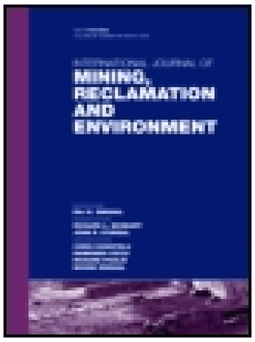

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Kamyar Tolouei[a], Ehsan Moosavi[a], Amir Hossein Bangian Tabrizi[a], Peyman Afzal[a] and Abbas Aghajani Bazzazi[b]

[a]Department of Petroleum and Mining Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran; [b]Department of Mining Engineering, University of Kashan, Kashan, Iran

ABSTRACT

In mines planning, the long-term production scheduling problem (LTPSP) in open-pit mines is considered as a significant issue. It also specifies the distribution of cash flow during the course of the mine-life. Actually, LTPSP is a large-scale optimisation problem including large data-sets, multiple constraints, and uncertainty in the input factors that, has to be solved in a reasonable time. LTPSP, despite the valuable efforts of researchers, has not yet been well resolved. In this paper, hybrid models have been offered by the Lagrangian relaxation (LR) method with meta-heuristic methods, bat algorithm and particle swarm optimisation for solving the LTPSP due to the deterministic assumption and concerning the grade uncertainty. To bring update the Lagrange multipliers, the meta-heuristic algorithms have been applied. In terms of cumulative net present value, average ore grade, and computational time in a 12-year production period, the consequences achieved from the case studies point out that a solution close to optimisation can be presented by the LR-bat algorithm hybrid strategy in comparison with other methods. The results analysis has shown that the proposed method produces a near-optimal solution with a rational time that can be a good suggestion for utilising in the mining industry.

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Open-pit mine; long-term production scheduling; grade uncertainty; Lagrangian relaxation; particle swarm optimisation algorithm; bat algorithm

1. Introduction

Long-term production scheduling in open-pit mines is very essential in the mining industry. The common (deterministic) methods and uncertainty-based approaches are the ones practiced to deal with this problem. In common methods, the planning process usually begins by making a geological block model divided into ore and adjacent ore inside a regular three-dimensional array. The size of the blocks is usually the same. Next, a collection of features such as grade, specific gravity is determined for each one of these blocks and they are estimated by some of the specific interpolation techniques such as kriging, the reverse distance method, etc. Finally, the information of samples taken from boreholes is evaluated. For the next step, these estimated features of the block are practiced to specify the economic value of these blocks and final mining processes. These geological and economic block models are remarked as the base inputs for the next production planning. The main drawback of this approach is that it assumes all the input parameters have been certainly identified. On the contrary, these parameters have always been accompanied by uncertainties. In case they are ignored, unreal and wrong decisions might be made.

CONTACT Ehsan Moosavi se.moosavi@yahoo.com; se_moosavi@azad.ac.ir

Uncertainties in input parameters may be caused by technical techniques and financial or environmental factors that influence the extraction planning of mines. Uncertainties caused by geological factors, which are known as geological or grade uncertainty, are considered as the most important source of uncertainty for the production planning processes in open-pit mines. The grade uncertainty is for the sake of estimating the grade amounts of individual blocks using the dispersed data sample of drilling logs. The geo-statistical simulating techniques provide a frame to specify the quality amount of the grade uncertainty based on simulating the ore multi-production amount. The accessibility to these techniques provides this situation that grade uncertainty has been integrated with the planning process for production. In this condition, optimum production and the annual net present value (NPV) will reach its maximum operational quantity. Within the last recent years, various planning models have been suggested to integrate grade uncertainty and planning. In fact, it led to a more comprehensive plan and a more applicable and optimum production.

Actually, the long-term production scheduling problem (LTPSP) in open-pit mines is a large-scale problem that should be solved in optimal time. Ore reserve might contain millions of blocks which usually necessitates more than one planning, whereas the operational and physical constraints bring about uncertainties in input data. This leads to an integrated optimum problem. Notably, solving it is very difficult and costly by the available traditional techniques.

Within the last few years, researchers have heeded less expensive calculating algorithms such as meta-heuristic techniques to solve problems, designs, and production scheduling difficulties. Although these techniques do not guarantee optimisation as a final solution for production, they can present suitable solutions with less expensive operational costs. Table 1 illustrates some of the models presented.

This paper focuses on developing an integer planning model in the problem related to the long-term production scheduling in open-pit mines and solving it by meta-heuristic methods. In this study, an optimum hybrid model has been developed by the Lagrangian relaxation method (LR) with bat algorithm (BA) and particle swarm optimisation (PSO) to solve the LTPSP of open-pit mines based on the deterministic assumption and regarding the grade uncertainty. The meta-heuristic algorithms have been used to update the Lagrange multipliers. The new suggested approaches have been compared with the results of the LR-genetic algorithm (GA), the traditional LR-sub-gradient (SG) and the conventional method without using the LR approach. The results obtained from the LR-meta-heuristic model indicate that this method is able to solve the linear optimisation problems faster and it can detect the best solution more carefully.

The following part of this paper is planned as below. Due to the condition of grade uncertainty, Section 2 model the objective functions and their related constraints. Section 3 presents a summary of the methodology and hybrid models and the suggested models will be advanced. For the next step, section 4 provides an assessment of the results. Validation of the established models is achieved. In section 5, the results are analysed and discussed. Lastly, section 6 presents the conclusion.

2. LTPSP formulation

The long-term production scheduling model is put into practice to estimate production targets and ore material current within several years. Totally, it takes an essential image of the production and executed it as a linear problem.

2.1. Deterministic approach

The most straightforward method is to illustrate a full space optimisation model, in which each period of the scheduling horizon, to consider decision-making fortitudes. Remarkably, the

Table 1. Review of presented models.

Features	Johnson [1]	Gershon [2]	Dagdelen and Johnson [3]	Ravenscroft [4]	Denby and Schofield [5]	Elevli [6]	Tolwinski [7]	Johnson et al. [8]	Godoy and Dimitrakopoulos [9]
Model									
Deterministic	✓	✓	✓			✓	✓	✓	
Uncertainty based				✓	✓				✓
Uncertainty Modelled									
Geological				✓	✓				✓
Equipment related									
Economic									
Objectives									
Maximise NPV	✓			✓	✓	✓	✓	✓	✓
Minimise risks				✓	✓				
Other purposes	✓	✓	✓			✓	✓		
Solution Models									
Traditional/Conventional	✓	✓		✓		✓	✓	✓	
Heuristics/Meta-heuristics			✓		✓				✓
LR/ALR			✓						
Meta-Heuristics									
Algorithm					GA				SA

Features	Ramazan and Dimitrakopoulos [10]	Gholamnejad et al. [11]	Ramazan and Dimitrakopoulos [12]	Bley et al. [13]	Lamghari and Dimitrakopoulos [14]	Sattarvand and Niemann-Delius [15]	Goodfellow and Dimitrakopoulos [16]
Model							
Deterministic	✓		✓	✓		✓	
Uncertainty based		✓			✓		✓
Uncertainty Modelled							
Geological		✓			✓		✓
Equipment related							
Economic							
Objectives							
Maximise NPV	✓	✓	✓	✓	✓	✓	
Minimise risks		✓	✓				✓
Other purposes				✓		✓	
Solution Models							
Traditional/Conventional	✓	✓	✓	✓			
Heuristics/Meta-heuristics					✓	✓	✓
LR/ALR							
Meta-Heuristics							
Algorithm					TS	ACO	SA

Features	Leite and Dimitrakopoulos [17]	Moosavi et al. [18]	Moosavi et al. [19]	Asad et al. [20]	Shishvan and Sattarvand [21]	Mokhtarian and Sattarvand [22]	Mokhtarian and Sattarvand [23]
Model							
Deterministic		✓	✓		✓	✓	
Uncertainty based	✓			✓			✓
Uncertainty Modelled							
Geological	✓			✓			
Equipment related							
Economic							✓
Objectives							
Maximise NPV		✓	✓		✓	✓	✓
Minimise risks	✓						✓
Other purposes				✓		✓	
Solution Models							
Traditional/Conventional	✓						✓
Heuristics/Meta-heuristics		✓	✓	✓	✓	✓	
LR/ALR		✓	✓	✓			
Meta-Heuristics							
Algorithm		GA	GA		ACO	ICA	

Features	Goodfellow and Dimitrakopoulos [24]	Lamghari and Dimitrakopoulos [25]	Khan [26]	Rahimi et al. [27]	Jelvez et al. [28]	Alipour et al. [29]	Chatterjee and Dimitrakopoulos [30]	Dimitrakopoulos and Senécal [31]
Model								
Deterministic					✓			
Uncertainty based	✓	✓	✓			✓	✓	✓
Uncertainty Modelled								
Geological	✓	✓	✓			✓	✓	✓
Equipment related						✓		
Economic						✓		
Objectives								
Maximise NPV	✓	✓	✓		✓	✓	✓	✓
Minimise risks	✓							✓
Other purposes					✓	✓		
Solution Models								
Traditional/Conventional								
Heuristics/Meta-heuristics	✓	✓	✓		✓	✓	✓	✓
LR/ALR							✓	
Meta-Heuristics								
Algorithm	SA, PSO, DE	TS, VND	PSO, BA			GA		TS

obtainability of restraints is bonded into the model. Hereafter, Equation (1) supplies the LTPSP objective function and Equations (2–8) show the constraints of the model.

