion Rials. Also, in Figs. 6 and 7, the NPV and optimal value of each year are compared together in each model, respectively. Similarly, in Figs. 8, 9, 10, 11, and 12, the average grade of each year and the production of various units, such as extraction, processing plant, and refining unit, as well as profit in the two models, have been compared (Table 5). With regard to the cutoff grade that has a direct relation to the average grade of mineral, Fig. 13 shows the effect of the optimal cutoff grade on the average grade of mineral in the ICA model.

Also, the advantages of the ICA model compared to the Lane model:

- The ICA model gets the optimal answer as quickly and easily as possible than the Lane model.
- Unlike the lane model, in the ICA method, the long and cyclic process will not been seen.
- The ICA model has a very low error and the results of the ICA model are good convergence in comparison with the Lane model.
- The ICA model is promising for solving nonlinear issues.

Conclusions

The NPV and cutoff grade play the critical role in the open pit mine production planning. In this research, the ICA algorithm is applied to determine the optimal cutoff grade with regard to the maximizing NPV. The efficiency of the ICA algorithm and

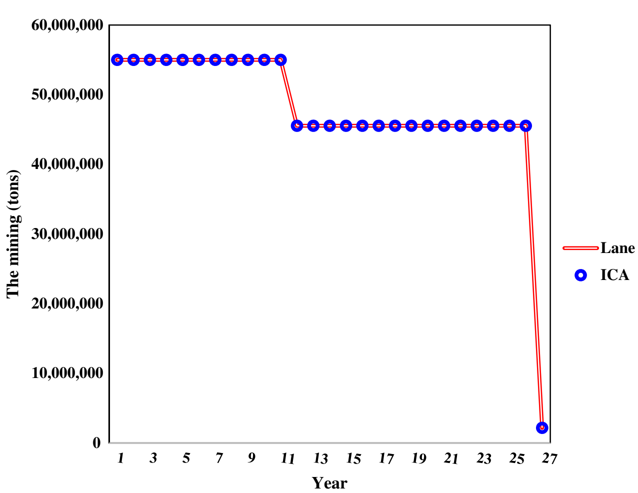

Fig. 9 Comparison of the production rate achieved from the two models

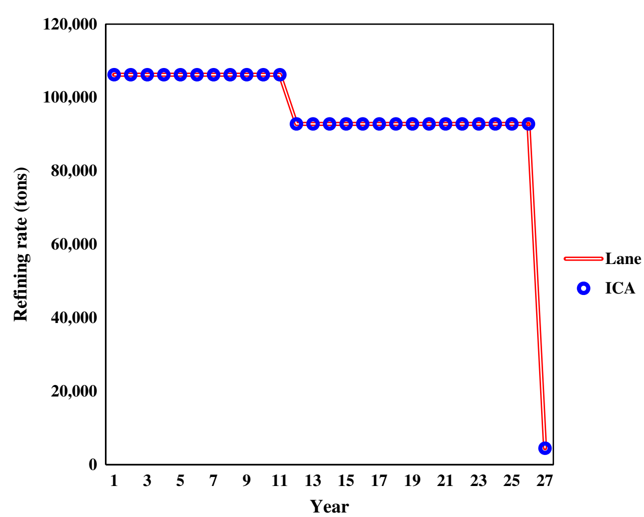

Fig. 11 Comparison of the refining unit achieved from the two models

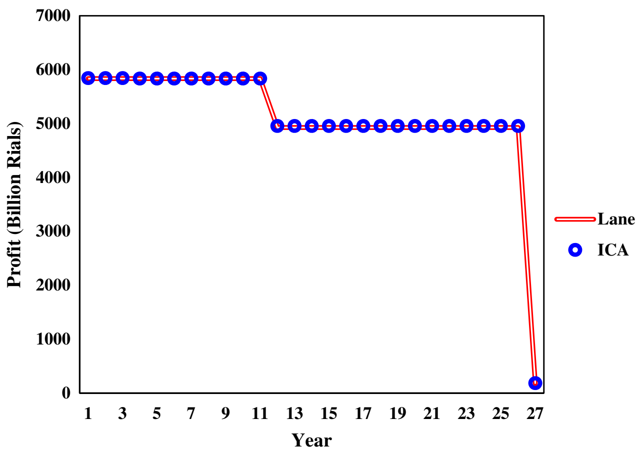

Fig. 12 Comparison of the profits achieved from the two models

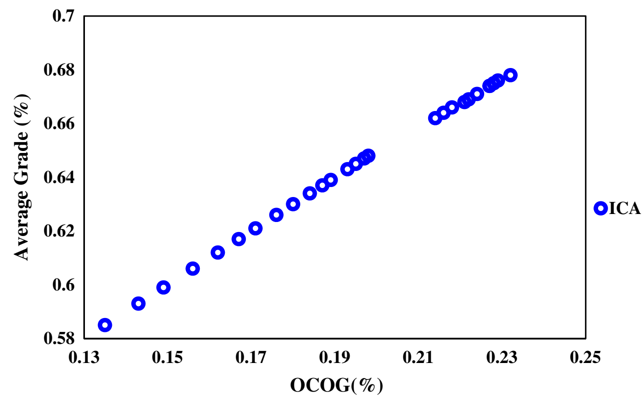

Fig. 13 Effect of the optimal cutoff grade on average grade in the ICA model

Lane's theory in the cutoff grade optimization problem was compared together. For this purpose, Sarcheshmeh copper mine was considered with different grades as case study in this research. The optimal cutoff grade for the different years of the mine life should be determined based on the objective function (NPV maximization). Then, the ICA method was implemented and according to the existing limitations, the optimal cutoff grade was calculated. Also, to validate the proposed model, the comparison of the results of the ICA model

Table 5 Optimal cutoff grade, production of different units, profit, and NPV in different years of project life by the Lane method

Year	OCOG (%)	Average grade (%)	The mining (tons)	The concentrate (tons)	Filtration rate (tons)	Profit (billion Rials)	NPV (billion Rials)
