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Prediction of the blastability designation of rock masses using fuzzy sets

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ABSTRACT

The main objective of rock blasting design is to achieve a balance among optimum powder factor, proper fragmentation, throws, ground vibration, etc. The in-situ rock mass properties are among the most important contributory factors in fragmentation. The term blastability is used to indicate the susceptibility of the rock mass to blasting and its characterization has become a pressing task for blasting operations. Several approaches have been used for estimating blastability. Despite their widespread use in practice, they have some common deficiencies leading to uncertainties in their practical applications through sharp transitions between two adjacent rating classes and the subjective uncertainties on data, which are close to the range boundaries of rock classes. In this study, the fuzzy set theory was applied to blastability designation (BD) classification systems. Furthermore, a new methodology in terms of ''Effective Rules'' is developed in construction of rule base part of the Mamdani fuzzy inference system structure, to efficiently solve fuzzy inference systems with a large number of fuzzy rules (e.g. nearly 400,000 rules). In comparison with the conventional methods, it was seen that the fuzzy model operated more consistently. Moreover, it was shown that the fuzzy set theory could effectively overcome the uncertainties encountered in the practical applications of conventional classification systems.

1. Introduction

Blasting is the most common method used for quarrying, mining and civil constructions and rock excavation. In all these activities the quality of rock mass fragmentation (degree and size distribution), because of its direct impact on efficiency of rock loading and crushing operations, is the major concern of any blasting operation. A blasting operation can be comprehensively described by intact rock and rock mass properties (the concern of this paper), explosive properties, blasting geometry or pattern and initiation sequences, etc.

The influence of in-situ rock mass property on blasting operations has long been studied by different researches [1–8], and it has been pointed out that it is one of the most important parameters influencing rock fragmentation. This influence is referred to as the blastability of a rock mass and its characterization is a pressing task for blasting operation. Blastability is a composite intrinsic property of a rock mass that represents the ease with which a rock mass can be fragmented by blasting [1].

Due to the complexity of the blasting process and the large number of involved parameters, approaches made for the determination of blastability are essentially empirical.

Fraenkel [9] proposed an empirical relationship for blastability based on blasthole and design parameters (height and diameter of the charge, hole depth and maximum burden). According to his equation, blastability is related to amount of charge. Hino [10] found that the number of slabs produced by tensile slabbing is related to tensile and compressive strength of rock and amplitude of the compressive stress wave. He named the ratio of compressive strength of rock to its tensile strength as blasting coefficient. Sassa and Ito [11] suggested the Rock Breakage Field Index (RBFI) and the Rock Breakage Laboratory Index (RBLI), by regression analysis of mechanical properties of rock measured in the laboratory and crack frequency studies at blast site. Heinen and Dimock [12] proposed a graphical method for assessment of blastability index according to seismic propagation velocity in rock mass. Based on the Pierce equation for burden calculation, Borquez [13] developed a blastability factor (KV) using Rock Quality Designation (RQD) and an alteration factor (indicating joint tightness and type of filling). Leighton et al. [14] and Lopez Jimeno [15] developed similar equations for determination of powder factor considering the information obtained from rock mass drillability quality and drilling parameters. Rakishev [16] devised five blastability classes according to the value of an index

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named critical fracture velocity. The critical fracture velocity can be obtained as a function of six geotechnical characteristics (density, longitudinal wave velocity, Poisson's ratio, elastic modulus, compressive and tensile strength of rock mass), and two structural parameters (dimension of a natural structure unit and the properties of fractures fillings and opening). Lilly [17] developed a blastability index based on rock mass description, joint density and orientation, specific gravity and hardness. This index relates to powder factor by a site-specific equation. Ghose [7] proposed a "blastability rating chart" similar to Bieniawski's geomechanics classification system of rock mass. It is categorized in five blastability classes and each class correlated with the powder factor in case of coal mine. Julius Kruttschnitt Mineral Research Center (JKMRC) approaches to classify the blastability of coal measure strata according to intact rock, structural feature of rock mass, blast design and environment parameters [8]. Latham and Lu [1] proposed a blastability designation (BD), as a part of their developed Energy-Block Transition (EBT) model, for prediction of Blasted Block Size Distribution (BBSD). This model is developed based on rock engineering systems (RES) approach and consideration of a comprehensive range of intact rock properties and discontinuity structures [1]. A feed forward back propagation neural network was developed by Han et al. [18] to classify rock mass blastability. The input vector consists of six parameters characterizing the structure of rock mass, strength of rock and fragmentation degree of blasting. The applied neural network output is a single vector denoting the rank of rock mass blastability.

Despite the usefulness and widespread use of rock classification systems (the concern of this paper) in the field of rock engineering and rock blasting, the existing conventional classification systems have some common practical deficiencies. As mentioned by Aydin [19], certain subjective uncertainties are encountered in rock classification systems. They are resulting from: (i) use of linguistic terms as input value of some parameters; (ii) predetermined and sharp class boundaries, whereas the rock mass quality is gradational in nature; (iii) prescribed rating scales representing contribution of each criterion to the overall quality; and (iv) reliability of input value of each parameter. Similar uncertainties are encountered when these systems are employed for determination of blastability classes, which will be discussed in detail later in the paper.