$$Maximize\ Z = \sum_{n=1}^{N} \sum_{t}^{T} \frac{NV_n^t}{(1+r)^t} \times X_n^t \tag{1}$$

$$\sum_{n=1}^{N} (g_n - G_{max}) \times O_n \times X_n^t \leq 0 \tag{2}$$

$$\sum_{n=1}^{N} (g_n - G_{min}) \times O_n \times X_n^t \geq 0 \tag{3}$$

$$\sum_{t=1}^{T} X_n^t \leq 1, \ \forall n = 1, 2, 3, \ \ldots \ , N \tag{4}$$

$$\sum_{=1}^{N} \left(O_n \times X_n^t\right) \leq PC_{max} \tag{5}$$

$$\sum_{n=1}^{N} (O_n + W_n) \times X_n^t \leq MC_{max} \tag{6}$$

$$X_k^t - \sum_{r=1}^{t} X_y^r \leq 0, \ \forall k = 1, 2, \ \ldots \ , N, \qquad \forall t = 1, 2, \ \ldots \ , T \tag{7}$$

$$YX_k^t - \sum_{y=1}^{l} \sum_{r=1}^{t} X_y^r \leq 0, \quad \forall y = 1, 2, \ \ldots \ , l, \quad \forall k = \ 1, 2, \ \ldots \ , N, \quad \forall t = 1, 2, \ \ldots \ , T \tag{8}$$

In the constructed model, the following indications were accepted: n: is the block identification number, $n = 1, 2, \ldots, N$; N is the total number of blocks to be scheduled; t is the scheduling periods index, $t = 1, 2, \ldots, T$; T is the total number of scheduling periods; NV_n^t is the net value to be generated by mining block n in period t; r is the discount rate in each period; X_n^t is the binary variable; G_{max} and G_{min} are targeted maximum and minimum average grade of the ore material to be processed in a period; g_n is the average grade of block n and O_n is the ore tonnage in block n; PC_{max} is the processing capacity; MC_{max} is the mining capacity; W_n is the tonnage of waste material within block n; k is the index of a block considered as extraction at certain period.

Equation (2) and Equation (3) show the grade blending constraints. One of the most important hitches in production scheduling is the ore grade which has to be set aside steady while leading to the processing plant. For this reason, the grade of ore that is being steered to the mill should be well-defined between two limits. Equation (2) demonstrates upper bound constraints. It is significant that the average grade of the material directed to the mill should be a lesser quantity or equal to the certain grade value for each period. Also, Equation (3) demonstrates lower bound constraints. Extraordinarily, the average grade of the material conducted to the mill has to be more or alike to the definite value for each period.

Equation (4) illustrates reserve constraints. Reserve restrictions made for each block specify that all measured blocks in the model should be mined on one occasion. Processing capacity constraints and mining capacity constraints are shown in Equation (5) and Equation (6). Processing capacity constraint means total tons of the treated ore should not surpass the processing capacity in every period. Also, mining capacity constraint assigns the entire quantity of material (waste and ore) to be mined cannot be more than the whole accessible mining capacity for each period. Furthermore, Equation (7) and Equation (8) define wall slope constraints. These restraints ratify that it is indispensable to mine all blocks limited directly through the mining of block k, a target block before the extraction of block k is begun. There are two methods to do these constraints: Using one constraint for each block per each period (Equation (7)); Using Y- constraints for each block at each period (Equation (8)).

2.2. Uncertainty-based approach

Basically, mining space is defined as a possible space based on the uncertainty that leads to this space. In mining engineering operations, the uncertainty makes a decision built on uncertain results. In 1998, Dimitrakopoulos [32] presented the classification of uncertainties in mining projects because of the importance of this subject. Amongst the uncertainties, grade uncertainty leads to a large share of probabilities caused by grade uncertainty.

At present, the indicator kriging (IK) is one of the most widely used methods for grade estimation in mining projects [33]. This technique was presented by Journel [34] to estimate the resources. The nature of the indicator method is binary data encoding depending on the cut-off value, Z_c. For the $Z(x)$ value, $i_k(x) = 1$ if $(x) \geq Z_C$, and otherwise, $i_k(x) = 0$. In fact, it is a nonlinear conversion of data value to binary system. Outcome values between 0 and 1 for each estimation point provide a set of indicators-converted quantity using kriging, that can be expounded as the proportion of the block overhead the determined cut-off on data support and the probability that the grade is overhead the determined indicator [35,36]. In the optimisation procedure of this paper, this probability is contemplated as the probability index (PI_n) for block n. The high probability blocks have less risk than low probability ones.

An integer programming-based model is considered in this section to inspect the grade uncertainty. In this method, a probability based on indicator kriging is allotted to each block (PI_n) which indicates the probability made from n for each block in the block model [37]. It is organised the objective function in such a way those earlier production periods are given to mine the blocks with higher certainty. When additional information usually becomes obtainable, the uncertain blocks are gone for later periods. Subsequently, one more objective function is presented to the objective function of the conventional model in the subsequent form of:

$$Maximize\ Z' = \sum_{n=1}^{N} \sum_{t}^{T} \frac{NV_n^t}{(1+r)^t} \times X_n^t \tag{9}$$

This objective function brings about the constraints (2–8).

3. LTPSP solution methodology

For the time being, the LR method is considered as one of the potential methods for make out the projected problem.

3.1. LTPSP reformulation with Lagrangian relaxation (LR)

The Lagrangian relaxation (LR) method is identified as one of the mathematical techniques for a mixed-integer programming problem. In the presentation of this method in LTPSP, Lagrangian multipliers relax the system constraints and introduce them to the objective function [37–41]. Next, the relaxed problem intensified into a more controllable sub-problem for separate units and solved through dynamic programming. Based on violations of system restraints, a sub-gradient method is applied to promote the multipliers. The convergence standard is satisfied in case convergence standard is achieved.

Fundamentally, LR is based on the viewpoint to relax the system restrictions as a result of Lagrangian multipliers. Then the relaxed problem is split into some smaller sub-problems [42]. The constant Lagrangian function can be made by dint of assigning non-negative Lagrangian multipliers λ^t, μ^t and v^t in terms of processing type at period t to the constraints (3), (5) and (6), respectively.

$$MaxL\ (X,\lambda,\mu,v) = Z'(X)\sum_{t=1}^{t}\lambda^t\left(\sum_{n=1}^{N}(g_n - G_{min})\times O_n\times X_n^t\right) + \sum_{t=1}^{T}\mu^t\left(PC_{max} - \sum_{n=1}^{N}\left(O_n\times X_n^t\right)\right) + \sum_{t=1}^{T}v^t\left(MC_{max} - \sum_{n=1}^{N}(O_n + W_n)\times X_n^t\right) \tag{10}$$

The LTPSP is illuminated through the Lagrangian relaxation method by relaxing or momentarily ignoring the preventing constraints and solving the problem as if they have never been. While maximising due to the control variable X_n^t in LTPSP, this is done over the dual optimisation process which strives to effect the constrained optimum by lessening the Lagrangian function L due to the Lagrangian multipliers λ^t, μ^t and v^t.

$$j^* = \underset{\lambda,\ \mu,\ v}{Min}\ j(\lambda,\ \mu,\ v)$$

where

$$j(\lambda,\ \mu,\ v) = \underset{X}{Max}\ L(X,\ \lambda,\ \mu,\ v)$$

Subjected to the constraint (4) and (7), assume that λ, μ and v are fixed, we maximise the Lagrangian function L as follows. From Equation (10), the formulation can be written as:

$$\begin{aligned} MaxL\ (X,\lambda,\mu,v) = & \sum_{n=1}^{N}\sum_{t}^{T}\frac{NV_n^t}{(1+r)^t}\times\ PI_n\times X_n^t\,\lambda^t \\ & -\sum\lambda^t{}_{t=1}^{t}\left(\sum_{n=1}^{N}(g_n - G_{min})\times O_n\times X_n^t\right) \\ & +\sum_{t=1}^{T}\mu^t\left(PC_{max} - \sum_{n=1}^{N}\left(O_n\times X_n^t\right)\right) \\ & +\sum_{t=1}^{T}v^t\left(MC_{max} - \sum_{n=1}^{N}(O_n + W_n)\times X_n^t\right) \end{aligned} \tag{11}$$

According to the Lagrangian multipliers, the modification of Lagrangian multipliers should be rationally done to make best use of the Lagrangian function. To regulate Lagrangian multipliers, most references practice a combination of sub-gradient method and several heuristics to achieve a fast solution [43,44]. In the current study, the BA and PSO are applied to amend the Lagrangian multipliers and improve the performance of LR technique.

3.2. Application of meta-heuristic algorithms to multipliers updating

The meta-heuristic algorithms are deliberated as the possible methods for making out the predicted problem. In the present study, BA and PSO are used to improve the Lagrangian multipliers.

3.2.1. Bat algorithm (BA)

One of the strongest optimisation procedures is the collective intelligence based on group behaviour. Yang [45] introduced an algorithm affected by the collective behaviour of bats in a natural environment based on the use of sound reflection by bats. Bats are able to navigate the precise trail and site of their bait via sending sound waves and receiving their reflections. The bat is able to draw a sound image of the obstacles connecting its sites and recognise them well when the sound waves turn back to the bat wave transmitter. This system makes it possible for the bats to identify moving objects such insects and trees. The micro-bats releases short-duration loud sound beats with continuous occurrence in the area of 25 kHz to 150 kHz and listen for the echo bouncing back from the nearby objects to find the food or avoid the barriers. Bats naturally issue 10 to 20 such sound pulses per second and are able to increase the pulse release rate to about 200 pulses per second as they approach their victim. Yang [45] presented the succeeding directions in order to convey these special properties of bats into an optimisation algorithm:

- All bats practice their echolocation abilities so as to detect their distance from a definite object and distinguish between food/prey and background obstacle in some way. The fact is that all bats use their echolocation skills.
- Since bats are able to change the wavelength λ and loudness A_0 of their released sound pulses to discover the food, they are capable of flying inadvertently with velocity v_i at position x_i with a frequency f_{min}. Likewise, bats can modify the rate and wavelength or frequency of their emitted pulse according to their distance from the prey.
- The loudness varies from a large positive value A_0 to the least persistent value A_{min}.

