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ORIGINAL ARTICLE

# TISS: a decision framework for tailing impoundment site selection

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**Abstract** Tailing dam is one of the most important mining operations interface with surrounding environment even as long as many years after ore reserve exhausted. Therefore, appropriate design and management respect to future limitations of environmental regulations is required. Recently, effect of multiple criteria on tailing impoundment site selection makes it complex as the conventional procedures unable to answer. The systematic approach of multi attribute decision-making helps decision-makers select the most preferable decision and provide the basis of a decision support system. This paper developed new strategy based on fuzzy multi attribute group decision-making methods including: technique for order preference by similarity to ideal solution and analytical hierarchy process in fuzzy group environment. A hypothetical case is processed to demonstrate the strategy's efficiency and results are compared and ranked so that the most preferable option is identified.

**Keywords** Tailing impoundment · Site selection · Multi attribute decision-making · Fuzzy sets

## Introduction

The increasing demand and ascending price of minerals in many cases make it possible to process lower grade ores, which means more production of tailing and more environmental disturbance. Tailing management apart from its necessary operating cost usually has a considerable portion in capital cost of a mining plan. On the other hand more countries enforce rigid rules to control the tailing dams' contaminations which in some cases shut the mines down (Akbari et al. 2007). Mine closure due to the intolerable tailing dam's disturbances, when technical and economical potentials are still survived, could be the most unfortunate end.

Tailings impoundments are probably the largest structures built by man. Unlike a water dam, they cannot be breached at the end of their service life; they will exist for many centuries and may impact the environment as they respond to the same natural forces which erode and level the surrounding landscape (Caldwell and Robertson 1983). As a consequence, nowadays, planning the tailing dams are engrossed in meticulous disciplines. It seems that detection of proper site to establish an impoundment structure is critical irreversible decision in earliest step of this way.

Formerly, tailings impoundment location was a simple procedure based on a relative distance from mill location. Since last two to three decades the environmental impacts have been taking into a great consideration. The mining engineers and environmental agencies usually have different objectives which come from their job and organization purposes. Mining engineers are often interested in increasing the productivity, decreasing the costs and maximizing the profit, while environmental managers believe that public health is the highest goal (Osanloo and Ataei 2003). The distinctions between the mining operation participants and such agencies are neither possible to solve nor eliminate completely. The best site is a location which meets both objectives.

The critical situation of decision on tailing impoundment site selection (TISS) triggers the demand of a new

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far-reaching strategy solution. This paper proposes a new framework by integrating the fuzzy group analytical hierarchy process (FGAHP) approach with Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS) method to address the problem of uncertainty and group-based decisions in TISS. The structure of the paper is organized as follows: Fundamental structure of the decision model include: guidelines for options survey and excluding analysis to generate possible alternatives; descriptions on governing factors in TISS and their relationships; and structure of impact matrix. Subsequently, required fuzzy operators are outlined and summarized. Theoretical background and sample applications of FGAHP and FTOPSIS methods are discussed in advance within next section. A short illustrative hypothetical case on impoundment site selection is performed in terms of performing the proposed model. The paper is concluded with expressing the results on this issue.

## Development of a new decision framework

Decision-making about tailing dam location is one of the key tasks in mine planning. TISS framework should satisfy required features of the problem including: (1) Critical situation due to long-term effect of decision, unchangeable as well as one time event, and high risk on mine closure; (2) Complexity root in multiple conflicting criteria where high quality of answer is needed; (3) Uncertainties; and (4) Governing by different authorities. Therefore unlike in the past, the solution strategy should be fit to the problem appropriately. This paper found fuzzy multi attribute group decision-making (FMAGDM) methods as suitable tools to handle this problem. Steps of this decision framework are shown in Fig. 1. There are two decision-makers (DMs) of mine authorities with different tendencies as DMs who interested in increasing the productivity, decreasing the

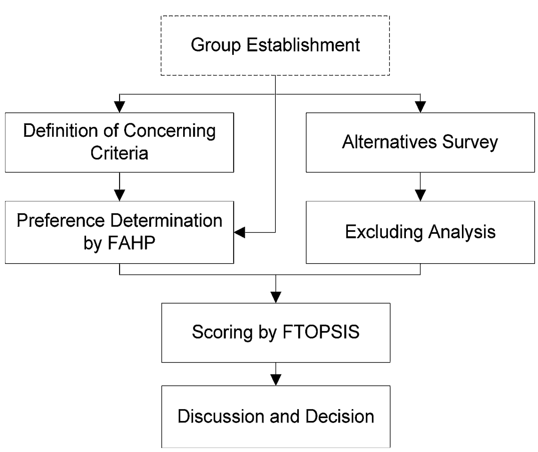


**Fig. 1** Structure of new TISS

costs, and maximizing the profit (DM A) and another DM relies on environmental sustainability (DM B). In the first step, concerning criteria about decision-making by DMs A and B are elucidated. Then, it refers to DMs to evaluate relative importance of attributes by FGAHP. Alternatives that are known in survey step, process by excluding analysis to achieve possible options. The final decision will be made by ranking the possible alternatives via FTOPSIS method and discussing about them.