As the fuzzy models can cope with the complexity of complicated and ill-defined systems in a flexible and consistent way, in the last two decades an increase in their applications to solve various problems in the field of mining geomechanics has been observed [3,4,19–32].

Nguyen [3]and Nguyen and Ashworth [4] proposed the first fuzzy rock mass classification approach. Ghose and Dutta [20] outlined a new classification model of roof strata cavability using fuzzy set methodology and linguistic variables. Den Hartog et al. [21] and Grima and Verhoef [22] used the fuzzy logic approach to predict the performance and bit consumption of rock cutting trenchers. Cebesoy [23] and Bascetin [24] used a fuzzy system for the selection of optimum equipment combination in surface mines. Recently many researchers employed the Mamdani fuzzy inference system on various problems in the field of mining and geomechanics. For instance for prediction of uniaxial compressive strength of Ankara agglomerates, Geological Strength Index (GSI), the possibility of sinkhole occurrences over abandoned mines, deformation modulus of rock masses, rock mass diggability, rippability and excavability and the fragmentation of bench blasting in an iron open pit mine [25–32]. These theoretical aspects together with practical field studies [1] have been used to develop a blastability classification system that is believed to hold significant advantages of the mentioned uncertainties over existing systems.

2. The blastability index rating method

Latham and Lu [1] developed the EBT model for BBSD prediction based on relating the area between the two curves of in-situ block size distribution (IBSD) and BBSD to the energy consumed in transforming bigger blocks to smaller ones. In addition, through this model, a blastability designation (BD) was designed and employed, which reflects the intrinsic resistance of the rock mass to blasting. In this study "Blastability Designation" rating method, proposed by Latham and Lu [1], was adopted as the reference blastability classification system of rock [1].

The considered parameters in the system fall into two groups. The first group is the intact rock properties, which includes strength, hardness, elasticity, deformability and density of rock,

Table 1
Suggested quantitative indications for the classification of the blastability of a rock mass associated with individual factor [1].

P_i	Description of ease of blasting		Blastability class				
	Factors affecting blastability	Depicting parameter	VE* 1	E** 2	M*** 3	D**** 4	VD***** 5
P_1	Strength	$UCS^{(a)}$ (MPa)	<25	25–60	60–100	100–180	180–> > 180
		$Is(50)^{(b)}$ (MPa)	<1	1–2.5	2.5–4	4–9	9–> > 9
P_2	Resistance to fracturing	$UTS^{(c)}$, (MPa)	<1.5	1.5–3	3–6	6–12	12–> > 12
P_3	Sturdiness of rock	$\rho^{(d)}$ (t/m^3)	<2.0	2.0–2.4	2.4–2.75	2.75–3.0	3.0–> > 3.0
P_4	Elasticity of rock	$E^{(e)}$ (GPa)	<25	25–50	50–100	100–150	150–> > 150
P_5	Resistance to dynamic loading	V_p $^{(f)}$ (km/s)	<1.5	1.5–2.5	2.5–3.0	3.0–4.0	4.0
P_6	Hardness of rock	$SHV^{(g)}$	<15	15–30	30–40	40–50	50–> > 50
P_7	Deformability	$\upsilon^{(h)}$	0.35–> > 0.35	0.3–0.35	0.25–0.30	0.25–0.20	<0.20
P_8	Resistance to breaking	$k_{Ic}^{(i)}$ (MPa m$^{1/2}$)	<0.5	0.5–1.5	1.5–2.5	2.5–3.5	3.5–> > 3.5
P_9	In-situ block sizes	*Mean IBSD* (m)	<0.25	0.25–0.75	0.75–1.5	1.5–2.5	2.50–> > 2.50
		Mean spacing (m)	<0.1	0.1–0.5	0.5–1.5	1.5–2.5	2.50–> > 2.50
P_{10}	Fragility of rock mass	$D^{(j)}$ (MPa m$^{0.5}$)	<1.50	1.50–2.00	2.00–2.50	2.50–2.75	2.75–> > 2.75
P_{11}	Integrity of rock mass	R_υ $^{(k)}$	<0.35	0.35–0.55	0.55–0.75	0.75–0.9	0.90–> > 0.90
		RQD (%)	<40	40–60	60–75	75–90	90–> > 90
P_{12}	Discontinuity plane's strength	$C^{(l)}$(MPa)	<0.05	0.05–0.15	0.15–0.25	0.25–0.50	0.50–> > 0.50
		$\varphi^{(m)}$ (deg.)	<7.5	7.5–15	15–20	20–30	30–> > 30