Each bat's current position is regarded as a possible solution to the optimisation problem [46–49].

According to the rules, the position x_i^t and the velocity v_i^t for each i-th virtual bat in the t repetition and also the frequency f_i are calculated as follows:

$$f_i = f_{min} + (f_{max} - f_{min})\beta \tag{12}$$

$$v_i^t = v_i^{t-1} + \left(x_i^t - x_*\right)f_i \tag{13}$$

$$x_i^t = x_i^{t-1} + v_i^t \tag{14}$$

where, $\beta \in [0, 1]$ is a random vector with uniform distribution and x_* is the best current position, which is selected in each replication after comparison with the position of the virtual bats. Usually, consider the frequency f with $f_{min} = 0$ and $f_{max} = 100$. In each replication, in the local search, one of the answers is selected as the best answer, and the new position of each bat is updated locally with the random step as follows:

$$x_{new} = x_{old} + \epsilon\overline{A^t} \tag{15}$$

where, $\epsilon \in [-1,\ 1]$ is a random number and $\overline{A^t}$ is the average loudness of the bats in the t repetition. Also, the loudness of the A_i loudness and the pulse rate r sent each time it is updated as follows:

$$A_i^{t+1} = \alpha A_i^t \tag{16}$$

$$r_i^{t+1} = r_i^0[1 - exp(-\gamma t)] \tag{17}$$

where α and γ are constant values and for $0 < \alpha < 1$ and $r > 0$, when $t \to \infty$,we have: $r_i^{t+1} \to r_i^0$ and $A_i^{t+1} \to 0$. The bat algorithm pseudo-code is shown in Figure 1.

3.2.2. Particle swarm optimisation (PSO)

In 1995, Eberhart and Kennedy [50,51] first introduced the PSO methodology is an optimisation method based on probability rules. Researchers have heeded the social behaviour of bird or fish groups during food search to direct the population to the promising area for space search. Definite rational procedures are applied for the manners of the beings of the ruling body. Birds are merely looking for their food through modifying their physical movements by escaping missions. Thus, each member of the group theoretically serves past experiences and other discoveries from members to catch food. Over a competitive search for food, this type of corporation is a positive improvement. The backbone of the PSO is to share information among the group members. In PSO, a particle is referred to each answer to a problem is the location of a bird in the search space. All particles have a degree of competence optimised by the quality of action. Additionally, every particle holds a component called the velocity specifying it in the search range [52–54].

```
Objective function f(x),   x = (x1, ..., xd)^T
Initialize the bat population xi (i = 1, 2, ..., n) and vi
Define pulse frequency fi at xi
Initialize pulse rates ri and the loudness Ai
while (t < Max number of iterations)
Generate new solutions by adjusting frequency,
and updating velocities and locations/solutions [equations (12) to (14)]
    if (rand > ri)
    Select a solution among the best solutions
    Generate a local solution around the selected best solution
    end if
    Generate a new solution by flying randomly
    if (rand < Ai & f(xi) < f(x*))
    Accept the new solutions
    Increase ri and reduce Ai
    end if
Rank the bats and find the current best x*
end while
Postprocess results and visualization
```

Figure 1. Pseudo-code of the BA.

The PSO begins with a group of accidental responses. For the next step, it seeks for the position and velocity of each particle to discover the finest answer in the problem space. According to the two most noteworthy values, each particle is reorganised at each stage of the population movement. It is demonstrated that the first value is the best answer in terms of suitability ever gained for each particle. This is the personal best and is called pbest. The global best, known as gbest, is the other best value ever achieved by means of the PSO. Swarm of particles is initialised at random over the search space and move through D-dimensional space to search for new solutions.

Authorise x_k^i and v_k^i respectively be the position and velocity of i-th particle in the search space at k-th iteration, then its velocity and position of this particle at $(k+1)$-th iteration are updated using the following equations [55]:

$$v_{k+1}^i = w.v_k^i + c_1.r_1.\left(p_k^i - x_k^i\right) + c_2.r_2.\left(p_k^g - x_k^i\right) \tag{18}$$

$$x_{k+1}^i = x_k^i + v_{k+1}^i \tag{19}$$

where r_1 and r_2 demonstrate accidental numbers between 0 and 1, c_1 and c_2 are constants, p_k^i demonstrate the best position of i-th particle, and p_k^g correlates with the global best position in the swarm up to k-th iteration. Figure 2 demonstrates the PSO algorithm pseudo-code [55].

3.3. The framework of the proposed hybrid method

Two steps are required for hybrid methods in the current paper. The first one states the Lagrangian function which brings update the Lagrange multipliers. The second step is the precise global extension of the stated LR function, in which the meta-heuristic algorithms are utilised to find out a new hybrid method near to the ideal maximum. Figure 3 illustrates the flowchart of the suggested approach.

Objective function: $f(x)$, $x = (x_1, x_2, \ldots, x_D)$;

Initialize particle position and velocity for each particle and set $k = 1$.

Initialize the particle's best known position to its initial position i.e. $p_k^i = x_k^i$.

do

Update the best known position (p_k^i) *of each particle and swarm's best known position* (p_k^g).

Calculate particle velocity according to the velocity equation (18).

Update particle position according to the position equation (19).

While maximum iterations or minimum error criteria is not attained

Figure 2. Pseudo-code of the PSO.

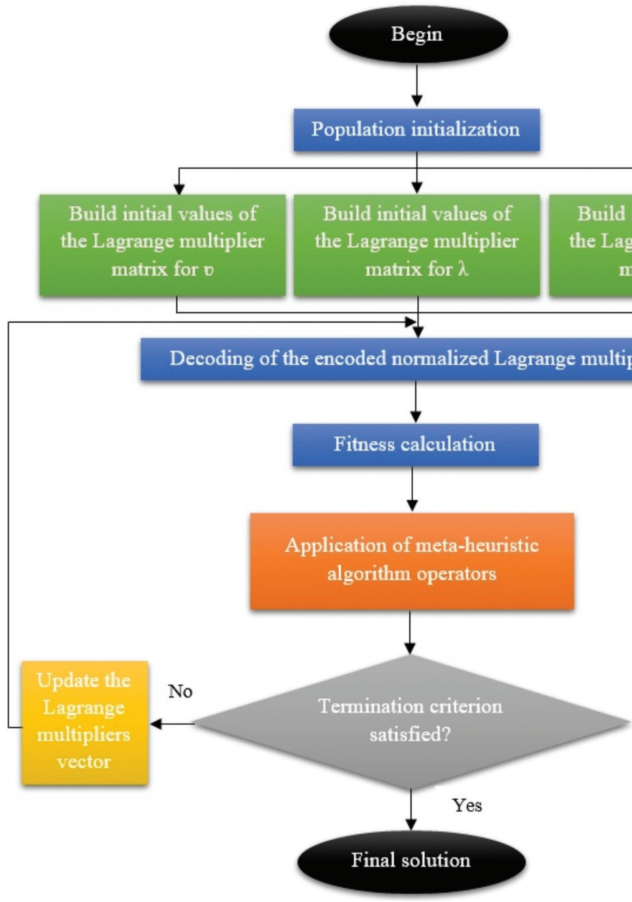

Figure 3. Flowchart of the proposed approach.

4. Numerical results

In this paper, we have developed, implemented, and tested the proposed model in MATLAB R2019a environment. Presented testing has been performed on an Intel Core i7-3770 K, 3.9 GHz and 16.0 GB RAM PC and MS Windows 7.

Table 2. Implementation of the proposed model for the synthetic data set containing 4750 blocks.

Iteration	Method	Optimality gap
1	LR-BA	0.094
	LR-PSO	0.112
	LR-GA	0.247
	LR-SG	2.07
2	LR-BA	0.069
	LR-PSO	0.098
	LR-GA	0.163
	LR-SG	1.245
3	LR-BA	0.029
	LR-PSO	0.047
	LR-GA	0.068
	LR-SG	0.621

4.1. Validation of the model with synthetic data

All the developed formulations are confirmed by the numerical experiments on the synthetic data set including 4750 blocks. As shown in Table 2, it is implied that the enactment by LR-BA is actually better than other methods from the view of the optimality gap.

4.2. Implementation of the model on the real case study

The iron ore push-back data including 6854 blocks and the gold deposit containing 15,276 blocks have selected as case studies to compare the suggested mathematical model for LTPSP. The proposed model was implemented on case studies in deterministic and uncertainty-based conditions.

4.2.1. The Chadormalu iron ore mine (CIOM)

Chadormalu has been recognised as the main iron ore one in the central part of Iran. Chadormalu is located at the epicentre of Persia (Iran) Desert, at the north of grey Chah-Mohammad mountains. Figure 4 illustrates the geographical location of the CIOM. Chadormalu deposit embraces some 400 million tons of resource and 320 million tons of reserves are divided between northern and southern ore bodies.