## Options survey and excluding analysis

A radius of 10–50 km around the mill, depending on project specifics, should be considered in the alternative survey (Robertson and Moss 1981). Candidate site may be feasible for one or more embankment options: (1) valley impoundments; (2) ring dikes; (3) in-pit disposal or open pit mines; (4) specially dug pits; (5) underground mines or mine backfilling; and (6) deep lake disposal (Vick 1990). Therefore, it may be feasible to have more than one alternative in particular sites. Some attention should be given to the capacity of the identified sites. Because of a marginal zone of disturbance around each tailing dam, it is preferable to have only one impoundment of adequate capacity instead of a cluster of small ones to avoid multiply this effect so centralization the environmental management programs (Robertson and Moss 1981).

Some of alternatives may take the top of ranking while being undesirable cause including unacceptable measures of an attributes that had not been very important. Unfortunately this failure was disregarded in many trade-off decision-making researches when the possibility of alternatives has not been verified manually so implementation of related alternatives is impossible. In this paper, an excluding analysis is regarded to development a systematic approach free from this fault. Robertson et al. (1980) introduced unacceptable excluding regards to detect impracticable sites such as visual, land-use, ecological, airborne release, seepage release, stability release, and operational cost. These fatal-flaw items are configured the screening step, therefore all passing alternatives are *possible* sites and appointed to detail study.

## Governing factors in TISS

Sixty-one numbers of extracted leading evaluative attributes from literatures and advices by experts were categorized into five main clusters as criteria to form first hierarchical level of overall goal that is optimum tailing dam site (Robertson et al. 1980; Robertson and Moss 1981; Caldwell and Robertson 1983). The attributes level consists of technical (T), stability (St), project economics (Pe), environmental (E), and social-

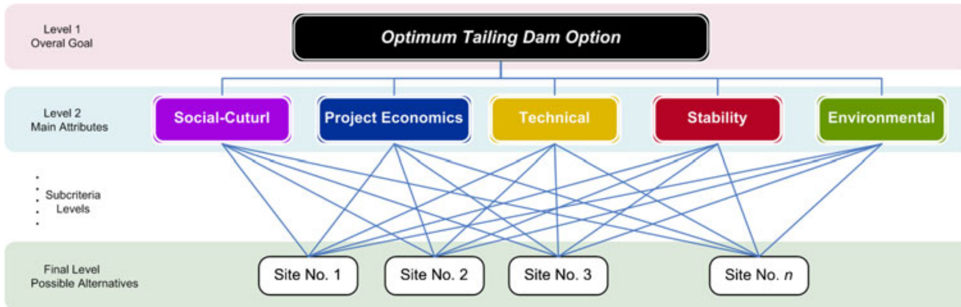


**Fig. 2** Structure of TISS hierarchy

economic (Se) factors (Fig. 2). Then, each main criterion was protracted to sub-attribute levels as described in Table 1. Note that the rest on Saaty and Ozdemir (2003), number or attributes in each hierarchy cluster should be less than seven plus or minus two.

## The TISS impact matrix

After constructing the hierarchy of TISS problem, to introduce MADM methods, candidate sites ($S_j$), attributes ($C_i$), and measures of performance of alternatives with respect to the attributes ($\tilde{x}_{i,j}$) must be established during an impact matrix (Fig. 3). Here, it is supposed that experts *A* and *B* on measures of performance in impact matrix have conjunct opinion. Impact matrix may consist of quantitative and qualitative measures of alternatives performance with respect to the attributes. In this case, the qualitative measures must be exchanged to quantities. For this attempt, as it has been shown in Table 2, fuzzy subjective measure scale has been used to exchange subjective measure of attribute.

