

ABBAS AGHAJANI BAZZAZI\*, MORTEZA OSANLOO\*, BEHROZ KARIMI\*\*

**OPTIMAL OPEN PIT MINING EQUIPMENT SELECTION USING FUZZY MULTIPLE ATTRIBUTE  
DECISION MAKING APPROACH**

**OPTYMALNYCH URZĄDZEŃ GÓRNICZYCH OTWARTYCH PIT FUZZY  
WIELOKROTNEGO WYBORU PRZY UŻYCIU ATRYBUTU DECYZYJNEGO PODEJŚCIE**

Equipment selection in mining engineering is one of the most important decision that is affected the mine design, production planning and economic parameters in open pit mining. Mine planning engineers generally use of their intuition and experiences in decision making even though equipment selection is a complex multi criteria decision problem. In real-world situation, because of incomplete or non-obtainable information, the data (attributes) are often not so deterministic, there for they usually are fuzzy-imprecise.

Combination of Analytical Hierarchy process (AHP) and entropy method applied to calculate global weights of the attributes. The weights then passed to the Technique for order by similarity to ideal solution (TOPSIS) method that the most efficient mining equipment alternative(s) could be appointed through distance measurement so that the best alternative has the nearest (distance) to the ideal solution and farthest from the negative-ideal solution in fuzzy environment. This approach is applied to select optimal loading-haulage equipment in Sungun open pit mine of Iran.

**Keywords:** mining equipment selection, open pit, multi criteria decision making (MCDM), analytical hierarchy process (AHP), TOPSIS, Fuzzy set theory

Wybór sprzętu górniczego w inżynierii jest jedną z najważniejszych decyzji, która ma wpływ na moje projektowania, planowania produkcji i parametrów ekonomicznych w otwartych pit górnictwa. Kopalnia planuje inżynierów powszechnie używać ich intuicji i doświadczenia w podejmowaniu decyzji, nawet jeśli wybór sprzętu jest złożonym problemem wielu kryteriów decyzji. W rzeczywistym świecie sytuacji, ze względu na niepełne lub nie do uzyskania informacji, danych (atrybuty) są często tak nie deterministyczny, bo tam zazwyczaj są fuzzy-nieprecyzyjne.

Połączenie Analytical Hierarchy procesu (AHP) oraz metodą entropii stosowany do obliczenia globalnego odważnika z atrybutów. Wagi następnie przeszedł do techniki na zamówienie przez podobieństwo

\* DEPARTMENT OF MINING AND METALLURGY ENGINEERING, AMIRKABIR UNIVERSITY OF TECHNOLOGY, TEHRAN, IRAN

\*\* DEPARTMENT OF INDUSTRIAL ENGINEERING, AMIRKABIR UNIVERSITY OF TECHNOLOGY, TEHRAN, IRAN

do idealne rozwiązanie (TOPSIS) metodę, która najbardziej wydajnych urządzeń górniczych alternatywnych ( $y$ ) może być wyznaczony poprzez pomiar odległości, tak że najlepszą alternatywę ma najbliższy (na odległość) do idealnym rozwiązaniem i najdalej od negatywnych-fuzzy doskonałym rozwiązaniem w środowisku. Takie podejście jest stosowane, aby wybrać optymalny załadunek-transportu drogowego sprzętu otwarty Sungun pit kopalni Iranu.

**Słowa kluczowe:** wybór urządzeń górniczych, otwarty dół, wielu kryteriów podejmowania decyzji (MCDM), proces hierarchii analitycznej (AHP), TOPSIS, teorii „zbioru rozmytego“

## 1. Introduction

Equipment selection is one of the most important aspects of open pit design. Mining costs are mainly affected by the number and capacity of equipment. Equipment selection for open-pit mines is definitely a major decision which will impact greatly the economic viability of an operation (Aghajani et al., 2007).

The selection of equipment for mining applications is not a well-defined process and because it involves the interaction of several subjective factors or criteria, decisions are often complicated and may even embody contradictions. Traditionally, procurement costs become elevated through a system of public tendering to appear as the primary criterion and the major costs of looking after the equipment during its useful life are not taken into account (Blanchard & Fabrycky, 1981).

Various types of cost model have been proposed for application to the selection of mining equipment. Expert system as decision aid in surface mine equipment selection was applied by Bandopadhyay & Venkatasubramanian (1987) and Denby & Schofield (1990). Hrebar (1990) and Sevim & Sharma (1991) used net present value analysis for selection of a dragline and surface transportation system.

Use of a linear breakeven model has been proposed by Cebesory (1997). Models for equipment selection and evaluation described by Celebi (1998) were aimed at selection of the equipment fleet on the basis of minimizing the unit stripping cost and maximizing production. Hall et.al (2003) illustrated how reliability analysis can provide mine management with quantitative information of value for decision making about surface mining equipment. Analytical hierarchy process has proposed for application to selection of equipment by some researchers (Samanta et al., 2002; Bascetin, 2004).

Equipment Selection (EQS) is computer software that used fuzzy logic for equipment selection in surface mines and proposed by bascetin et al (2006). Application of AHP-TOPSIS method for loading- haulage equipment selection in open pit mines was used by Aghajani & Osanloo (2007).

Most of these decision-making tools either rely on objective input data, with little or no subjective judgment, or spotlight on a single parameter. Also, because of incomplete or non-obtainable information, the data (attributes) are often not so deterministic; there for they usually are fuzzy-imprecise and application of fuzzy logic for surface mine equipment selection is exigent. Fuzzy multi-criteria decision-making (Fuzzy-MCDM)

techniques can be very useful in encompassing several subjective criteria with conflicting objectives to arrive at an eclectic decision. Combination of fuzzy set theory and technique for order performance by similarity to ideal solution (TOPSIS) is developed to solve open pit mining equipment multi-attribute selection problem.

## 2. Multiple Attribute Decision Making (MADM) method

### 2.1. Defining MADM

MADM methods are developed to handle concept selection problems. In this class of problems, the “best” solution is determined from a finite and usually small set of alternatives. The selection is performed based on the evaluation of the attributes and their preference information.