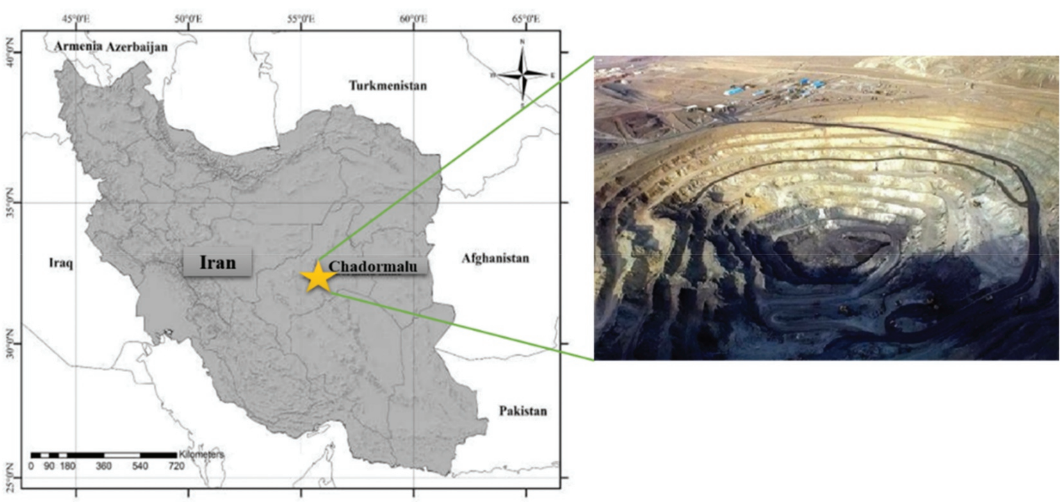

Figure 4. The geographical location of the CIOM.

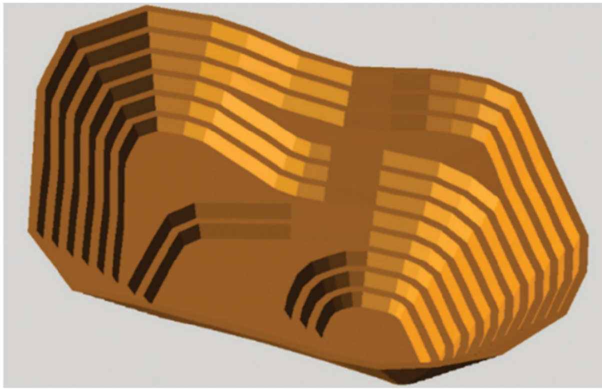

Figure 5. A 3D view of the second push-back in the CIOM [37].

Four push-backs are scheduled for the CIOM that the presented mathematical model in this paper is practiced in the second push-back. The 3D view of the second push-back is displayed in Figure 5. This push-back includes 6854 blocks of which 2754 are ore blocks and 4100 are waste blocks. The tonnage of waste and ore presented in the aforementioned push-back shall be 103.8 and 110.2 million tons, respectively. The technical parameters and the number of model variables for CIOM are demonstrated in Tables 3 and 4.

The proposed framework is applied to the CIOM in both deterministic and uncertainty-based conditions. The results of NPV and average ore grade (AOG) in deterministic and uncertainty-based conditions for the CIOM using presented models in the twelve production periods are illustrated in Figures 6 and 7, respectively.

Figures 8 and 9 are shown the comparison of cumulative NPV and AOG in the total periods between deterministic and uncertainty-based approaches in the CIOM. As outcomes are shown, the LR–BA method is better than other presented methods. As disclosed in Figures 8 and 9, the cumulative NPV and the AOG (Fe) in the 12-year period with assuming deterministic and considering grade uncertainty using the LR–BA method are 61.711 M$, 61.413 M$, 57.67 %, and

Table 3. Technical parameters for CIOM.

Parameters	Value	Unit
Total block	6854	-
Block dimension	$25 \times 25 \times 15$	[m^3]
Discount rate	10	[%]
Average grade	52.8	[%]
Mining capacity	25	[Mtone/year]
Processing capacity	8.1	[Mtone/year]
Mining recovery	90	[%]
Processing recovery factor	76	[%]
Mine life	12	[year]

Table 4. Number of model variables for CIOM.

Variables	Value
Reserve constraints	6854
Iron ore grade constraints	12
Processing capacity constraints	12
Mining capacity constraints	12
Wall slope constraints	82,248
Binary constraints	6854
Total variables of model	95,992

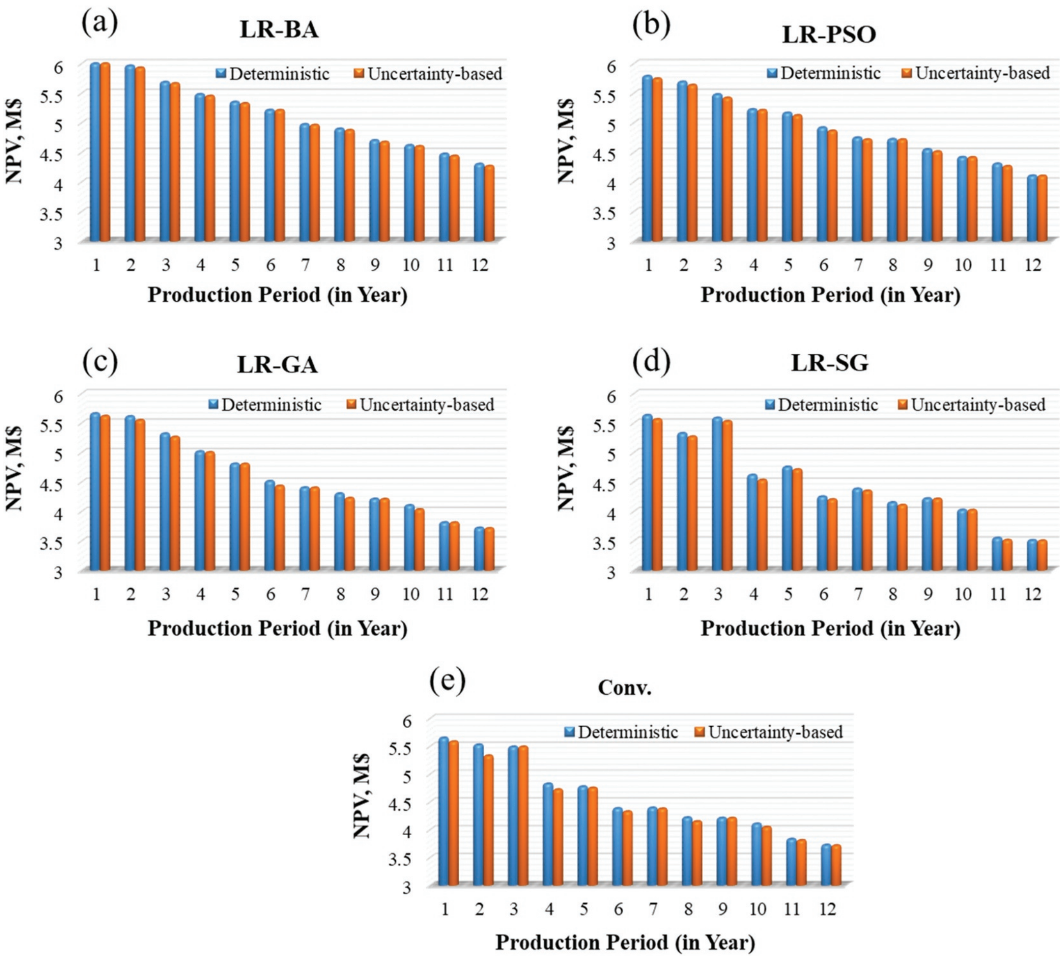

Figure 6. Comparison of NPV in deterministic and uncertainty-based conditions for the CIOM using presented models in the 12-year period (a) LR-BA (b) LR-PSO (c) LR-GA (d) LR-SG (e) Conventional method.

57.49 %. These results are 11.98 %, 12.72 %, 0.94 %, and 0.82% higher than the conventional method (Conv.), respectively.

Cross-sectional views of the four schedules for 9 periods are shown in Figure 10. The figure shows that there are obvious differences in the schedules in terms of the position of each extraction period. One of the most important factors in decision-making is the operational feasibility of accessing equipment to all blocks that must be mined at any period and satisfying slope constraints. In Figures 10(a,b), the cross-sections of the schedules are displayed by the LR approach assuming deterministic conditions and concerning grade uncertainty. Also, the cross-sections of the schedules are illustrated by the conventional method with the deterministic assumption and with considering grade uncertainty in Figure 10(c,d).

In Figure 11, the cross-sectional views of the production schedules for 12-year periods are shown using the proposed model (Figure 11(a)) and the conventional method (Figure 11(b)) under grade uncertainty. According to Figure 11, it is observed from the generated schedules that while satisfying the slope constraint, the size of push-back by the proposed method is relatively bigger than the conventional approach and the number of push-back blocks using the proposed model is 5.57% more than the conventional approach. As well as, given the uncertainty of the grade, the upgraded production schedule is closer to the real and has become more operational.