(*) very easy; (**) easy; (***) moderate; (****) difficult; (*****) very difficult; (a) uniaxial compressive strength; (b) point-load strength index; (c) uniaxial tensile strength; (d) density; (e) elastic modulus; (f) *P*-wave velocity; (g) Schmidt hardness value; (h) Poisson's ratio; (i) fracture toughness of rock; (j) fractal dimension of in-situ rock mass; (k) ratio of *P*-wave in field to that in laboratory; (l) cohesion; (m) friction angle.

etc. They are dependent on rock texture, internal bonds, composition and distribution of minerals forming the rock. The second group is the discontinuity structure consisting of orientation, spacing and extent of discontinuities, and IBSD, which are created by long-term geological processes [1]. So the assessment of blastability of the rock mass could be made according to the following equation, which forms the basis for the proposed blastability classification chart (Table 1) [1]:

$$BD=\sum_{j=1}^{n} W_j R_j \quad (1)$$

where BD = the blastability designation that collectively quantifies the resistance to fragmentation by blasting of a rock mass; j = index; R_j = the rating value of the jth factor obtained from Table 1 (a value between 0 and 1 according to jth factor class); W_j = the weighting coefficient of jth factor can be determined from rock engineering systems and interaction matrix approach. It is obvious that the value of BD is in the range 0–1 (Table 2) and that the greater the BD, the more the difficulty in blasting the rock [1].

As mentioned before, there are some common deficiencies existing in practical applications of conventional classification systems. A precise examination of Table 1 reveals that there are some uncertainties on data close to the range boundaries of rock classes. For instance, it is not clear whether a rock having a uniaxial compressive strength of 100 MPa should be included in class 3 or 4, leading to subjective decision-making. The other parameters in Table 1 are also related to this type of uncertainty.

Sometimes, uncertainty arises from the fixed numerical score rating on each input parameter for a given rock class interval. In other words, the same numerical scores were applied in the regions of both the lower and upper boundaries of class intervals. This may result in similar situations. Consider two hypothetical rock masses in Table 3. Let us suppose the parameters of rock masses 1 and 2 to be close to the lower and upper boundaries of rock class interval (Table 1). Rating each input parameter based on average class value method (described in Section 4.2), a situation is reached where the same blastability is attributed for both rock masses. However, from the point of view of an experienced field engineer, it is expected that the quality of rock mass 2 is much more than that of rock mass 1. This type of uncertainty can be evaluated by using continuous rating charts to some extent.

Another deficiency, which is common in the conventional classification schemes, is the existence of sharp transition between two adjacent classes (Tables 1 and 2), because transition between rock classes is not so sharp but gradational in the field. This type of uncertainty is significantly important in the conventional classification systems such as RMR, Q, diggability, rippability and so on, in which according to the corresponding class of each index, some design parameters and construction facilities will be prescribed. For example, in Table 4 the determining RMR values between Rock mass class I and Rock mass class II are 81 and 80, respectively. Consequently, just for the rating difference of only 1 (i.e. 81–80), the average stand-up time of an 8 (m) span for a tunnel roof extends from 6 months to 10 years with possibility of widening the span from 8 to 15 (m) [33]. As the value of BD would not be used in obtaining design parameters or selection of construction facilities, this type of uncertainty is not an important concern here. When dealing with input parameters in Table 1 for BD calculation, we will be faced with this type of uncertainty.

Table 2
Blastability classification according to the E-B-T coefficient B_i and BD [1].

Description of ease of blasting	VE (a)	E (b)	M (c)	D (d)	VD (e)
Blastability class	1	2	3	4	5
BD	<0.25	0.25–0.50	0.50–0.70	0.70–0.85	>0.85

(a) very easy; (b) easy; (c) moderate; (d) difficult; (e) very difficult.

Table 3
Comparison between the two different rock masses in terms of blastability.

Classification parameter	Weight, W_i	Rock mass properties		Ratings	
		Rock mass 1	Rock mass 2	Rock mass 1	Rock mass 2
UCS (MPa)	0.1475	110	170	0.75	0.75
UTS (MPa)	0.1344	7	11	0.75	0.75
E (GPa)	0.1273	25	45	0.5	0.5
ρ (t/m^3)	0.1249	2.76	2.95	0.75	0.75
SHV	0.1225	41	49	0.75	0.75
V_p (m/s)	0.1208	3100	3900	0.75	0.75
D	0.1131	2.10	2.45	0.5	0.5
$MIBS$ (m)	0.1095	0.8	1.4	0.5	0.5
BD	–	–	–	0.66	0.66
Ease of BD	–	–	–	*M*	*M*

Table 4
Design parameters and engineering properties of rock mass [33].