In the decision making process, many MADM techniques use decision matrix (or goal achievement matrix)  $D$  to describe the states of the attributes of each alternative. In decision matrix format, columns indicate attributes considered in a given problem; and in which rows list the competing alternatives. Specifically, a MADM problem with  $m$  alternatives ( $A_1, A_2 \dots A_m$ ) that are evaluated by  $n$  attributes ( $C_1, C_2 \dots C_n$ ) can be viewed as a geometric system with  $m$  points in  $n$ -dimensional space. An element  $x_{ij}$  of the matrix indicates the performance rating of the  $i$ th alternative,  $A_i$ , with respect to the  $j$ th attribute,  $C_j$ , as shown in following equation (Hwang & Yoon, 1981):

$$D = \begin{matrix} & C_1 & C_2 & C_3 & \dots & C_n \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{matrix} & \left[ \begin{array}{cccccc} x_{11} & x_{12} & x_{13} & \dots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \dots & x_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & x_{m3} & \dots & x_{mn} \end{array} \right. \end{matrix} \quad (1)$$

Generally, MADM methods can be classified into compensatory and non-compensatory methods based on the treatment of the attribute information. The compensatory methods allow trade-offs between criteria, assigning a number to each multidimensional representation of an alternative. The non-compensatory methods do not permit the trade-off between criteria, i.e. one unfavorable criterion value cannot be offset by reducing a favorable value of another criterion (Hwang & Yoon, 1981). In figure 1 several MADM methods are listed (Sen & yang, 1998).

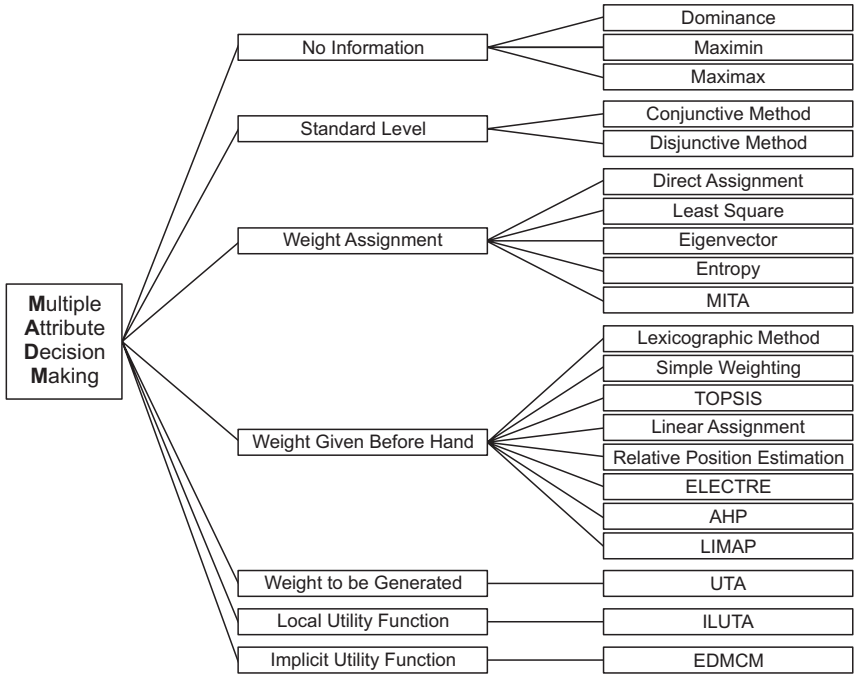


Fig. 1. Classification of MADM methods

## 2.2. Attribute weighting

Each appraisal criterion cannot be assumed to be of equal importance because the appraisal criteria have various meanings. There are many methods that can be employed to determine weights, such as the eigenvector method, weighted least square method, entropy method, AHP, as well as linear programming techniques for multidimensional of analysis preference (LINMAP). The method you choose depends on the nature of the problem. In this study, AHP and entropy method were chosen for attribute weighting because a potential problem with using methods such as the AHP to determine attribute weights is that there is no way to predict shifts in the perceived importance that a decision-maker places on each attribute. Furthermore, traditional methods of determining attribute importance by directly questioning decision-makers assume that attribute importance weights are somehow fixed in a decision-makers head independent of the situation (Starr & Zeleny, 1977).

Inconsistency between decision-makers' stated preferences and their actual choices can be addressed by viewing decision-making as a type of information processing activity, where relevant information about available alternatives is transmitted, perceived, and processed via decision attributes. Thus, attributes are defined as sources of information

such that the more information is emitted by a specific attribute the more relevant is the attribute for a given decision situation. A means to assess the information generated as a result of following a structured decision-making process can be found using the concept of entropy. Within the context of information theory, entropy is a criterion for assessing the amount of uncertainty represented by a discrete probability distribution. Entropy analysis is based on the assumption that there is a direct relationship between uncertainty and the information provided by a distribution of data points, where complete certainty is associated with the absence of information (Hwang & Yoon, 1981; Zeleny, 1982).

### 2.3. Entropy weighting

Entropy weighting is a MADM method used to determine the importance weights of decision attributes by directly relating a criterion's importance weighting relative to the information transmitted by that criterion. For example, given a MADM decision matrix with column vector  $x_j = (x_{1j}, x_{2j}, \dots, x_{mj})$  that shows the contrast of all alternatives with respect to  $j^{\text{th}}$  attribute, an attribute has little importance when all alternatives have similar outcomes for that attribute. Moreover, if all alternatives are the same in relation to a specific attribute then that attribute should be eliminated because it transmits no information about decision-makers preferences. In contrast, the attribute that transmits the most information should have the greatest importance weighting. Mathematically this means that the projected outcomes of attribute  $j$ ,  $p_{ij}$ , are defined as:

$$P_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}} \quad (2)$$

The entropy  $E_j$  of the set of projected outcomes of attribute  $j$  is:

$$E_j = -\left(\frac{1}{\ln m}\right) \sum_{i=1}^m P_{ij} \ln P_{ij} \quad (3)$$

Where  $m$  is the number of alternatives and guarantees that  $E_j$  lies between zero and one. The degree of diversification  $d_j$  of the information provided by outcomes of attribute  $j$  can be defined as  $d_j = 1 - E_j$ . Hence, the entropy weighting of an attribute is calculated as follows:

$$W_j = \frac{d_j}{\sum_{j=1}^n d_j} \quad (4)$$

In situations where a decision-maker has an a priori  $\lambda_j$  subjective weighting for an attribute, a compromise weighting,  $w_j^0$ , that takes into account both a decision maker's

subjective preference and the objective entropy weighting of the attribute is calculated as follows:

$$w_j^0 = \frac{\lambda_j w_j}{\sum_{j=1}^n \lambda_j w_j} \tag{5}$$

Whereas entropy weighting provides a dynamic and objective assessment of a decision maker’s attribute preference relative to the decision-making process, a priori weighting methods such as the AHP deceptively determine attribute importance statically and independently of the decision-making process.