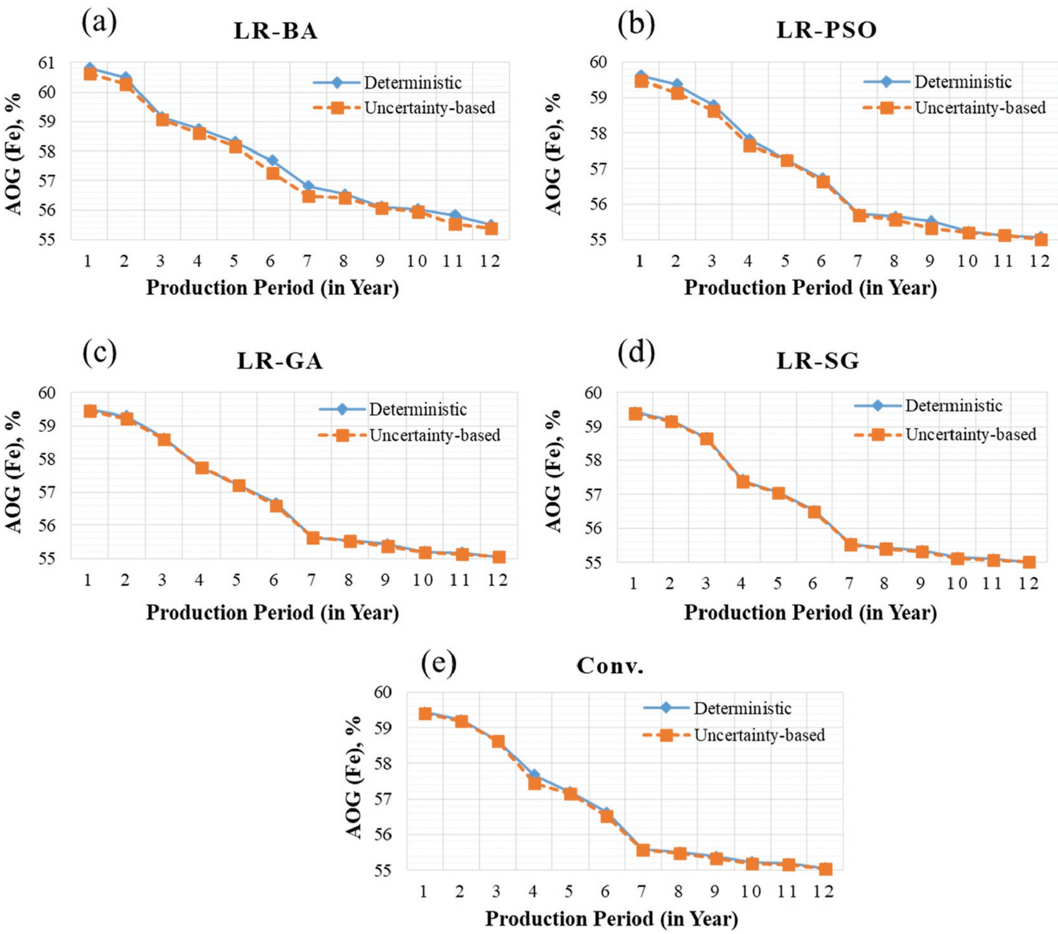

Figure 7. Comparison of AOG (Fe) in deterministic and uncertainty-based conditions for the CIOM using presented models in the 12-year period (a) LR-BA (b) LR-PSO (c) LR-GA (d) LR-SG (e) Conventional method.

Figure 12 illustrates ore production of the proposed model and the conventional method in the deterministic and uncertainty-based conditions. The results show that the proposed method while satisfying the constraints of mining capacity and processing capacity is better than the conventional method both in the deterministic assumption and with considering grade uncertainty. The total ores in the 12-year periods, in the deterministic condition, using the proposed LR-BA model and conventional method, are 99.81 Mt and 97.82 Mt, respectively. Also, in the uncertainty-based approach, the total ores in the 12-year periods by the proposed LR-BA model and the conventional method, are respectively, 98.19 Mt and 96.64 Mt.

4.2.2. The gold deposit (GD)

The proposed integrated mine scheduling optimisation framework has implemented on the real gold deposit. The aforementioned gold deposit has 15,276 blocks and is located in Iran. The 3D view of the GD is displayed in Figure 13. Tables 5 and 6 demonstrate the technical parameters and the number of model variables for GD.

Figures 14 and 15 are shown results of NPV and AOG in deterministic and uncertainty-based conditions for the GD using presented models in the twelve production periods. Additionally, the comparison of cumulative NPV and AOG in the total periods between deterministic and uncertainty-based approaches in GD are demonstrated in Figure 16 and Figure 17, respectively.

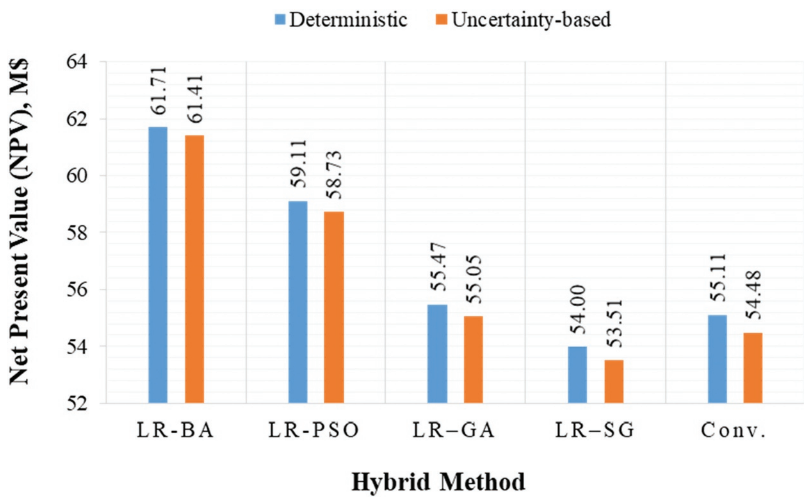

Figure 8. Comparison of cumulative NPV in the total periods between deterministic and uncertainty-based approaches in CIOM.

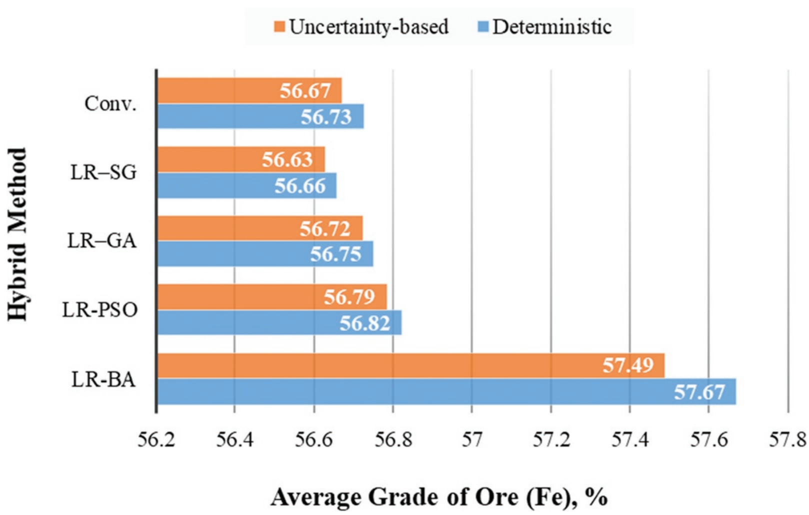

Figure 9. Comparison of AOG (Fe) in the total periods between deterministic and uncertainty-based approaches in CIOM.

As outcomes are shown, the LR–BA method is better than other presented methods. The cumulative NPV and the AOG (Au) in the 12-year period with assuming deterministic assumption and concerning grade uncertainty using the LR–BA method are 18.94 M$, 18.64 M$, 1.948 g/t, and 1.940 g/t, as shown in Figures 16 and 17. These outcomes are 8.41 %, 10.75 %, 3.40 %, and 3.91% better than the conventional method (Conv.), respectively.

In Figure 18, The cross-sectional views of the production schedules with considering grade uncertainty for 12-year periods using the proposed model (Figure 18(a)) and the conventional method (Figure 18(b)) are displayed. Pursuant to Figure 18, it is perceived from the generated schedules that while satisfying the slope constraint, the size of push-back by the proposed method is relatively bigger than the conventional approach and the number of push-back blocks using the proposed model is 4.46% more than the conventional approach.

As well as, ore production of the proposed model and the conventional method in the deterministic and uncertainty-based conditions for GD are demonstrated in Figure 19. The results show

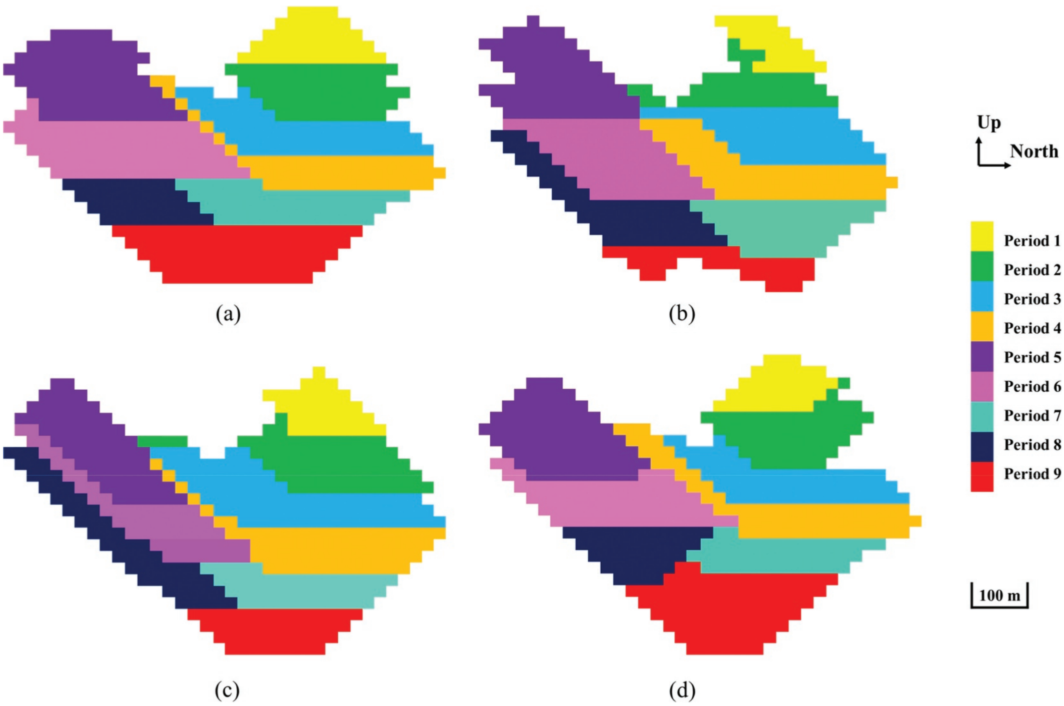

Figure 10. The cross-sectional views of the proposed schedule (a) uncertainty-based (b) deterministic and the schedule obtained by the conventional method (c) uncertainty-based (d) deterministic for CIOM.