## Fuzzy sets and fuzzy numbers

In many real-life cases of decision making, especially when the alternatives are related to qualitative aspects, it may be

**Table 1** Description of main attribute in sub-levels

| | | |
|---|---|---|
| (Pe) Project Economics | (E) Environmental | (Se) Social–economic |
| ($Pe_1$) Capital costs | ($E_1$) Air borne | ($Se_1$) Land-use |
| ($Pe_2$) Operational costs | ($E_{11}$) Exposure to prevailing winds | ($Se_{11}$) Recreation land-use |
| ($Pe_3$) Preservation costs | ($E_{12}$) Wind speed | ($Se_{12}$) Economic land-use |
| ($Pe_4$) Reclamation and closure cost | ($E_{13}$) Dump surface area | ($Se_{13}$) Ownership |
| ($Pe_5$) Economic risk | ($E_2$) Water contamination | ($Se_{14}$) Roads or other public facilities |
| ($Pe_6$) Reuse feasibility | ($E_{21}$) Underlying rock types | ($Se_{15}$) Proximity and size of human habitation |
| ($Pe_7$) Preparation costs | ($E_{22}$) Surface soils | |
| ($Pe_8$) Perfectibility and taxes | ($E_{23}$) Permeability of foundation | ($Se_2$) Special values |
| | ($E_{24}$) Groundwater discharge area | ($Se_{21}$) Archeological resource |
| (T) Technical | ($E_{25}$) Flood plain | ($Se_{22}$) Historic value |
| ($T_1$) Horizontal distance of pit | ($E_{26}$) Groundwater quality | ($Se_{23}$) Aesthetics |
| ($T_2$) Elevation change with respect to pit | ($E_{27}$) Depth of groundwater | ($Se_{24}$) Elevation |
| ($T_3$) Storage capacity | ($E_{28}$) Surface water quality | ($Se_{25}$) Scenic value |
| ($T_4$) Access from mine | ($E_{29}$) Stream crossed by delivery system | ($Se_{26}$) Sacred or traditional sites |
| ($T_5$) Disposal option possible | ($E_3$) Ecology | ($Se_3$) Visibility |
| ($T_6$) Availability of construction and reclamation materials | ($E_{31}$) Endangered species | ($Se_{31}$) Prominence |
| ($T_7$) Technical feasibility | ($E_{32}$) Wildlife habitat and quality | ($Se_{32}$) Proximity to public |
| ($T_8$) Expansion capability | ($E_{33}$) Fishery habitat and quality | ($Se_4$) Social |
| | ($E_{34}$) Sensitive or unique ecosystems | ($Se_{41}$) Transportation and traffic |
| (St) Stability | ($E_{35}$) Watershed quality | ($Se_{42}$) Navigable waters |
| ($St_1$) Topography | ($E_{36}$) Vegetation types | ($Se_{43}$) Housing |
| ($St_2$) Faults | ($E_{37}$) Area of disturbance | ($Se_{44}$) Labor market |
| ($St_3$) Upstream drainage area | | ($Se_{45}$) Government expenditures |
| ($St_4$) Strength and compressibility of upstream drainage area foundation | | ($Se_{46}$) Regional government development |
| ($St_5$) Proximity to perennial streams of rivers | | |
| ($St_6$) Location of flood plains | | |
| ($St_7$) Embankment height | | |

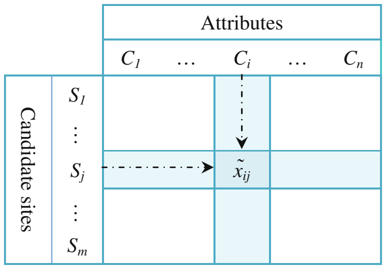


**Fig. 3** Representation of decision matrix (Hwang and Yoon 1981)

**Table 2** Fuzzy subjective measure scale after Durán and Aguilo (2008)

| Qualitative measure | Relative importance | Fuzzy symbol | Assigned value |
|---|---|---|---|
| Very low | Equal | $\tilde{1}$ | (1, 1, 1) |
| Low | Weak | $\tilde{3}$ | (2, 3, 4) |
| Average | Fairly strong | $\tilde{5}$ | (4, 5, 6) |
| High | Very strong | $\tilde{7}$ | (6, 7, 8) |
| Very high | Absolute | $\tilde{9}$ | (9, 9, 9) |
| Intermediate values | Intermediate values | $\tilde{2}, \tilde{4}, \tilde{6}, \tilde{8}$ | (a − 1, a, a + 1) |

difficult for a decision maker to specify exact attribute preference or quantify them using precise values. In fact, the uncertainty can be induced in two ways: incomplete data, and imprecise judgments. These kinds of uncertainty were expressed in the preference and performance values by the way of real values assessed in a predefined range. Soon other approaches based on interval values and linguistic labels were proposed (Sun and Li 2009). That means a decision maker can easily provide the form of linguistic variables by fuzzy numbers to express their preference or performance. The concepts of the fuzzy numbers and fuzzy set were introduced by Zadeh (1965). He presented ideas concerning fuzzy decision-making and fuzzy optimization. The applications of the fuzzy set theory in this study are elaborated in uncertainly of impact matrix entries and linguistic variable judgments of DMs. There are some types of fuzzy numbers such as triangular and trapezoid in shape of ($a_1$, $a_2$, $a_3$) and ($a_1$, $a_2$, $a_3$, $a_4$) respectively. In this study, uncertainties have been shown by triangular fuzzy number (TFN) because of its simplicity and wider implications. The TFN membership functions are usually described as Eq. (1):