### 2.4. Analytic Hierarchy Process

This method has been developed by Saaty (1990; 1994). The AHP structures the decision problem in levels which correspond to one understands of the situation: goals, criterion, sub-criterion, and alternatives. By breaking the problem into levels, the decision-maker can focus on smaller sets of decisions. In AHP technique the elements of each level compared to its related element in upper level inform by pair-wise comparison method.

It must be noted that, in pair comparison of criterion if the priority of element *i* compared to element *j* is equal to  $w_{ij}$  then the priority of element *j* compared to element *i* is equal to  $1/w_{ij}$ . The priority of element compared to it is equal to one.

AHP method is applied in this research for criteria weighting. So, at first, set up *n* criteria in the rows and columns of  $n \times n$  matrix. Then, Perform pair wise comparisons of all the criteria according to the goal. The fundamental scale used for this purpose is shown in Table 1. For a matrix of order *n*,  $((n) \times (n - 1)/2)$  comparisons are required. Use average over normalized columns to estimate the Eigen values of the matrix. The redundancy of the pair wise comparisons (Table 1) makes the AHP much less sensitive to judgment errors; it also lets one measure judgment errors by calculating the consistency index of the comparison matrix, and then calculating the consistency ratio.

TABLE 1

Scale for pair wise comparisons

Numerical assessment	Linguistic meaning
1	Equal important
3	Moderately more important
5	Strongly more important
7	Very strongly important
9	Extremely more important
2,4,6,8	Intermediate values of importance

## 2.5. Combination of AHP and entropy method

Since entropy weighting passively determines attribute weights without a decision-makers conscious intention, the opportunity to learn during the attribute weighting process is eliminated which in turn may reduce both decision-maker understanding and expectance. Thus, neither entropy weighting nor the AHP are entirely adequate for determining attribute importance weights in complex real-world situations. However, the innovative integration of the AHP and entropy weighting could potentially serve as a critical component of a comprehensive solution for classifying and prioritizing product requirements. In AHP and entropy method are often criticized for its inability to adequately handle the inherent uncertainty and imprecision associated with the mapping of the decision-makers perception to crisp values.

In many practical cases the human preference model is uncertain and decision makers might be reluctant or unable to assign crisp values to the comparison judgments (Zadeh, 1965). The use of fuzzy set theory allows the decision-makers to incorporate unquantifiable information, incomplete information, non-obtainable information, and partial facts into the decision model.

## 3. Fuzzy MADM method

### 3.1. Fuzzy TOPSIS method

It is often difficult for a decision-maker to assign a precise performance rating to an alternative for the attributes under consideration. The merit of using a fuzzy approach is to assign the relative importance of attributes using fuzzy numbers instead of precise numbers. This section extends the TOPSIS to the fuzzy environment. We briefly review the rationale of fuzzy theory before the development of fuzzy TOPSIS; as follows:

**Definition 3.1.** A fuzzy set  $\tilde{a}$  in a universe of discourse  $X$  is characterized by a membership function  $\mu_a(x)$  which associates with each element  $x$  in  $X$ , a real number in the interval  $[0, 1]$ . The function value  $\mu_a(x)$  is termed the grade of membership of  $x$  in  $\tilde{a}$  (Zadeh, 1965).

The present study uses triangular fuzzy numbers. A triangular fuzzy number  $\tilde{a}$  can be defined by a triplet  $(a_1, a_2, a_3)$ . Its conceptual schema and mathematical form are shown by Equation 6 (Kaufmann & Gupta, 1985):

$$\mu_{\tilde{a}}(x) = \begin{cases} 0 & x \leq a_1 \\ \frac{x - a_1}{a_2 - a_1} & a_1 < x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2} & a_2 < x \leq a_3 \\ 0 & x > a_3 \end{cases} \tag{6}$$

**Definition 3.2.**  $\tilde{a} = (a_1, a_2, a_3)$  and  $\tilde{b} = (b_1, b_2, b_3)$  are two triangular fuzzy numbers, then the vertex method is defined to calculate the distance between them, as Equation 7:

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]} \tag{7}$$

Let  $\tilde{a}, \tilde{b}, \tilde{c}$  be three triangular fuzzy numbers. The fuzzy number  $\tilde{b}$  is closer to fuzzy number  $\tilde{a}$  than the other fuzzy number  $\tilde{c}$  if and only if  $d(\tilde{a}, \tilde{b}) < d(\tilde{a}, \tilde{c})$ .

The basic operations on fuzzy triangular numbers are as follows:

$$\tilde{a} \times \tilde{b} = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3) \quad \text{for multiplication} \tag{8}$$

$$\tilde{a} + \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \quad \text{for addition} \tag{9}$$

The fuzzy MADM can be concisely expressed in matrix format as Equations 10 and 11:

$$\tilde{D} = \begin{matrix} & & C_1 & C_2 & C_3 & \dots & C_n \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{matrix} & \left[ \begin{array}{cccccc} \tilde{x}_{11} & \tilde{x}_{12} & \tilde{x}_{13} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \tilde{x}_{23} & \dots & \tilde{x}_{2n} \\ \tilde{x}_{31} & \tilde{x}_{32} & \tilde{x}_{33} & \dots & \tilde{x}_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \tilde{x}_{m3} & \dots & \tilde{x}_{mn} \end{array} \right] \end{matrix} \tag{10}$$

$$\tilde{W} = [\tilde{w}_1, \tilde{w}_2, \tilde{w}_3, \dots, \tilde{w}_n] \tag{11}$$

where  $\tilde{x}_{ij}, i = 1, 2, \dots, m, j = 1, 2, \dots, n$  and  $\tilde{w}_j = 1, 2, \dots, n$  are linguistic triangular fuzzy numbers,  $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$  and  $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$ . Note that  $\tilde{x}_{ij}$  is the performance rating of the  $i^{\text{th}}$  alternative,  $A_i$ , with respect to the  $j^{\text{th}}$  attribute,  $C_j$  and  $\tilde{w}_j$  represents the weight of the  $j^{\text{th}}$  attribute,  $C_j$ .