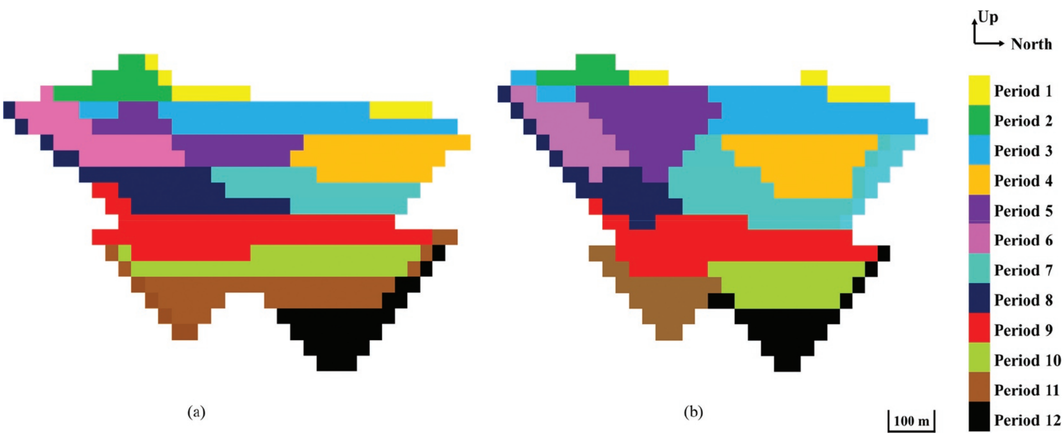

Figure 11. The cross-sectional views of uncertainty-based schedule for CIOM obtained by (a) the proposed approach and (b) the conventional method.

that the proposed method while satisfying the constraints of mining capacity and processing capacity generates a better solution than the conventional method.

The model on CIOM and GD in both scenarios (assuming deterministic and considering grade uncertainty) shows that in the early years of mining, grade extractors were obtained and the ore was sent to the processing plant. It has a higher value that results in higher net worth in the early years. The net value obtained in the scenario with the grade uncertainty indicates that this particular value is achievable; in the scenario with assuming deterministic, the net value obtained is only theoretically increased and not achievable. In order to model the vein deposits, the proposed models were

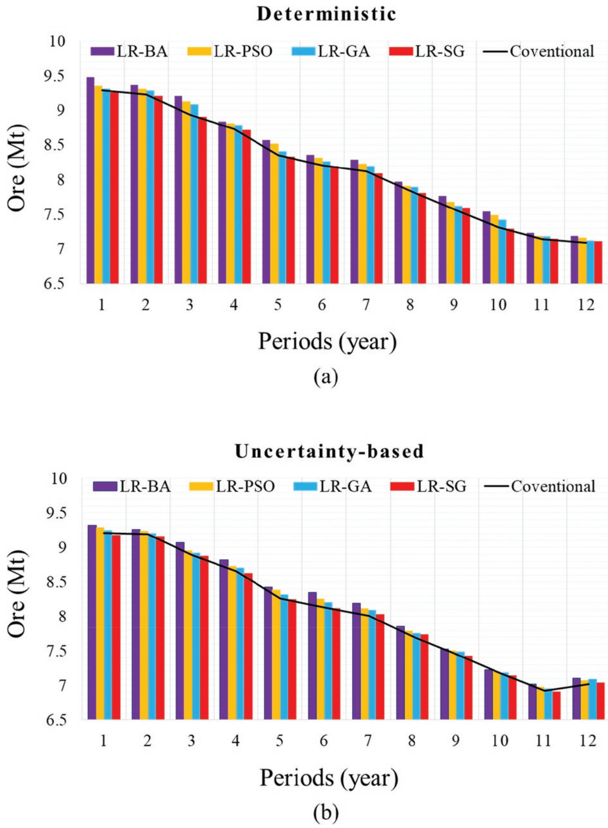

Figure 12. Ore production obtained by the proposed model and the conventional method in (a) deterministic and (b) uncertainty-based conditions for CIOM.

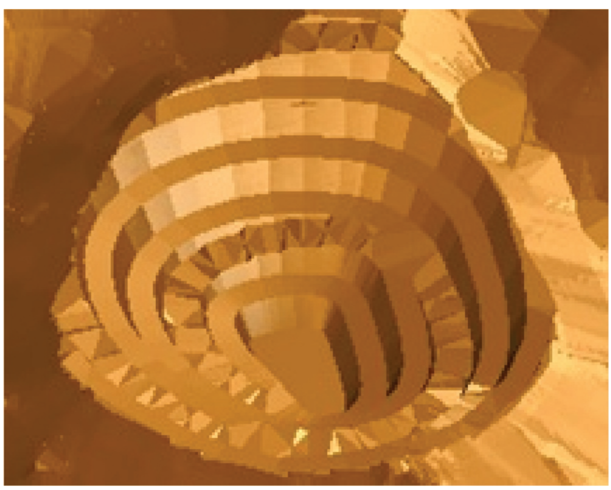

Figure 13. Three-dimensional view of the study area.

Table 5. Technical parameters for GD.

Parameters	Value	Unit
Total block	15,276	-
Block dimension	$8 \times 8 \times 10$	[m^3]
Discount rate	10	[%]
Average grade	1.7	[g/t]
Mining capacity	4.5	[Mtone/year]
Processing capacity	2.2	[Mtone/year]
Mining recovery	74	[%]
Processing recovery	85	[%]
Mine life	12	[year]

Table 6. Number of model variables for GD.

Variables	Value
Reserve constraints	15,276
Gold grade constraints	12
Processing capacity constraints	12
Mining capacity constraints	12
Wall slope constraints	183,312
Binary constraints	15,276
Total variables of model	213,900

implemented on the gold case study, and the results showed that the average grade obtained in the scenario decreased with the uncertainty of grade in the early years of mining. The life of mine has increased dramatically. The results of the scenario with assuming deterministic are quite the opposite. The results demonstrate that the decision-maker has obtained near-optimal results with respect to the policy drawn in consecutive years of mining. In other words, minimisation of deviation from predetermined production targets and minimisation of risk have been important results of the proposed models.

5. Discussion

A block model is presented by open-pit mines, which distinguishes the whole ore body. The equipment size, geology, data spacing, and the selected blasting pattern govern block sizes. The life-of-mine optimisation of open-pit mine long-term production scheduling problem (LTPSP) is remarked as one of the computationally-intensive processes. LTPSP is classified into three sub-problems: extraction time of a block, the decision on the destination of the extracted block, and the amount of material extracted in a given period. Large scale LTPSPs are both combinatorial and hard to solve. Heuristic methods are still the only feasible method for large-scale industrial applications. Nevertheless, Lagrangian relaxation has been verified to be mainly powerful for integer programming problems. Regarding such a problem, the hard coupling constraints are first relaxed over the introduction of Lagrangian multipliers. The relaxed problem can be decoupled into separate sub-problems. These sub-problems can be solved for a certain set of multipliers if they are not NP-hard. Based on the level of a constraint violation, multipliers are iteratively in tune. Simple heuristics are practiced to modify the relaxed problem solutions to form a viable result satisfying all the constraints at the termination of such updating iterations.

The sub-gradient optimisation method is a mostly used approach to Lagrangian multipliers. Indeed, the sub-gradient method can be measured as a well-matched version of the gradient method. This method has been tested only for small-scale problems even though it represents good convergence features. To specify the multiplier values based on the past calculation outcomes, the authors use the sub-gradient method that is usually employed. According to the zigzag phenomenon and small steps, the sub-gradient method may join gradually on large problems.