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & a_1 < x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & a_2 < x \le a_3 \\ 0 & Otherwise \end{cases} \tag{1}$$

Some of utilized operations in the solution strategy are described below according to researches by Zadeh (1965), Zimmermann (1991), and Kaufmann and Gupta (1985):

Let $\tilde{a} = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3)$ and $\tilde{b} = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3)$ be two triangular fuzzy numbers, then:

a. Fuzzy numbers summation and difference are defined by:

$$\tilde{a} \pm \tilde{b} = (a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3) \tag{2}$$

b. Also, fuzzy numbers multiplication is defined by:

$$\tilde{a} \times \tilde{b} = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3) \tag{3}$$

c. On the other hand, fuzzy numbers division is defined as follows:

$$\frac{\tilde{a}}{\tilde{b}} = \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right) \tag{4}$$

d. Vertex method is defined to calculate the distance between them, according to Eq. (5):

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}\left[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2\right]} \tag{5}$$

## Priority derivation in the FGAHP

An essential feature of decision problems is the DM's preferences that refer to expressions of the relative importance of criteria. The Eigenvector method of AHP developed by Saaty (1980, 2000) probably is the best-known and most widely used in subjective weighing of attributes, because of its simplicity, easy determination of the maximum Eigen value, and reduction in inconsistency of judgments (Saaty 2000). Especially where the subjective judgments of different individuals constitute an important part of the Eigenvector method, it is a powerful weighing methodology in order to determine the priorities among different criteria. It decompose the decision-making problem into a system of hierarchies of objectives, attributes (or criteria), and alternatives. Hence, by breaking the problem into levels, the DM can focus on smaller sets of decisions (Venkata Rao 2007). Its power and flexibility cause to be the most highly regarded and widely used tool in various area of decision making problems, especially sustainable development issues such as reclamation (Bascetin 2007), landfill location (Şener et al. 2006; Banar et al. 2007; Gemitzi et al. 2007), and land use management (Lamelas et al. 2008; Soltanmohammadi et al. 2009). Though the aim of AHP is to capture a DM's knowledge, the conventional AHP can not fully reflect the human thinking style. Linguistic and vague descriptions could not be solved easily by AHP until the recent development in fuzzy decision-making (Cheng 1999). Besides, in most situations, the decisions are made by a decision group not only because of the opportunities at stake but also because of wider implications of the decision in terms of responsibility (Yu et al. 2009). By incorporating fuzzy set theory with AHP under multiple-expert position, FGAHP allows a more accurate description of the decision-making process. The steps of FGAHP method for determination of the criteria's priorities encompasses five steps as summarized as follows:

**Step 1:** Determine the objective and the evaluation attributes by decomposing the decision problem into a hierarchy with a goal at the top, criteria and sub-criteria at levels and sub-levels (Fig. 2).

**Step 2:** Construct the fuzzy judgment matrices. The DMs use the defined scale (Table 2) to assess the priority score. If the decision-making problem consists of $n$ criteria, and $K$ DMs, each fuzzy judgment matrix $(\tilde{A}^1, \tilde{A}^2, \ldots, \tilde{A}^k)$ take the form:

$$\tilde{A}^k_{n\times n} = \begin{matrix} & C_1 & C_2 & C_3 & \cdots & C_n \\ C_1 & \tilde{a}_{11k} & \tilde{a}_{12k} & \tilde{a}_{13k} & \cdots & \tilde{a}_{1nk} \\ C_2 & \tilde{a}_{21k} & \tilde{a}_{22k} & \tilde{a}_{23k} & \cdots & \tilde{a}_{2nk} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ C_n & \tilde{a}_{n1k} & \tilde{a}_{n2k} & \tilde{a}_{n3k} & \cdots & \tilde{a}_{nnk} \end{matrix},$$

$$\tilde{a}_{nnk} = 1,\ \tilde{a}_{jik} = 1/\tilde{a}_{ijk},\ \tilde{a}_{ijk} \neq 0 \tag{6}$$

where, $a_{ijk}$ denotes the comparative importance of attribute $i$th with respect to attribute $j$th in function of $k$th DM opinion.