The normalized fuzzy decision matrix denoted by  $\tilde{R}$  is shown as Equation 12.

$$\tilde{R} = [r_{ij}]_{m \times n} \quad (12)$$

The weighted fuzzy normalized decision matrix is shown as Equation 13.

$$\tilde{v} = \begin{bmatrix} \tilde{v}_{11} & \tilde{v}_{12} & \cdots & \tilde{v}_{1j} & \cdots & \tilde{v}_{1n} \\ \tilde{v}_{21} & \tilde{v}_{22} & \cdots & \tilde{v}_{2j} & \cdots & \tilde{v}_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{v}_{i1} & \tilde{v}_{i2} & \cdots & \tilde{v}_{ij} & \cdots & \tilde{v}_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{v}_{m1} & \tilde{v}_{m2} & \cdots & \tilde{v}_{mj} & \cdots & \tilde{v}_{mn} \end{bmatrix} = \begin{bmatrix} \tilde{w}_1 \tilde{r}_{11} & \tilde{w}_2 \tilde{r}_{12} & \cdots & \tilde{w}_j \tilde{r}_{1j} & \cdots & \tilde{w}_n \tilde{r}_{1n} \\ \tilde{w}_1 \tilde{r}_{21} & \tilde{w}_2 \tilde{r}_{22} & \cdots & \tilde{w}_j \tilde{r}_{2j} & \cdots & \tilde{w}_n \tilde{r}_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{w}_1 \tilde{r}_{i1} & \tilde{w}_2 \tilde{r}_{i2} & \cdots & \tilde{w}_j \tilde{r}_{ij} & \cdots & \tilde{w}_n \tilde{r}_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{w}_1 \tilde{r}_{m1} & \tilde{w}_2 \tilde{r}_{m2} & \cdots & \tilde{w}_j \tilde{r}_{mj} & \cdots & \tilde{w}_n \tilde{r}_{mn} \end{bmatrix} \quad (13)$$

Given the above fuzzy theory, the fuzzy TOPSIS procedure is then defined as follows:

**Step 1:** Choose the linguistic ratings ( $\tilde{x}_{ij}, i = 1, 2 \dots m, j = 1, 2 \dots n$ ) for alternatives with respect to criteria and the appropriate linguistic variables  $\tilde{w}_j, j = 1, 2 \dots n$  for the weight of the criteria.

The fuzzy linguistic rating  $\tilde{x}_{ij}$  preserves the property that the ranges of normalized triangular fuzzy numbers belong to  $[0, 1]$ ; thus, there is no need for a normalization procedure. For this instance, the  $D$  defined by Equation 10 is equivalent to the  $\tilde{R}$  defined by Equation 12.

**Step 2:** Construct the weighted normalized fuzzy decision matrix. The weighted normalized value  $\tilde{v}$  is calculated by Equation 13.

**Step 3:** Identify positive ideal ( $A^*$ ) and negative ideal ( $A^-$ ) solutions. The fuzzy positive-ideal solution (FPIS,  $A^*$ ) and the fuzzy negative-ideal solution (FNIS,  $A^-$ ) are shown as Equations 14 and 15:

$$A^* = (\tilde{v}_1^*, \tilde{v}_2^*, \tilde{v}_3^*, \dots, \tilde{v}_n^*) = \left\{ \max_i v_{ij} \mid (i = 1, 2, \dots, m, j = 1, 2, \dots, n) \right\} \quad (14)$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \tilde{v}_3^-, \dots, \tilde{v}_n^-) = \left\{ \min_i v_{ij} \mid (i = 1, 2, \dots, m, j = 1, 2, \dots, n) \right\} \quad (15)$$

**Step 4:** Calculate separation measures. The distance of each alternative from  $A^*$  and  $A^-$  can be currently calculated using Equations 16 and 17.

$$d_i^* = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_j^*), i = 1, 2, \dots, m \quad (16)$$

$$d_i^- = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_j^-), i = 1, 2, \dots, m \tag{17}$$

**Step 5:** Calculate similarities to ideal solution. This step solves the similarities to an ideal solution by Equation 18:

$$CC_i = \frac{d_i^-}{d_i^- + d_i^*} \tag{18}$$

**Step 6:** Rank preference order. Choose an alternative with maximum  $CC_i^*$  or rank alternatives according to  $CC_i^*$  in descending order.

### 3.2. Fuzzy membership function

The decision makers use the linguistic variables to evaluate the importance of attributes and the ratings of alternatives with respect to various attributes. The present study has only precise values for the performance ratings and for the attribute weights. In order to illustrate the idea of fuzzy MACD, we deliberately transform the existing precise values to five-levels, fuzzy linguistic variables-very low (VL), low (L), medium (M), high (H) and very high (VH).

Among the commonly used fuzzy numbers, triangular and trapezoidal fuzzy numbers are likely to be the most adoptive ones due to their simplicity in modeling and easy of interpretation. Both triangular and trapezoidal fuzzy numbers are applicable to the present study. We feel that a triangular fuzzy number can adequately represent the five-level fuzzy linguistic variables and thus, is used for the analysis hereafter.

As a rule of thumb, each rank is assigned an evenly spread membership function that has an interval of 0.30 or 0.25. Based on these assumptions, a transformation table can be found as shown in Table 2.

TABLE 2

Transformation for fuzzy membership functions

Rank	Attribute grade	Membership function
Very Low (VL)	1	(0.00,0.10,0.25)
Low (L)	2	(0.15,0.30,0.45)
Medium (M)	3	(0.35,0.50,0.65)
High (H)	4	(0.55,0.70,0.85)
Very High (VH)	5	(0.75,0.90,1.00)

For example, the fuzzy variable-Very Low has its associated triangular fuzzy number with minimum of 0.00, mode of 0.10 and maximum of 0.25. The same definition is then applied to the other fuzzy variables-Low, Medium, High and Very High. Fig. 2 illustrates the fuzzy membership functions.