Properties of rock mass	Rock mass rating (rock class)				
	100–81 (I)	80–61 (II)	60–41 (III)	40–21 (IV)	<20 (V)
Classification of rock mass	Very good	Good	Fair	Poor	Very poor
Average stand-up time	10 years for 15-m span	6 months for 8-m span	1 week for 5-m span	10 h for 2.5-m span	30 min for 1-m span
Cohesion of rock mass (MPa)	>0.4	0.3–0.4	0.2–0.3	0.1–0.2	<0.1
Angle of internal friction of rock mass	$>45^\circ$	35–45°	25–35°	15–25°	15°

Finally, the above mentioned uncertainties encountered in the practical application of conventional rock blastability classification systems can be processed by using the fuzzy set theory, which enables a soft approach to handle such uncertainties [22,29].

3. Fuzzy set theory

The fuzzy set theory was introduced in 1965 by Zadeh [34] as a mathematical way to represent linguistical vagueness. In the classical set theory, a given element either belongs or does not belong to a set. The membership of an element is crisp (0,1) and an '*A*' crisp set of real objects is described by a unique membership function such as X_A in Fig. 1. On the other hand, in fuzzy set theory a membership function that can vary from 0 to 1 is specified in Fig. 1. That is, the transition from 'belong to a set' to 'not belong to a set' is gradual and this is characterized by a membership function. This particular characteristic of fuzzy membership functions provides a robust mathematical tool to handle nonlinear and complex problems smoothly, imitating the human brain when managing inexact information. In addition, fuzzy set theory can be used for developing rule-based models, which combine physical insights, expert knowledge and numerical data in a transparent way that closely resembles the real world.

Fuzzy set theory provides a systematic calculus to deal with linguistic information, and it performs numerical computation by using linguistic labels stipulated by membership functions [35]. Moreover, fuzzy "if–then" rules form the key component of a Fuzzy Inference System (FIS), that can effectively model human expertise in a specific application.

3.1. Fuzzy if–then rules

To infer in a rule based fuzzy model, the fuzzy proposition needs to be represented by an implication function. The implication function is called fuzzy "if–then" rule. A fuzzy if–then rule, also known as the fuzzy rule, assumes the form "if *x* is *A* then *y* is *B*" where *A* and *B*, are linguistic values defined by fuzzy sets on universes of discourse *X* and *Y*, respectively. Often "*x* is *A*" is called the antecedent or premise, while "*y* is *B*" is called the consequence or conclusion. Examples of fuzzy if–then rule are widespread in daily linguistic expressions such as "If pressure is high, then volume is small" [35].

Each rule in a fuzzy model is a relation such as $R_i=(X\times Y\rightarrow[0,1])$, which is calculated using the following equation [36]:

$$\mu_{Ri}(x,y)=I(\mu_{Ai}(x),\mu_{Bi}(y)) \tag{2}$$

where $\mu_{Ri}(x,y)$ is the *R* relation's membership degree of rule "*i*" according to "*x*" and "*y*" inputs; $\mu_{Ai}(x)$ and $\mu_{Bi}(x)$ are the membership degrees of "*x*" and "*y*" inputs, respectively; and "*I*" denotes the "*and*" or "*or*" operator [29]. Most rule-based systems involve more than one rule. The process of obtaining the overall consequent from the individual consequents contributed by each rule in the rule base is known as aggregation of rules. There are two aggregation strategies, namely conjunctive system of rules and disjunctive system of rules [37]. In the case of conjunctive system of rules that must be jointly satisfied, the rules are connected by "*and*" connectives. In this case aggregated output, *y*, membership function is

$$\mu_y(y)=\min(\mu_y^1(y),\mu_y^2(y),\ldots,\mu_y^r(y)) \quad \text{for } y\in Y \tag{3}$$

Furthermore, in the case of disjunctive system of rules where the satisfaction of at least one rule is required, the rules are connected by "*or*" connectives. In this case aggregated output, *y*, membership function is

$$\mu_y(y)=\max(\mu_y^1(y),\mu_y^2(y),\ldots,\mu_y^r(y)) \quad \text{for } y\in Y \tag{4}$$

3.2. Fuzzy inference system

The FIS is a popular computing framework based on the concepts of fuzzy set theory, fuzzy if–then rules and fuzzy reasoning. FISs have been successfully applied in fields such as automatic control, data classification, decision analyses, expert systems and computer vision [35,37].

The basic structure of an FIS consists of three conceptual components: a rule (knowledge) base, which contains the selection of rules; a database, which defines the membership functions used in the fuzzy rules; and a reasoning mechanism, which performs the inference procedure upon the rules and given facts to derive a reasonable output or conclusion (Fig. 2). Basic FIS can take either fuzzy inputs or crisp inputs, but the outputs it produces are almost always fuzzy sets. In cases where a crisp value is needed, defuzzification method should be carried out. There are several FISs that have been employed in various applications. The most commonly used include: the Mamdani fuzzy model; the Takagi–Sugeno–Kang fuzzy (TSK) model; the