**Step 3:** Establish aggregate matrix $(\tilde{P}_{ij})$ by mix the various judgment matrices from all DMs. Weighted geometric mean for average of individual judgments (AIJ) method under unequal relative importance about $k$th group's members $(\beta_k)$ could be formulated as follow (Sun and Li 2009):

$$\tilde{P}_{ij} = \left[\prod_{k=1}^{K} \tilde{a}_{ijk}^{\beta_k}\right]^{1/\sum \beta_k} \tag{7}$$

As a rule, for group decision-making with multiple experts, there are two basic ways to obtain group priorities in AHP, the AIJ and aggregation of individual priorities (AIP) method (Escobar and Moreno Jiménez 2007). The choice of method depends on whether the group is assumed to act together as a unit or as separate individuals. In the former case, the geometric average of individual judgments, AIJ, satisfies the reciprocity requirement which it involved in the TISS problem, implying a synergistic aggregation of individual preferences in such a way that the group becomes a new individual and behaves like one (Forman and Peniwati 1998).

**Step 4:** Find the fuzzy relative normalized weight $(\tilde{V}_j)$ of each attribute by calculating the geometric mean of the $j$th row as follow:

$$\tilde{V}_j = \left[\prod_{i=1}^{n} \tilde{p}_{ij}\right]^{1/n} \tag{8}$$

And then normalize the geometric means of rows in the comparison matrix. This can be represented as:

$$\tilde{w}_j = \frac{\tilde{V}_j}{\sum_{j=1}^{m} \tilde{V}_j} \tag{9}$$

Therefore a set of $n$ fuzzy weights, $\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n$, is obtained.

**Step 5:** Do consistency analysis to ensure of results accuracy by following:

$$\tilde{A} \times \tilde{w}_j = \lambda_{\max} \times \tilde{w}_j,\ i = 1, 2, \ldots, n \tag{10}$$

Then consistency index ($CI$) is calculated as:

$$CI = \frac{\lambda_{\max} - n}{n - 1} \tag{11}$$

The final item of Step 5 that has to be calculated is consistency ratio ($CR$). The formulation of $CR$ is:

$$CR = \frac{CI}{RI} \tag{12}$$

If the $CR$ is less than 10%, the comparisons are acceptable, otherwise not. $RI$ represents the average index for randomly generated weights. The table of random indexes of the matrices of order 1–15 can be seen in Saaty (1980).

Since $\lambda_{\max}$ is a triangular number, it has to be defuzzified into a crisp number to compute the $CI$. Durán and Aguilo (2008) suggest central value of $\lambda_{\max}$ to bypass the defuzzification procedure because of the symmetry of the triangular number; the central number corresponds to defuzzified value appropriately.

## Discrimination of the sites by FTOPSIS

The FTOPSIS method is an extension of fuzzy set theory on Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method which was developed by Hwang and Yoon (1981). They explained that a multiple criteria decision-making problem may be viewed as a geometric system. The $m$ alternatives that are evaluated by $n$ attributes are similar to $m$ points in an $n$-dimensional space. This method is based on the concept that the chosen alternative should have the shortest Euclidean distance from the *ideal* solution which consists of all best data attainable, and the farthest from the *worst* solution that consists of all worst data attainable. The advantages of using this concept have been highlighted by (1) its intuitively appealing logic, (2) its simplicity and comprehensibility, (3) its computational efficiency, (4) its ability to measure the relative performance of the alternatives with respect to individual or all evaluation criteria in a simple mathematical form, and (5) its applicability in solving various practical MCDM problems (Yeh and Chang 2009; Shih et al. 2007). Besides, according to the simulation comparison from Zanakis et al. (1998), TOPSIS has the fewest rank reversals among the eight methods in the category. Despite its merits in comparison with other MCDM methods, the TOPSIS method does not consider the relative importance (weight) of the distances from the positive and the negative ideal solutions (Yeh and Chang 2009). This drawback has been tackled in this paper by cross-breeding into the FGAHP method to provide predefined weights. Thus, the FTOPSIS is chosen as ranking tool. The main procedure of the FTOPSIS method for the selection of the best alternative from among those available is described below:

**Step 1:** The impact matrix components ($\tilde{x}_{ij}$) need to be normalized in order to obtain the dimensionless environment ($\tilde{r}_{ij}$) for inter-attribute comparison by Eq. (13).

$$\tilde{r}_{ij} = \frac{\tilde{x}_{ij}}{\sqrt{\sum_{i=1}^{m} \tilde{x}_{ij}^2}} \tag{13}$$