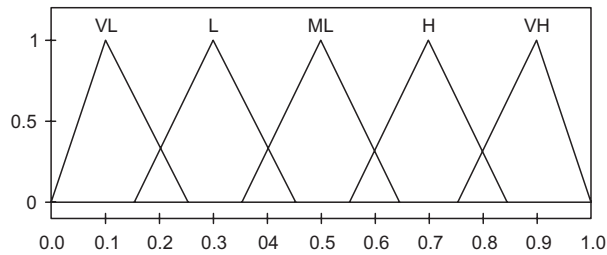


Fig. 2. Fuzzy triangular membership functions

## 4. Application of fuzzy TOPSIS method in Sungun copper mine

### 4.1. Sungun copper mine location

Sungun mine is one of the largest copper deposits of Iran which is located in the north-west of the country close to Azerbaijan, Armenia and Turkey borders (Fig. 3). Technical and economical studies were shown that the most appropriate of mining method for this deposit is open pit mining method. By this method 384 million tons of ore with 0.665 percentage of copper grade can be mined. Total mine's life estimated to be 31 years with annual production of 7 million tons in first 5 years and 14 million tons for remaining years. During this period 680 million tons of waste must be removed. So, the waste

to ore ratio in this mine is 1.8:1 (Hoseinie et al., 2006). Three potential transportation system alternatives have been evaluated for ore transportation. These are loader-truck (A1), shovel-truck (A2) and shovel-truck-belt conveyor (A3) systems.



Fig. 3. Geographical location of Sungun copper mine

### 4.2. Weighting criteria by AHP-entropy method for haulage-loading equipment selection

The structure of the problem according to Saaty’s hierarchy is given in Fig. 4. The goal is to select the loading–hauling system that can meet optimal production requirements. This goal is placed on the first level of the hierarchy. Two strategic factors, namely cost and operational/technical factors, are identified to achieve this goal, which form the second level of the hierarchy. The third level of the hierarchy covers the criteria defining the two strategic factors of cost and operational/technical factors of the second level. There are two criteria related to cost, namely capital and operating cost. The criteria associated with operational/technical factors are operating conditions and equipment technical parameters. Some criteria are divided into some sub criteria (Fig. 4). Expert Choice software is used to determine the global priority weights.

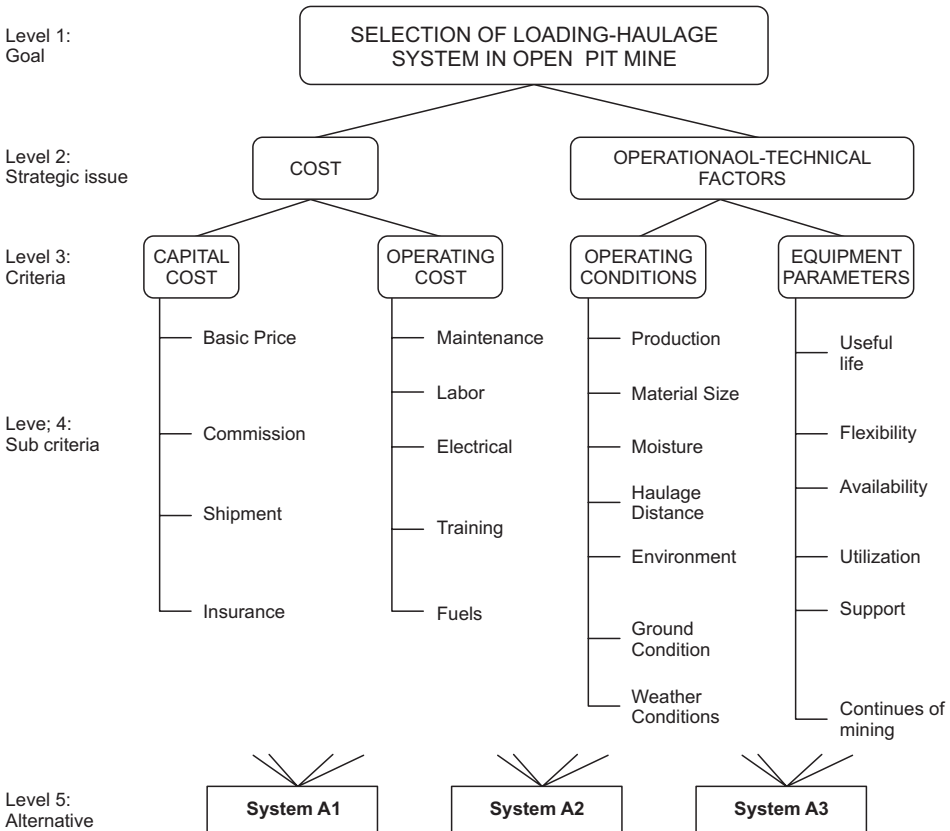


Fig. 4. AHP model for loading–hauling system selection

These matrices are constructed by an expert team. Using this approach, an evaluation team of four members who are frequently involved in equipment selection in the particular open pit mine operation was used. It deserved mention; all of them have equal impression in decision making process. In this study, twenty two attributes and three alternatives were considered. AHP model and entropy method were used to attribute weighting because Weight of attribute should be given to decision makers for application in fuzzy TOPSIS method.

For the first step of this methodology, the decision matrix, representing the performance values of each alternative with respect to each criterion, is computed.

Table 3 is showing thirteen attributes are the smaller the better type criteria (italic) and nine attributes are the larger the better type criteria. Criteria weighting with entropy method were calculated by using equations 2 to 4 and combination of AHP and entropy method were calculated by equation 5 and results are summarized in Table 3, also Global weights of the evaluation attributes calculated using AHP and entropy method are illustrated in Fig. 5.