1	0.233	0.679	55,000,000	20,417,000	106,210	5834	37,434
2	0.233	0.679	55,000,000	20,417,000	106,210	5834	37,214
3	0.231	0.678	55,000,000	20,417,000	106,210	5834	36,961
4	0.230	0.677	55,000,000	20,417,000	106,210	5834	36,670
5	0.230	0.677	55,000,000	20,417,000	106,210	5834	36,336
6	0.228	0.675	55,000,000	20,417,000	106,210	5834	35,951
7	0.226	0.673	55,000,000	20,417,000	106,210	5834	35,509
8	0.223	0.670	55,000,000	20,417,000	106,210	5834	35,001
9	0.221	0.668	55,000,000	20,417,000	106,210	5834	34,416
10	0.218	0.666	55,000,000	20,417,000	106,210	5834	33,743
11	0.217	0.665	55,000,000	20,417,000	106,210	5834	32,970
12	0.200	0.650	45,538,000	20,417,000	92,807	4926	28,843
13	0.200	0.650	45,538,000	20,417,000	92,807	4926	28,232
14	0.200	0.650	45,538,000	20,417,000	92,807	4926	27,541
15	0.198	0.648	45,538,000	20,417,000	92,807	4926	26,745
16	0.195	0.645	45,538,000	20,417,000	92,807	4926	25,831
17	0.190	0.640	45,538,000	20,417,000	92,807	4926	24,779
18	0.187	0.637	45,538,000	20,417,000	92,807	4926	23,569
19	0.185	0.635	45,538,000	20,417,000	92,807	4926	22,178
20	0.181	0.631	45,538,000	20,417,000	92,807	4926	20,578
21	0.177	0.627	45,538,000	20,417,000	92,807	4926	18,739
22	0.173	0.623	45,538,000	20,417,000	92,807	4926	16,623
23	0.170	0.620	45,538,000	20,417,000	92,807	4926	14,190
24	0.167	0.617	45,538,000	20,417,000	92,807	4926	11,392
25	0.163	0.613	45,538,000	20,417,000	92,807	4926	8174
26	0.160	0.610	45,538,000	20,417,000	92,807	4926	4474
27	0.155	0.605	2,176,000	975,000	4438	172	149

and the Lane model was compared. After the implementation of the program, the optimum cutoff grade at the early years of the mine life is from 0.232% by ICA method and by the end of the mine life to 0.135%, and with the Lane model at the early years of the mine life from 0.233 to 0.155% at the end of mine life. The total NPV of the earnings during the mine life in the ICA method and the Lane model was 37,750 billion Rials and 37,434 billion Rials, respectively. Also, the production rate, the processing rate and the refining rate obtained with the method of the ICA model are compared with the Lane model. In the ICA, the correct adjustment of the factors and their values affect the performance and speed of convergence of the imperialist competitive algorithm's. The results of the research showed that a very low error of the ICA model and the convergence of its results in comparison with the Lane method. As a result, this meta-heuristic algorithm converges more rapidly to the optimal answer.

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