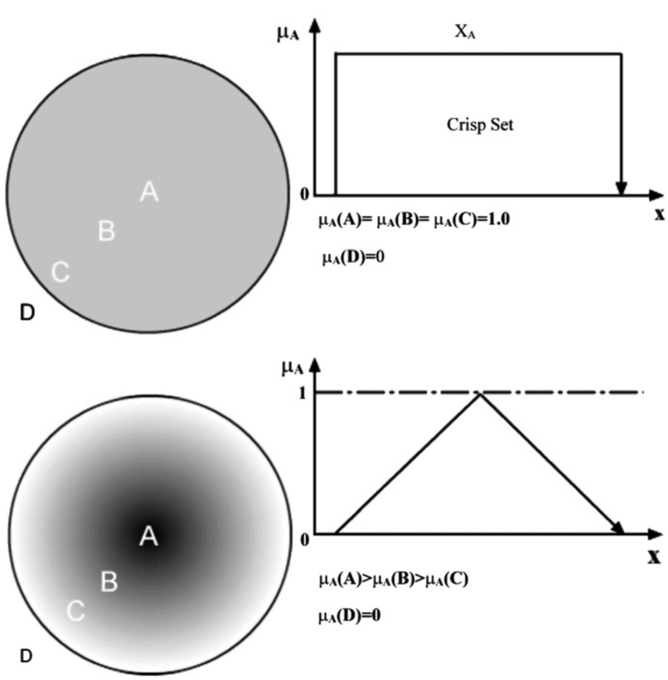


Fig. 1. Crisp and fuzzy sets [30].

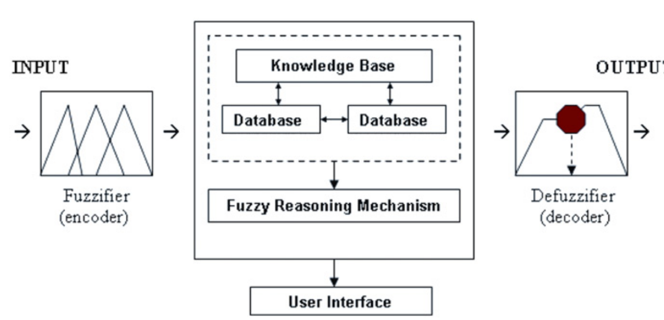


Fig. 2. A typical architecture of a fuzzy model [39].

Tsukamoto fuzzy model; and the Singleton fuzzy model [22,25–32, 36,38].

The differences between these FISs lie in the consequents of their fuzzy rules, and thus their aggregation and defuzzification procedures differ accordingly. In this paper, the Mamdani fuzzy model is widely used since this model is easier to interpret and analyze when compared with the others [22,25–32,36].

In this study, the Mamdani fuzzy algorithm was selected to express the blastability system by fuzzy sets. As mentioned by Grima [36], the Mamdani algorithm is perhaps the most appealing fuzzy method to employ in engineering geological problems. Fig. 3 depicts a two-rule Mamdani FIS which derives the overall output "z" when subjected to two crisp inputs "x" and "y" [40].

The general "if–then" rule structure of the Mamdani algorithm is given in the following equation:

$$R_i: \text{ if } x_i \text{ is } A_{il} \text{ and} \ldots \\ \text{then} \quad y \text{ is } B_i \text{ (for } i=1,2,\ldots,k), \tag{5}$$

where k is the number of rules, x_i is the input variable (antecedent variable) and y is the output variable (consequent variable). Although many methods of composition of fuzzy relations (e.g. min–max, max–max, min–min, max–mean, etc.) exist in the literature, max–min and max–product methods are the two most commonly used techniques [37]. The basic form of a fuzzy composition process is given by the following expression:

$$\underset{\sim}{B} = \underset{\sim}{A} o \underset{\sim}{R} \tag{6}$$

where $\underset{\sim}{A}$ is the input or antecedent, defined on universe X; $\underset{\sim}{B}$ is the output or consequent defined on universe Y; and $\underset{\sim}{R}$ is the fuzzy relation characterizing the relationship between specific inputs (x) and specific outputs (y). The following stages are the components of the Mamdani inference algorithm [36]:

1. Compute the degree of fulfillment, α_i (weight of each rule), of the input for each rule i by considering the degree of membership (μ), where '$\wedge$' is the minimum operator:

$$\alpha_i = \mu\underset{\sim}{A}_{i1}(x_1) \wedge \mu\underset{\sim}{A}_{i2}(x_1) \wedge \mu\underset{\sim}{A}_{i3}(x_1) \wedge \cdots \wedge \mu\underset{\sim}{A}_{in}(x_1), \quad 1 \le i \le k \tag{7}$$

2. For each rule derive output fuzzy set using the minimum normt-norm:

$$\mu\underset{\sim}{B}_i(y) = \alpha_i \wedge \mu\underset{\sim}{B}_i(y) \tag{8}$$