**Step 2:** A set of weights $\{\tilde{W}_j\}$, where $\tilde{w}_j$ is the weight of the *j*th attribute (according to results of previous section), is incorporated to form a weighted normalized impact matrix ($\tilde{V}$) by Eq. (14).

$$\tilde{v} = [\tilde{v}_{ij}] = [\tilde{w}_j \tilde{r}_{ij}] \;,\; i = 1,2,\ldots,m \;,\quad j = 1,2,\ldots,n \tag{14}$$

**Step 3:** Obtain the fuzzy positive ideal solution (*FPIS*) and fuzzy negative ideal solution (*FNIS*) through Eqs. (15) and (16).

$$FPIS_j = \{(\max_j \tilde{V}_{ij} | i \in I), (\min_j \tilde{V}_{ij} | i \in I') | j = 1,2,\ldots,n\} \tag{15}$$

$$FNIS_j = \{(\min_j \tilde{V}_{ij} | i \in I), (\max_j \tilde{V}_{ij} | i \in I') | j = 1,2,\ldots,n\} \tag{16}$$

where $I$ and $I'$ represent benefit and cost attributes, respectively.

**Step 4:** Evaluate the separation measures. The separation of each alternative from the ideal one is given by the Euclidean distance in the following equations:

$$PD_i = \sum_{j=1}^{n} d(\tilde{v}_{ij}, FPIS_j), \quad i = 1,2,\ldots,m \tag{17}$$

$$ND_i = \sum_{j=1}^{n} d(\tilde{v}_{ij}, FNIS_j), \quad i = 1,2,\ldots,m \tag{18}$$

The $PD_i$ and $ND_i$ represent crisp Euclidean distance from fuzzy positive and negative ideal solution respectively.

**Step 5:** The relative closeness of a particular alternative to the ideal solution ($CC_i$) can be expressed in this step as follows:

$$CC_i = \frac{ND_i}{ND_i + PD_i} \tag{19}$$

Finally, the alternatives can be ranked according to descending order and the one with the maximum overall or composite performance score is the most preferable solution. Scope of obtained scores of alternatives is relative to 0 and 100% for hypothetical negative and positive ideal solutions, respectively.

## A short illustrative example

In order to benchmark the proposed approach, a numerical example is illustrated herein. It is assumed that a company is looking for selecting a location to build a new tailing dam in five alternative sites. With consensus of two DMs, entries of impact matrix obtained in form of linguistic terms. Linguistic verbal measures have been converted to fuzzy numbers to building the numerical impact matrix based on Table 2. Due to large size of data, it is illustrated by a broken matrix in Table 3. Some of entries are benefit attributes and others are cost as superscripted with plus and minus in impact matrix, respectively.

According to the FGAHP, preferences of DMs on attributes started with establishing thirteen individual judgments matrix [Eq. (2)] for each DM, respect to every sub-criteria cluster which indicated in Table 1. The AIJ method [Eq. (7)] is desirable to aggregate individual opinions of DMs. Aggregation of different DMs judgments done among their equal values ($\beta_A = \beta_B$) assumption. Afterwards, the local weights of attributes are estimated according to Eqs. (8) and (9). Thus, by multiplying the local weights in upper level ones, global weights are generated. In the last step of FGAHP, inconstancy ratios of judgments are verified. Table 4 represents judgments of DMs, fuzzy local weights, and inconsistency check results for main level of attributes as a synopsis.

In the same way, the outcomes of FGAHP for all attributes in lowest level of clusters are shown in Fig. 4. Note that for understanding the role of each main attribute in ranking results, cumulative effect of its cluster's population should be regarded as well. This concept is demonstrated in Fig. 5 with reference to each DM's opinions. Therefore, the developed TISS model can aggregate incompatible opinions of DMs to make final traded-off weights as both regards are satisfied.

After identification attributes weights, ranking process via FTOPSIS put forward. First, fuzzy impact matrix normalized by Eq. (13) and then weighted normalized impact matrix is constructed (Eq. 14). The fuzzy positive ideal solution, *FPIS*, and the fuzzy negative ideal solution, *FNIS*, can be found using Eqs. (15) and (16) so theirs separation