TABLE 3

Criteria weighting by AHP and entropy method

<b>Criteria \ Alternative</b>	<b>System A1</b>	<b>System A2</b>	<b>System A3</b>	<b>AHP weight</b>	<b>Entropy weight</b>	<b>AHP-entropy weight</b>
Basic price (c1)	0.261	0.304	0.435	0.167	0.026	0.123
Commission (c2)	0.214	0.286	0.500	0.015	0.069	0.030
Shipment (c3)	0.292	0.292	0.417	0.052	0.016	0.024
Insurance (c4)	0.273	0.273	0.455	0.047	0.034	0.046
Maintenance (c5)	0.421	0.263	0.316	0.091	0.020	0.054
Labor (c6)	0.308	0.308	0.385	0.051	0.006	0.009
Electrical (c7)	0.263	0.316	0.421	0.074	0.020	0.044
Training (c8)	0.250	0.375	0.375	0.029	0.018	0.015
Fuels (c9)	0.438	0.250	0.313	0.131	0.029	0.109
Production (c10)	0.143	0.381	0.476	0.026	0.107	0.080
Material size (c11)	0.385	0.154	0.462	0.024	0.092	0.064
Moisture (c12)	0.353	0.235	0.412	0.019	0.027	0.015
Haulage distance (c13)	0.364	0.273	0.364	0.060	0.009	0.016
Environment (c14)	0.385	0.308	0.308	0.016	0.006	0.003
Ground condition (c15)	0.471	0.294	0.235	0.046	0.047	0.062
Weather condition (c16)	0.450	0.250	0.300	0.018	0.034	0.018
Useful life (c17)	0.154	0.346	0.500	0.010	0.104	0.030
Flexibility (c18)	0.500	0.333	0.167	0.028	0.093	0.076
Availability (c19)	0.278	0.333	0.389	0.019	0.010	0.005
Utilization (c20)	0.143	0.381	0.476	0.036	0.107	0.111
Support (c21)	0.316	0.263	0.421	0.025	0.020	0.015
Continues of mining (c22)	0.150	0.350	0.500	0.017	0.107	0.053

### 4.3. Evaluation procedure By Fuzzy Topsis in Sungun copper mine

In order to transform the performance ratings to fuzzy linguistic variables as discussed in the previous section, the performance ratings in Table 3 are normalized into the range of [0, 1] by Equations 19 and 20 (Cheng, 1999):

$$r_{ij} = \frac{x_{ij} - \min\{x_{ij}\}}{[\max\{x_{ij}\} - \min\{x_{ij}\}]} \quad \text{the larger the better type} \quad (19)$$

$$r_{ij} = \frac{\max\{x_{ij}\} - x_{ij}}{[\max\{x_{ij}\} - \min\{x_{ij}\}]} \quad \text{the smaller the better type} \quad (20)$$

Normalized decision matrix for fuzzy TOPSIS analysis is illustrated in Table 4.

TABLE 4

Normalized decision matrix for fuzzy TOPSIS analysis

Alternative Criteria	System A1	System A2	System A3	Criteria weight (wj)
Basic price	0.670	0.337	0.196	0.123
Commission	0.800	0.419	0.000	0.030
Shipment	0.583	0.393	0.250	0.024
Insurance	0.636	0.477	0.136	0.046
Maintenance	0.221	0.519	0.553	0.054
Labor	0.538	0.323	0.346	0.009
Electrical	0.663	0.287	0.237	0.044
Training	0.700	0.026	0.375	0.015
Fuels	0.175	0.577	0.563	0.109
Production	0.000	1.000	0.929	0.080
Material size	0.677	0.000	0.885	0.064
Moisture	0.412	0.641	0.265	0.015
Haulage distance	0.382	0.477	0.409	0.016
Environment	0.677	0.677	0.423	0.003
Ground condition	0.918	0.618	0.206	0.062
Weather condition	0.140	0.577	0.600	0.018
Useful life	0.031	0.847	1.000	0.030
Flexibility	1.000	0.790	0.000	0.076
Availability	0.378	0.790	0.667	0.005
Utilization	0.000	1.000	0.929	0.111
Support	0.516	0.519	0.237	0.015
Continues of mining	0.020	0.864	1.000	0.053

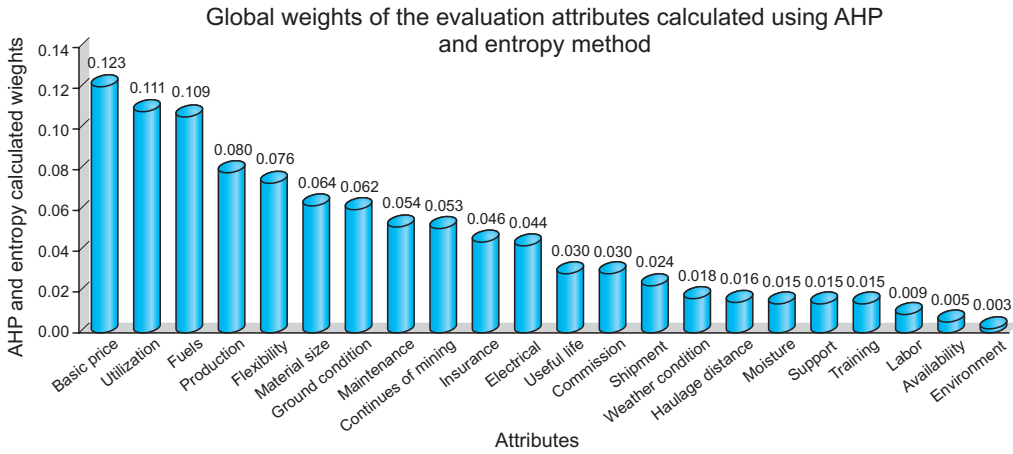


Fig. 5. Global weights of the evaluation attributes calculated using AHP and entropy method

By using fuzzy membership function discussed in Section 3.2, the crisp values of global weight and performance of each alternative, exchange to fuzzy number (Table 5).

The fuzzy linguistic variable is then transformed into a fuzzy triangular membership function as shown in Table 6. This is the first step of the fuzzy TOPSIS analysis. The fuzzy attribute weight is also collected in Table 6.