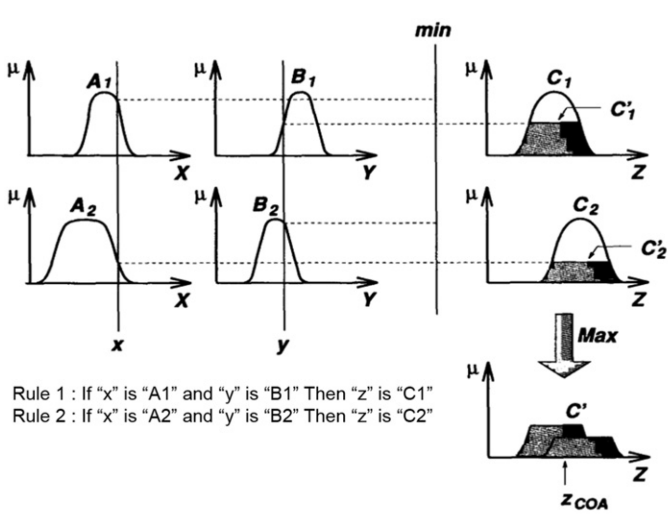


Fig. 3. The Mamdani FIS [40].

3. Aggregate the output fuzzy sets by taking the maximum:

$$\mu\underset{\sim}{B}_i = \max_{i=1,2,\ldots,k} (\mu\underset{\sim}{B}_i(y)) \tag{9}$$

Taking into consideration the stages given above, a Mamdani algorithm was constructed for the blastability system.

3.3. Defuzzification methods

Defuzzification refers to the way a crisp value is extracted from a fuzzy set as a representative value. Although there are a number of defuzzification methods in the literature such as centroid of area (COA) or center of gravity, mean of maximum, smallest of maximum, etc., the most widely adopted defuzzification method is the COA method [36]. In this study, the crisp value adopting the COA defuzzification method was obtained by

$$z^*_{COA} = \frac{\int_Z \mu_A(z) z \, dz}{\int_Z \mu_A(z) \, dz} \tag{10}$$

where z^*_{COA} is the crisp value for the "z" output and μ_A is the aggregated output membership function.

4. Construction and application of fuzzy sets to the Blastability Index Rating Method

4.1. Construction of input–output sets and rule consequents

This section presents the application of the Mamdani fuzzy identification framework described in Section 3 for construction of a fuzzy rule-based model to assess the blastability. The principal components of the fuzzy model were fuzzy inference, fuzzy sets for input/output variables and fuzzy if–then rules. The architecture of the fuzzy modeling presented in Fig. 4 has fuzzy rules representing a nonlinear mapping between input and outputs.

In this study, the input variables of the fuzzy model were the uniaxial compression and tensile strength, density, P-wave velocity, hardness, elasticity, deformability, fracture toughness, in-situ block sizes, fractal dimension of in-situ rock, integrity of rock mass and discontinuity plane's strength parameters of the blastability designation rating method (Table 1). In the next section, it will be shown that according to RES and interaction matrix analysis of geological field data of G cutting site [1], these twelve input variables will be decreased to eight. The output of the fuzzy model is a final index rating, indicating the ease of blastability of rock mass. The mentioned parameters were then represented by fuzzy sets as the input/output variables of the fuzzy model.

In the present fuzzy model, triangular and trapezoidal membership functions were developed as they are the most common type of membership functions used in rule-based fuzzy modeling [22,29,36]. These fuzzy sets represent the 'VE' (very easy), 'E' (easy), 'M' (moderate), 'D' (difficult) and 'VD' (very difficult) classes that are given in Tables 1 and 2. Fig. 5 depicts input and output variables. Fuzzy Inference System (FIS) Editor in Matlab environment was used to establish input and output variables. Each input and output variable was fuzzified with membership function (MF) graphically designed with the toolbox. These fuzzy sets represent the 'VE' (very easy), 'E' (easy), 'M' (moderate), 'D' (difficult) and 'VD' (very difficult) classes that are given in Tables 1 and 2.

The final stage of defining model is the construction of the if–then rules. The if–then rules were introduced to the fuzzy model by considering the rating probabilities, which could be