**Table 3** Impact matrix of case study in shape of fuzzy numbers

| Criteria | | | Alternatives | | | | |
|---|---|---|---|---|---|---|---|
| Main Level | Sub L. 1 | Sub L. 2 | Site 1 | Site 2 | Site 3 | Site 4 | Site 5 |
| Social-economic (Se) | Se1 | $^{-}$Se11 | (4,5,6) | (9,9,9) | (2,3,4) | (6,7,8) | (4,5,6) |
| | | $^{-}$Se12 | (4,5,6) | (6,7,8) | (4,5,6) | (4,5,6) | (2,3,4) |
| | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| | Se4 | $^{+}$Se46 | (5,6,7) | (2,3,4) | (6,7,8) | (4,5,6) | (2,3,4) |
| Environmental (E) | E1 | $^{-}$E11 | (2,3,4) | (4,5,6) | (2,3,4) | (9,9,9) | (2,3,4) |
| | | $^{-}$E12 | (4,5,6) | (6,7,8) | (1,1,1) | (3,4,5) | (2,3,4) |
| | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| | E3 | $^{-}$E37 | (3,4,5) | (1,1,1) | (4,5,6) | (6,7,8) | (6,7,8) |
| Stability (St) | $^{-}$St1 | | (2,3,4) | (4,5,6) | (4,5,6) | (2,3,4) | (1,1,1) |
| | $^{-}$St2 | | (3,4,5) | (6,7,8) | (9,9,9) | (1,2,3) | (1,1,1) |
| | ⋮ | | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| | $^{-}$St7 | | (4,5,6) | (6,7,8) | (9,9,9) | (1,2,3) | (4,5,6) |
| Project-economics (Pe) | $^{-}$Pe1 | | (6,7,8) | (9,9,9) | (2,3,4) | (5,6,7) | (2,3,4) |
| | $^{-}$Pe2 | | (4,5,6) | (6,7,8) | (1,1,1) | (6,7,8) | (3,4,5) |
| | ⋮ | | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| | $^{-}$Pe8 | | (3,4,5) | (9,9,9) | (1,2,3) | (2,3,4) | (5,6,7) |
| Technical (T) | $^{-}$T1 | | (6,7,8) | (9,9,9) | (2,3,4) | (1,1,1) | (6,7,8) |
| | $^{-}$T2 | | (3,4,5) | (7,8,9) | (2,3,4) | (1,1,1) | (6,7,8) |
| | ⋮ | | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| | $^{+}$T8 | | (4,5,6) | (1,1,1) | (9,9,9) | (9,9,9) | (5,6,7) |

$^{+}$ Benefit attribute, $^{-}$ cost attribute

**Table 4** Judgments of DMs, fuzzy local weights, and inconsistency check results for main level of attributes

| | DM A: Productive | | | | | DM B: Environmentalist | | | | | Local aggregated weight (%) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Pe | E | T | St | Se | Pe | E | T | St | Se | |
| Pe | $\tilde{1}$ | $\tilde{6}$ | $\tilde{2}$ | $\tilde{5}$ | $\tilde{7}$ | $\tilde{1}$ | $\widetilde{1/4}$ | $\tilde{2}$ | $\widetilde{1/2}$ | $\widetilde{1/3}$ | (15.9, 27.8, 47.2) |
| E | $\widetilde{1/6}$ | $\tilde{1}$ | $\widetilde{1/5}$ | $\widetilde{1/2}$ | $\tilde{2}$ | $\tilde{4}$ | $\tilde{1}$ | $\tilde{5}$ | $\tilde{3}$ | $\tilde{2}$ | (14.0, 22.4, 38.9) |
| T | $\widetilde{1/2}$ | $\tilde{5}$ | $\tilde{1}$ | $\tilde{4}$ | $\tilde{6}$ | $\widetilde{1/2}$ | $\widetilde{1/5}$ | $\tilde{1}$ | $\widetilde{1/3}$ | $\widetilde{1/4}$ | (11.1, 18.2, 31.5) |
| St | $\widetilde{1/5}$ | $\tilde{2}$ | $\widetilde{1/4}$ | $\tilde{1}$ | $\tilde{3}$ | $\tilde{2}$ | $\widetilde{1/3}$ | $\tilde{3}$ | $\tilde{1}$ | $\widetilde{1/2}$ | (9.50, 17.3, 30.3) |
| Se | $\widetilde{1/7}$ | $\widetilde{1/2}$ | $\widetilde{1/6}$ | $\widetilde{1/3}$ | $\tilde{1}$ | $\tilde{3}$ | $\widetilde{1/2}$ | $\tilde{4}$ | $\tilde{2}$ | $\tilde{1}$ | (8.10, 14.4, 22.7) |

For aggregated matrix: $\lambda_{max} = 5.050$, CI $= 0.012$, and CR $= 0.01 \leq 0.1$ (RI $= 1.12$ for $n = 5$)

measures from every alternative accomplished. In final step of FTOPSIS, relative closeness of a particular alternative to the ideal solution, $CC_i$, is expressed by Eq. (19). See final ranking result and cumulative portion of each main criterion in Fig. 6.