TABLE 5

Decision matrix using fuzzy linguistic variables

Criteria \ Alternative	System A1	System A2	System A3	Criteria weight (wj)
1	2	3	4	5
Basic price	H	L	VL	VH
Commission	H	M	VL	L
Shipment	M	L	L	VL
Insurance	H	M	VL	L
Maintenance	L	M	M	M
Labor	M	L	L	VL
Electrical	H	L	L	L
Training	H	VL	L	VL
Fuels	VL	M	M	VH
Production	VL	VH	VH	H
Material size	H	VL	VH	M
Moisture	M	H	L	VL
Haulage distance	L	M	M	VL
Environment	H	H	M	VL
Ground condition	VH	H	L	M

1	2	3	4	5
Weather condition	VL	M	M	VL
Useful life	VL	VH	VH	L
Flexibility	VH	H	VL	H
Availability	L	H	H	VL
Utilization	VL	VH	VH	VH
Support	M	M	L	VL
Continues of mining	VL	VH	VH	M

The second step in the analysis is to find the weighted fuzzy decision matrix. Using Equation 8, the fuzzy multiplication equation, the resulting fuzzy weighted decision matrix is shown as Table 7.

According to Table 7, we know that the elements  $v_{ij}, \forall i, j$  are normalized positive triangular fuzzy numbers and their ranges belong to the closed interval  $[0, 1]$ . Thus, we can define the fuzzy positive-ideal solution (FPIS,  $A^*$ ) and the fuzzy negative-ideal solution (FNIS,  $A^-$ ) as:  $\tilde{v}_j^* = (1,1,1)$  and  $\tilde{v}_j^- = (1,1,1) j = 1,2,\dots,n$ . This is the third step of the fuzzy TOPSIS analysis. For the fourth step, the distance of each alternative from  $A^*$  and  $A^-$  can be currently calculated using equations 16 and 17. The fifth step solves the similarities to an ideal solution by equation 18. The resulting fuzzy TOPSIS analyses are summarized in Table 8.

TABLE 6

Decision matrix using fuzzy linguistic variables

Criteria \ Alternative	System A1	System A2	System A3	Criteria weight ( $w_j$ )
	2	3	4	5
Basic price	(0.55,0.70,0.85)	(0.15,0.30,0.45)	(0.00,0.10,0.25)	(0.75,0.90,1.00)
Commission	(0.55,0.70,0.85)	(0.35,0.50,0.65)	(0.00,0.10,0.25)	(0.15,0.30,0.45)
Shipment	(0.35,0.50,0.65)	(0.15,0.30,0.45)	(0.15,0.30,0.45)	(0.00,0.10,0.25)
Insurance	(0.55,0.70,0.85)	(0.35,0.50,0.65)	(0.00,0.10,0.25)	(0.15,0.30,0.45)
Maintenance	(0.15,0.30,0.45)	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.35,0.50,0.65)
Labor	(0.35,0.50,0.65)	(0.15,0.30,0.45)	(0.15,0.30,0.45)	(0.00,0.10,0.25)
Electrical	(0.55,0.70,0.85)	(0.15,0.30,0.45)	(0.15,0.30,0.45)	(0.15,0.30,0.45)
Training	(0.55,0.70,0.85)	(0.00,0.10,0.25)	(0.15,0.30,0.45)	(0.00,0.10,0.25)
Fuels	(0.00,0.10,0.25)	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.75,0.90,1.00)
Production	(0.00,0.10,0.25)	(0.75,0.90,1.00)	(0.75,0.90,1.00)	(0.55,0.70,0.85)
Material size	(0.55,0.70,0.85)	(0.00,0.10,0.25)	(0.75,0.90,1.00)	(0.35,0.50,0.65)
Moisture	(0.35,0.50,0.65)	(0.55,0.70,0.85)	(0.15,0.30,0.45)	(0.00,0.10,0.25)
Haulage distance	(0.15,0.30,0.45)	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.00,0.10,0.25)
Environment	(0.55,0.70,0.85)	(0.55,0.70,0.85)	(0.35,0.50,0.65)	(0.00,0.10,0.25)
Ground condition	(0.75,0.90,1.00)	(0.55,0.70,0.85)	(0.15,0.30,0.45)	(0.35,0.50,0.65)
Weather condition	(0.00,0.10,0.25)	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.00,0.10,0.25)
Useful life	(0.00,0.10,0.25)	(0.75,0.90,1.00)	(0.75,0.90,1.00)	(0.15,0.30,0.45)



1	2	3	4	5
Flexibility	(0.75,0.90,1.00)	(0.55,0.70,0.85)	(0.00,0.10,0.25)	(0.55,0.70,0.85)
Availability	(0.15,0.30,0.45)	(0.55,0.70,0.85)	(0.55,0.70,0.85)	(0.00,0.10,0.25)
Utilization	(0.00,0.10,0.25)	(0.75,0.90,1.00)	(0.75,0.90,1.00)	(0.75,0.90,1.00)
Support	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.15,0.30,0.45)	(0.00,0.10,0.25)
Continues of mining	(0.00,0.10,0.25)	(0.75,0.90,1.00)	(0.75,0.90,1.00)	(0.35,0.50,0.65)