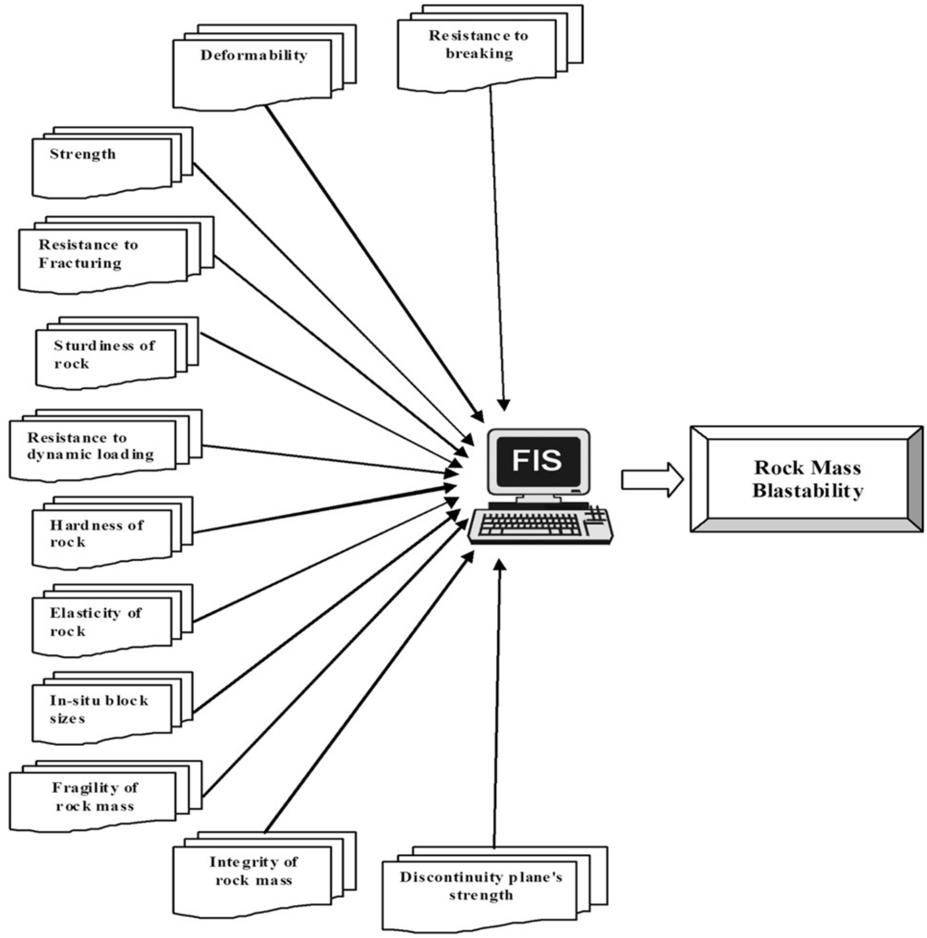


Fig. 4. Inputs and output sets of constructed fuzzy inference systems.

obtained from the adopted *BD* rating method. For the construction of the if–then rules, mainly field data and experience were utilized. As the rating system has eight main parameters and each parameter has five subclasses, theoretically the number of if–then rules is $5^8 = 390{,}625$. However, because of the nature of rock mass and based on expert opinion, some impossible rules must be ignored when extracting if–then rules. For example, if the class of rock compressive strength is easy, it should not be expected the tensile strength to be medium, difficult, or very difficult and so on [29,36].

Nearly 400,000 number of rules is significantly bigger than the number of employed rules in previous studies (for example 20, 19, 126, 400, 625 and 9450 fuzzy rules were used in references [26–31], respectively).

Furthermore, construction of nearly 400,000 rules with eight input parameters and one output does not seem logical, practical and manageable. Moreover, there is high potential of making mistakes in controlling validity of combinations of input parameters, which do not conflict with the nature of rock mass among nearly 400,000 possible combinations. In addition, an extremely high processing time is required to solve the fuzzy inference system with such a large number of rules. For example a Mamdani FIS with 8 input and 1 output variables consisting of 3000 rules needs 8 h CPU time to predict the output in a computer with a P4 Intel processor.

To reduce the complexity of defining and controlling the possibility of a large number of rules (nearly 400,000 rules) and processing time, a new strategy was developed in construction of rule (knowledge) base part of Mamdani FIS structure. In this method contrary to the usual Mamdani FIS, the applied fuzzy rules in the rule base part of fuzzy system structure are not fixed and predefined, and relevant to each crisp input set fuzzy rules are defined and used in extracting the final output value. Therefore, an equivalent fuzzy system adoptable to each input parameters set, resulting in a small number of rules, was used instead of the system with a large number of unmanageable rules.

In essence, this type of strategy in employing rule base part in Mamdani FIS was developed based on the fuzzy mechanism (''Min–Max'' fuzzy relation) employed in Mamdani FIS. When a Mamdani fuzzy inference system with all its possible rules subjects to a crisp input parameters is set, it is seen that only some of the rules have non-zero degree of fulfillment ''α_i''. Let us name these rules as ''Effective Rules''. Therefore, for a specific crisp input set elimination of rules with zero degree of fulfillment from the system will not change the final output value. This is illustrated schematically in Fig. 6. In Fig. 6, an example of Mamdani FIS is shown with two input and one output sets. The input and output variables were represented by fuzzy sets with a combination of triangular and trapezoidal membership functions (Fig. 6a–c). These fuzzy sets represent the ''E'' (easy), ''M'' (moderate) and ''D'' (difficult) classes, so that the maximum number of possible rules that can be constructed is $3^2 = 9$ (Fig. 6d).