Based on results from the TISS method, *Site 3* with the score of 71.7% is the best alternative for designing tailing impoundment in this case. Figure 6 reflect useful information about decision-making responses; *Site 3* interested in technical, project economic, and social-economic characters and should be included as main reasons of top ranking situation. However essential treatment efforts about probable stability problems should be considered.

## Conclusions

The TISS framework is a technique for evaluating limited number of feasible alternatives to select the best one under uncertain environment. The consideration of the objective,

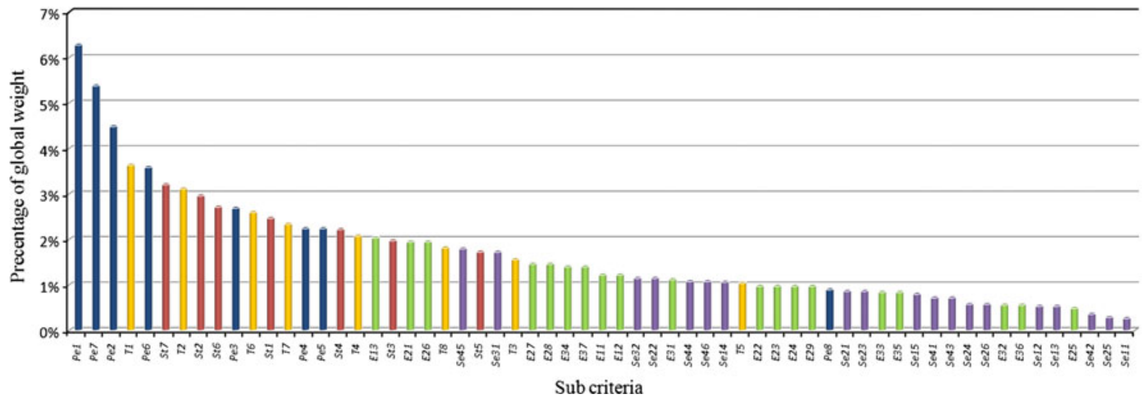


**Fig. 4** Global weights of the sub criteria calculated using FGAHP method

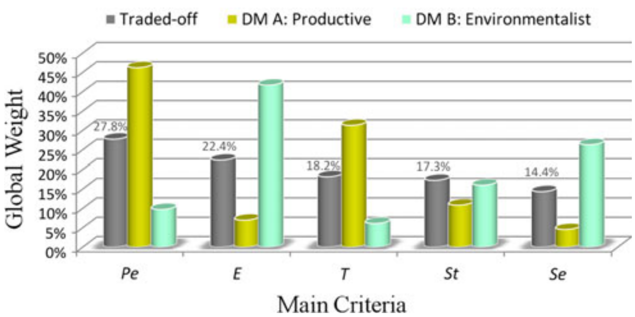


**Fig. 5** Global weights of the main criteria calculated using FGAHP method

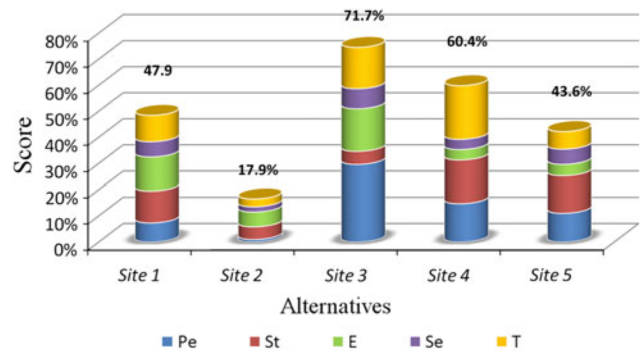


**Fig. 6** Preference order of alternatives by TISS method

subjective and critical factors can be more acquaint with the adoption of the mathematical model with the real-world problems. Facilities of the both quantitative and qualitative type of inputs powered by fuzzy definition provide a wide range of application in various levels of evaluation from preliminary rule-of-thumb assessments to ultimate decision makings. Besides applying the 49 governing factors in TISS model make it a comprehensive tool to support all aspects of decision problem. The modeling of the impoundment site selection problem through the TISS framework provided a common language between difference authorities with opposite views. So its analytical decision support structure could provide the logical justifying of the made decisions for those in charge in the project. The partial evaluation on five main attributes can address strength and weakness points of each site; so the TISS remarks concerns of particular site based on obvious results.

The solution procedure was illustrated through a numerical example. At the beginning of the process the DMs had very strong opinions about the goodness of each alternative. These opinions, however, were based on only one or two points of view; namely economical versus environmental ways of thinking. This seemed to create discussion between DMs with different right opinions in the beginning. The multidimensional consideration of the problem broadened their views, decreasing the conflict.

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