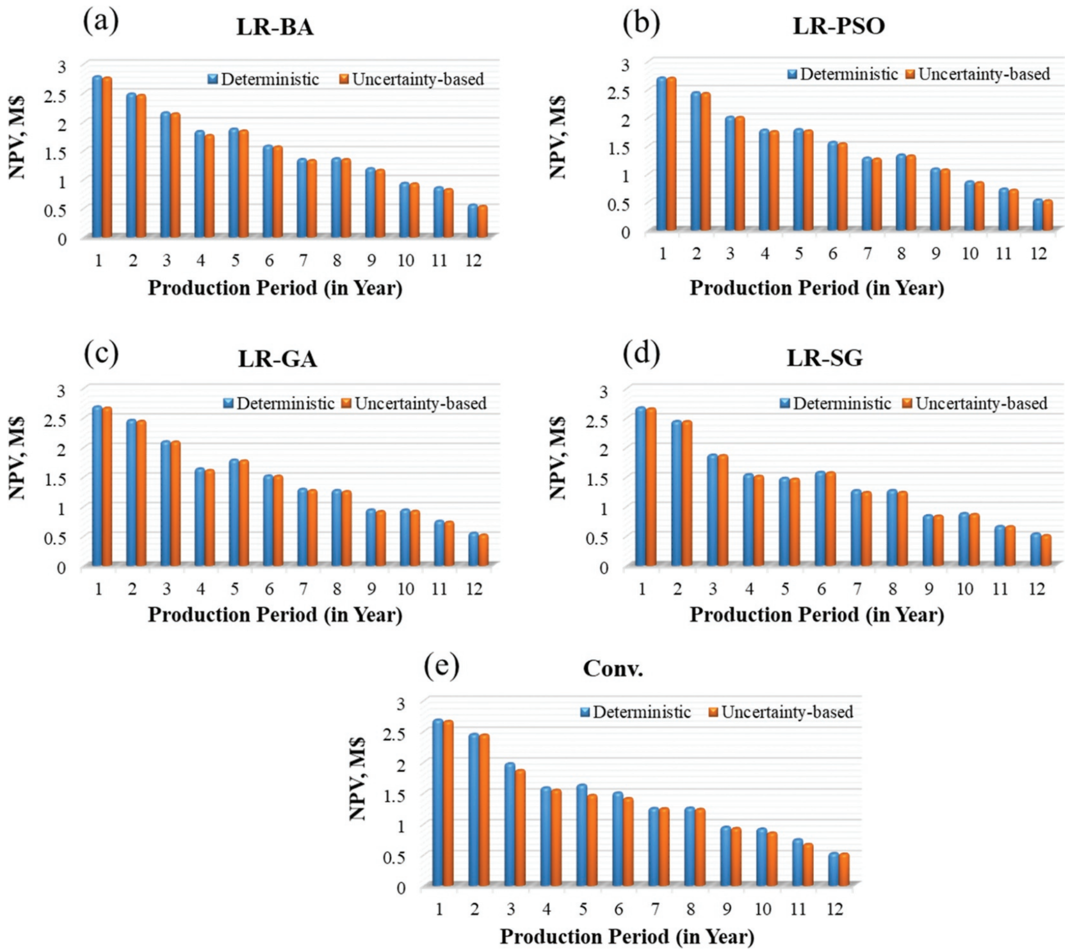

Figure 14. Comparison of NPV in deterministic and uncertainty-based conditions for the GD using presented models in the 12-year period (a) LR-BA (b) LR-PSO (c) LR-GA (d) LR-SG (e) Conventional method.

Clearly, the sub-gradient directions often cause the multipliers to zigzag across sharp ridges. Since the direction is found by a weighted combination of the gradients of nearby facets, zigzagging is expressively decreased.

Meta-heuristics algorithms are owned by the larger class of evolutionary algorithms, which create solutions to optimisation problems by means of techniques stimulated by natural evolution. They can be simply applied for the solution of hard optimisation problems and they are responsible for great modelling flexibility. Within the framework of an LR solution to the LTPSP, meta-heuristics approaches can be practiced for the dual variables. LR–meta-heuristics is grounded on the idea that meta-heuristics algorithms are incorporated into the Lagrangian relaxation method to update the Lagrangian multipliers. This approach directs to progress the presence of the Lagrangian relaxation method in deciphering combinatorial optimisation problems such as the LTPSP. Outcomes gained by means of LR with BA, PSO, and GA algorithms specify that the speeding up the convergence and highly near-optimal solution to the LTPSP can be completed by the LR–meta-heuristics. The numerical results demonstrate the LR-BA method generates a better solution than other methods for the LTPSP in terms of NPV and AOG. As well as, the general approach of applying the LR and bat algorithm in the light of the open-pit mine scheduling problem using this

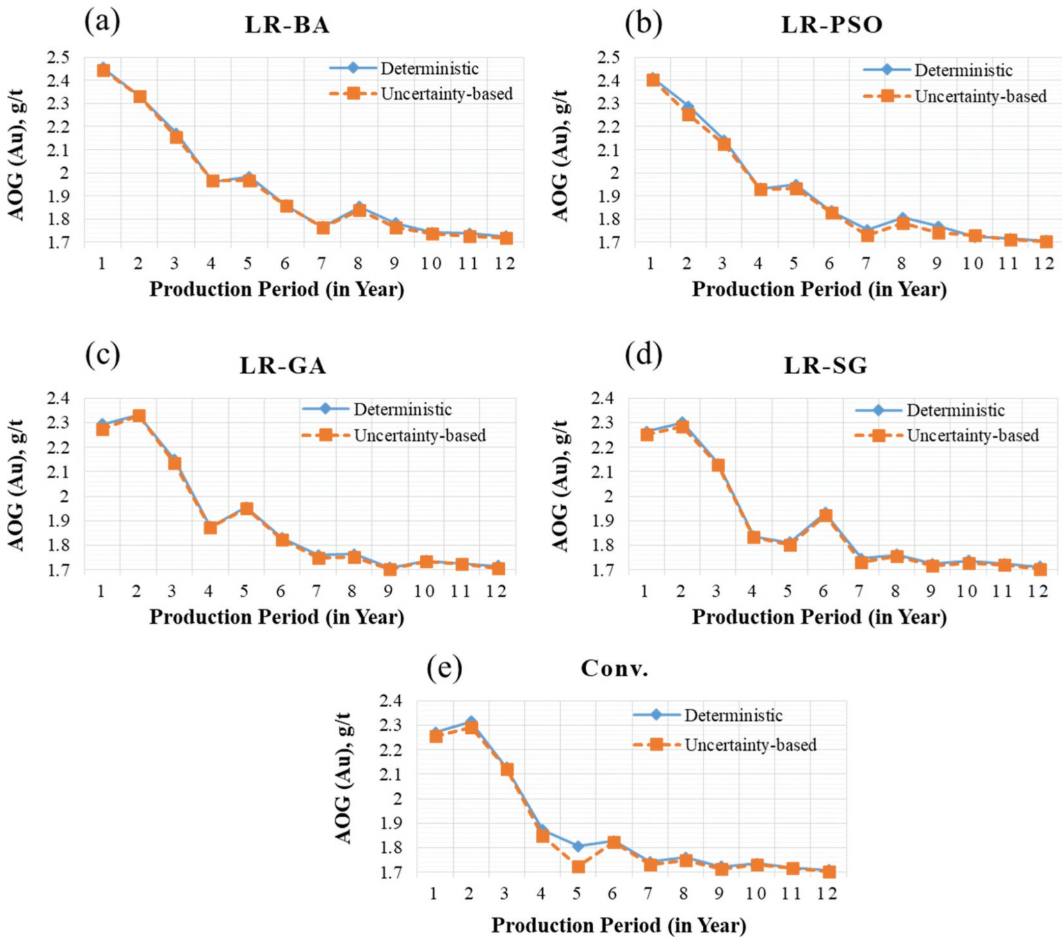

Figure 15. Comparison of AOG (Au) in deterministic and uncertainty-based conditions for the GD using presented models in the 12-year period (a) LR-BA (b) LR-PSO (c) LR-GA (d) LR-SG (e) Conventional method.

proposed procedure which was presented in the flowchart in Figure 3, could be used for solving long-term production scheduling problems.

Figure 20 demonstrates that the iteration number according to the problem size of BA is the minimum. It is worth noting that the iteration number of SG fluctuates widely with the growing problem scale. This proposes that BA is more robust than other methods. Besides, the optimality gap and computational time of each method for CIOM and GD are shown in Table 7. It is assessed that the CPU times by means of the LR-BA hybrid suggested method in the present study, are nearly 11.24% and 9.33% higher than that of the other methods for CIOM and GD. The computational times are considerably less with the presented methods compared to the traditional methods, especially when the problem size increases. These results demonstrate that the presented methods diminish the computational time significantly while retaining a small optimality gap. The suggested method appears to be a feasible option for solving the mine production scheduling problem where the number of integer variables is huge. Also, the small optimality gap demonstrates the effectuality of the suggested approach.

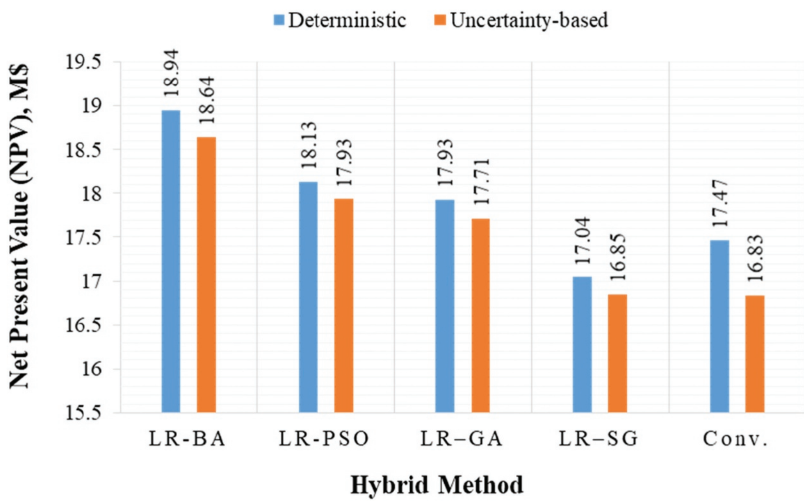

Figure 16. Comparison of cumulative NPV in the total periods between deterministic and uncertainty-based approaches in GD.