TABLE 7

## Fuzzy-weighted decision matrix

Criteria \ Alternative	System A1	System A2	System A3
Basic price	(0.41,0.63,0.85)	(0.11,0.27,0.45)	(0.11,0.27,0.45)
Commission	(0.08,0.21,0.38)	(0.00,0.03,0.11)	(0.02,0.09,0.20)
Shipment	(0.00,0.07,0.21)	(0.00,0.03,0.11)	(0.00,0.03,0.11)
Insurance	(0.08,0.21,0.38)	(0.00,0.03,0.11)	(0.02,0.09,0.20)
Maintenance	(0.19,0.35,0.55)	(0.00,0.05,0.16)	(0.05,0.15,0.29)
Labor	(0.00,0.05,0.16)	(0.00,0.03,0.11)	(0.00,0.05,0.16)
Electrical	(0.11,0.27,0.45)	(0.00,0.03,0.11)	(0.00,0.03,0.20)
Training	(0.00,0.07,0.21)	(0.00,0.01,0.06)	(0.00,0.05,0.16)
Fuels	(0.00,0.09,0.25)	(0.26,0.45,0.65)	(0.41,0.63,0.65)
Production	(0.00,0.07,0.21)	(0.41,0.63,0.85)	(0.30,0.49,0.55)
Material size	(0.12,0.25,0.42)	(0.00,0.05,0.16)	(0.26,0.45,0.42)
Moisture	(0.00,0.09,0.25)	(0.00,0.03,0.11)	(0.00,0.01,0.06)
Haulage distance	(0.00,0.03,0.11)	(0.00,0.05,0.16)	(0.00,0.05,0.16)
Environment	(0.00,0.07,0.21)	(0.00,0.07,0.21)	(0.00,0.03,0.11)
Ground condition	(0.26,0.45,0.65)	(0.19,0.35,0.55)	(0.05,0.15,0.29)
Weather condition	(0.00,0.03,0.11)	(0.00,0.05,0.16)	(0.00,0.05,0.16)
Useful life	(0.00,0.03,0.11)	(0.11,0.27,0.45)	(0.11,0.27,0.45)
Flexibility	(0.41,0.63,0.85)	(0.41,0.63,0.85)	(0.00,0.07,0.21)
Availability	(0.00,0.03,0.11)	(0.00,0.09,0.25)	(0.00,0.07,0.16)
Utilization	(0.00,0.09,0.25)	(0.56,0.81,1.00)	(0.41,0.63,0.85)
Support	(0.00,0.07,0.21)	(0.00,0.05,0.16)	(0.00,0.03,0.06)
Continues of mining	(0.00,0.05,0.16)	(0.26,0.45,0.65)	(0.26,0.45,0.65)

An example is used in order to illustrate Steps 4 and 5 calculations as follows:

$$\begin{aligned}
 d_1^* = & \sqrt{\frac{1}{3}[(1-0.41)^2 + (1-0.63)^2 + (1-0.85)^2]} + \sqrt{\frac{1}{3}[(1-0.08)^2 + (1-0.21)^2 + (1-0.38)^2]} \\
 & + \sqrt{\frac{1}{3}[(1-0.00)^2 + (1-0.07)^2 + (1-0.21)^2]} + \dots + \sqrt{\frac{1}{3}[(1-0.00)^2 + (1-0.07)^2 + (1-0.21)^2]} \quad (21) \\
 & + \sqrt{\frac{1}{3}[(1-0.0)^2 + (1-0.05)^2 + (1-0.16)^2]} = 21.77
 \end{aligned}$$

$$\begin{aligned}
 d_1^- &= \sqrt{\frac{1}{3}[(0-0.41)^2+(0-0.63)^2+(0-0.85)^2]} + \sqrt{\frac{1}{3}[(0-0.08)^2+(0-0.21)^2+(0-0.38)^2]} \\
 &+ \sqrt{\frac{1}{3}[(0-0.00)^2+(0-0.07)^2+(0-0.21)^2]} + \dots + \sqrt{\frac{1}{3}[(0-0.00)^2+(0-0.07)^2+(0-0.21)^2]} \quad (22) \\
 &+ \sqrt{\frac{1}{3}[(0-0.0)^2+(0-0.05)^2+(0-0.16)^2]} = 7.60
 \end{aligned}$$

CC<sub>1</sub> is calculated for this example as follows:

$$CC_1 = \frac{d_1^-}{d_1^- + d_1^+} = \frac{7.60}{7.60 + 21.77} = 0.26 \quad (23)$$

TABLE 8

Fuzzy TOPSIS analysis

Alternative	di <sup>+</sup>	di <sup>-</sup>	CC <sub>i</sub>	Normalized CC <sub>i</sub>	Ranking
A1	21.77	7.60	0.26	0.33	2
A2	21.28	8.10	0.28	0.35	1
A3	21.92	7.21	0.25	0.31	3

In conclusion, Shovel-Truck (A2) has become the most desirable system among three alternatives with the normalized final performance value of 0.35; while loader-Truck and Shovel-Truck-Conveyor belt have positioned at the second and third ranks with 0.33 and 0.31 as the final performance values, respectively (Fig. 6).

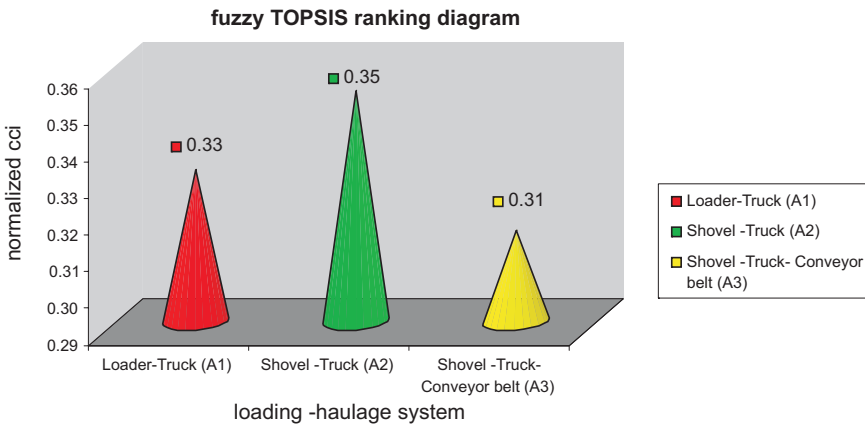


Fig. 6. Fuzzy TOPSIS ranking

## 5. Conclusions

The open pit equipment selection problem is a strategic issue and has significant impacts to the open-pit design and production planning. Most of exiting open pit equipment selection rely on objective input data, with little or no subjective judgment, or focus on a single parameter; and therefore lead to a poor equipment selection due to the MADM nature of equipment selection problem.

In this study, combination of AHP and entropy method is used for attributes weighting and TOPSIS-fuzzy set theory techniques are introduced to select the suitable loading-haulage equipment in large open pit mines. The methods and experiences learned from the study can be valuable to the open pit mines future strategic planning. Empirical results showed that the proposed methods are viable approaches in solving the proposed mining equipment selection problem. TOPSIS is a viable method for the proposed problem and is suitable for the use of precise performance ratings. When the performance ratings are vague and inaccurate, then the fuzzy TOPSIS is the preferred technique. There exists other worth investigating MADM methods for mining equipment selection problem. This becomes one of the future research opportunities in this classical yet important research area.

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*Received: 14 October 2008*