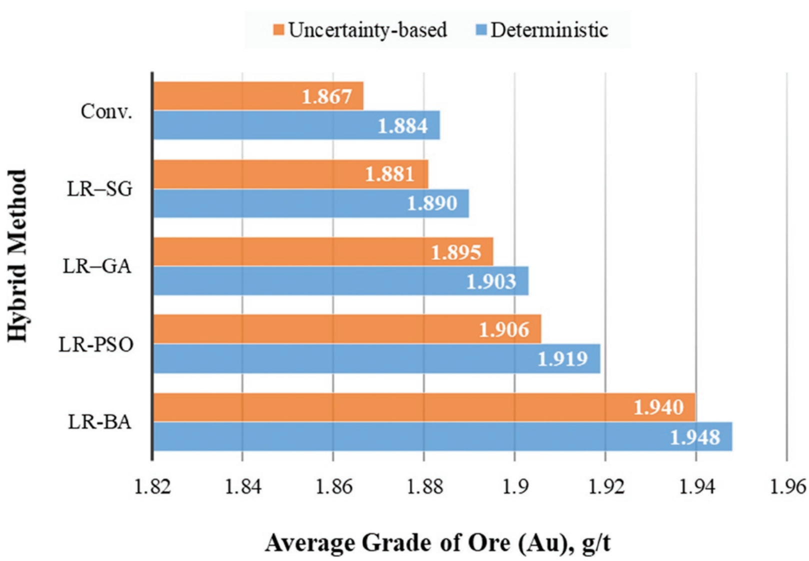

Figure 17. Comparison of AOG (Au) in the total periods between deterministic and uncertainty-based approaches in GD.

6. Conclusions

In the mining industry, long-term production scheduling in open-pit mines is very indispensable. The common (deterministic) methods and uncertainty approaches are the ones experienced to cope with this problem. The chief shortcoming of the deterministic method is that it supposes all the input factors that have been definitely recognised. In contrast, these constraints have always been attended by uncertainties. Unreal and wrong decisions might be made if they are ignored. Therefore, in this study, the following is analysed:

(1) This paper concentrates on developing an integer planning model in the problem associated with the long-term production scheduling in open-pit mines through bearing in mind grade

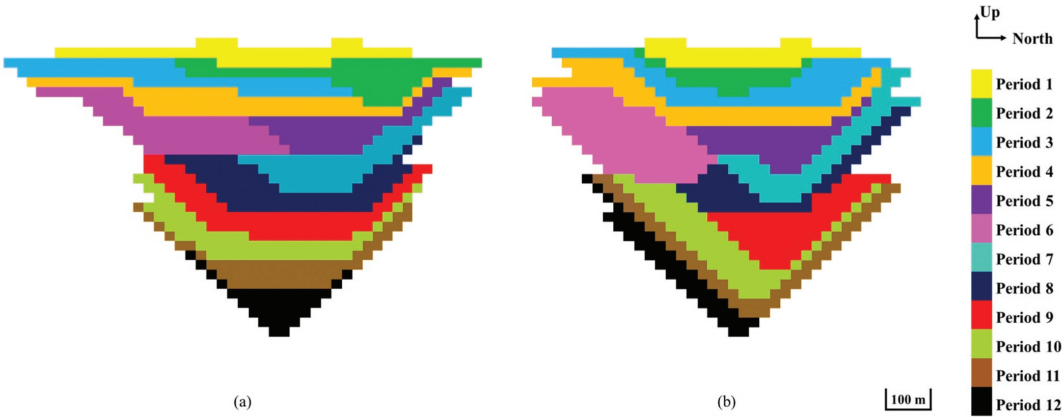

Figure 18. The cross-sectional views of uncertainty-based schedule for GD obtained by (a) the proposed approach and (b) the conventional method.

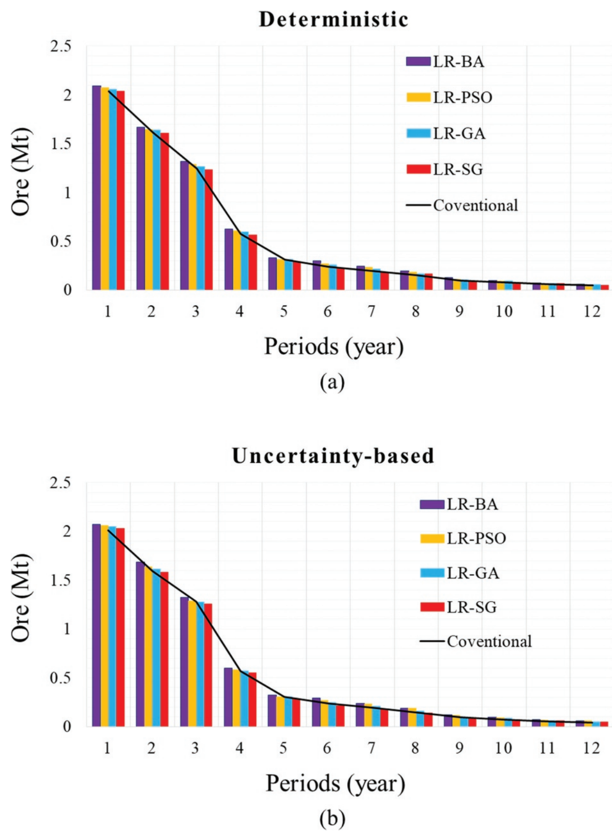

Figure 19. Ore production obtained by the proposed model and the conventional method in (a) deterministic and (b) uncertainty-based conditions for GD.

uncertainty. The enclosure of grade uncertainty in the model creates the outcomes closer to reality. The outcomes with the deterministic supposition are compared with the uncertainty-based condition.

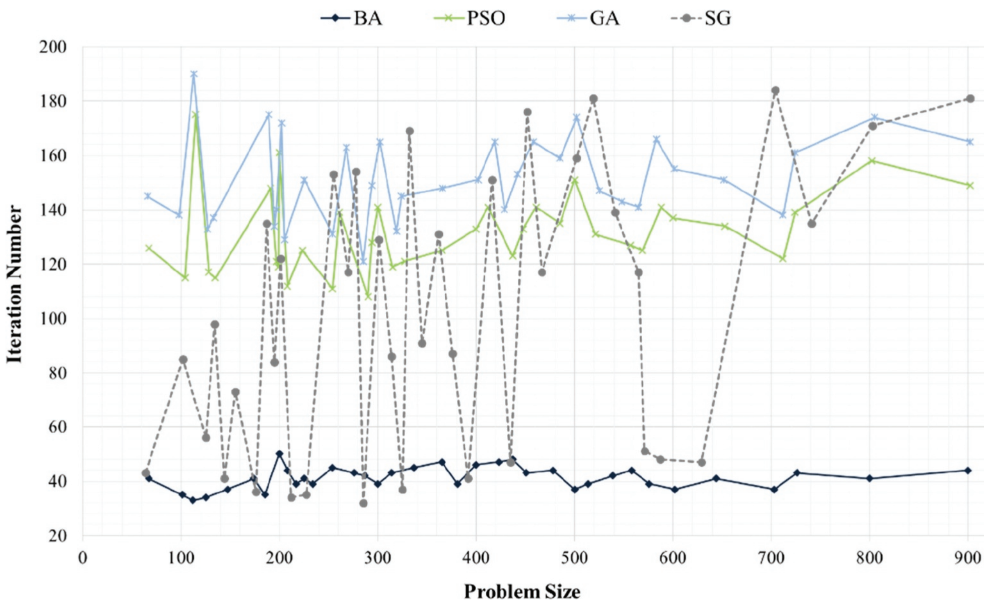

Figure 20. Comparison of iteration number of proposed models.

Table 7. General information about the solution found by MATLAB for proposed models.

Case study	Methods	Number of blocks (*N*)	Number of periods (*T*)	Optimality gap	Computational time in minutes
CIOM	LR-BA	6854	12	0.054	29.08
	LR-PSO	6854	12	0.076	32.35
	LR-GA	6854	12	0.082	33.42
	LR-SG	6854	12	0.219	37.76
	Conv.	6854	12	2.451	228.37
GD	LR-BA	15,276	12	0.063	44.04
	LR-PSO	15,276	12	0.074	48.15
	LR-GA	15,276	12	0.081	51.03
	LR-SG	15,276	12	0.203	52.57
	Conv.	15,276	12	3.419	627.46

(1) This work solves the long-term production scheduling problem through the Lagrangian relaxation and meta-heuristic algorithms which combined the meta-heuristic algorithms into the Lagrangian relaxation (LR) method to bring up to date the Lagrangian multipliers and develop the performance of LR method. The consequences of the case study demonstrate that the LR method can perform an appropriate solution to the main problem. This is chiefly striking in large scale problems.
(2) The accessible mixed-integer programming model is practiced in numerical experiments on two real case studies. The numerical outcomes display that a better solution of the LTPSP can be attained by means of the LR-BA method than other methods in terms of NPV, AOG, and computational time.
(3) The applications of other population-based meta-heuristics techniques are proposed to study and compare their behaviours by means of the framework projected in this paper.

Disclosure statement

No potential conflict of interest was reported by the author(s).

ORCID

Amir Hossein Bangian Tabrizi http://orcid.org/0000-0002-4387-0343

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