The nine-rule Mamdani FIS mechanism, when subjected to two crisp inputs of ''$X=55$'' and ''$Y=50$'', is illustrated in Fig. 6e. It can be seen that only four of nine rules (effective rules: 4, 5, 7 and 8) have non-zero consequences and the rest of the rules are

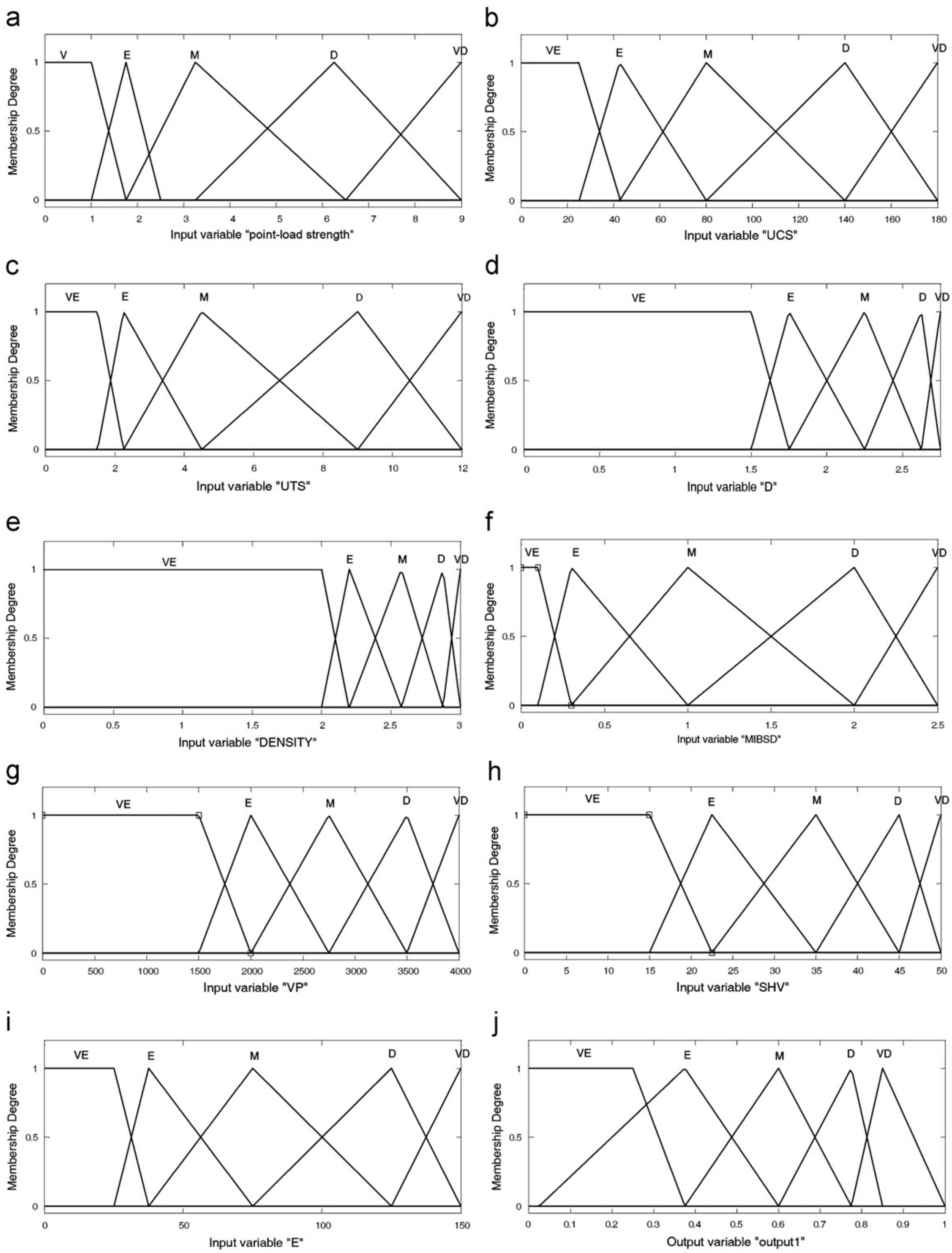


Fig. 5. Input and output membership functions for: (a) point load strength; (b) *UCS*; (c) *UTS*; (d) fractal dimension of in-situ rock mass; (e) density; (f) mean in-situ block size distribution; (g) *P*-wave velocity; (h) Schmidt hardness; (i) Young modulus; (j) final rating index. VE: very easy; E: easy; M: medium; D: difficult; VH: very difficult [43].

effectless (neutral) and can be omitted from the knowledge base of the system, without affecting the final value.

The point is that, effective rules could be determined before defining all rules and running the whole system. Considering all possible combinations of classes of input variables, which intersected by relevant crisp input parameters, it is possible to determine all the effective rules in a systematic way. For instance according to Fig. 6a and b the classes of input variables intersected by input values of "$X=55$" and "$Y=50$" are (M, D) and (E, M), respectively. Therefore, as depicted in Fig. 6f all effective rules for the mentioned example were determined by constructing all combinations of input variables $X=\{M, D\}$ and $Y=\{E, M\}$. Generally for a system with n fuzzy inputs represented by a combination of triangular and trapezoidal membership functions, the number of effective rules will be in the range of 2^n and $1^n=1$. Therefore, in this method for each input set, the

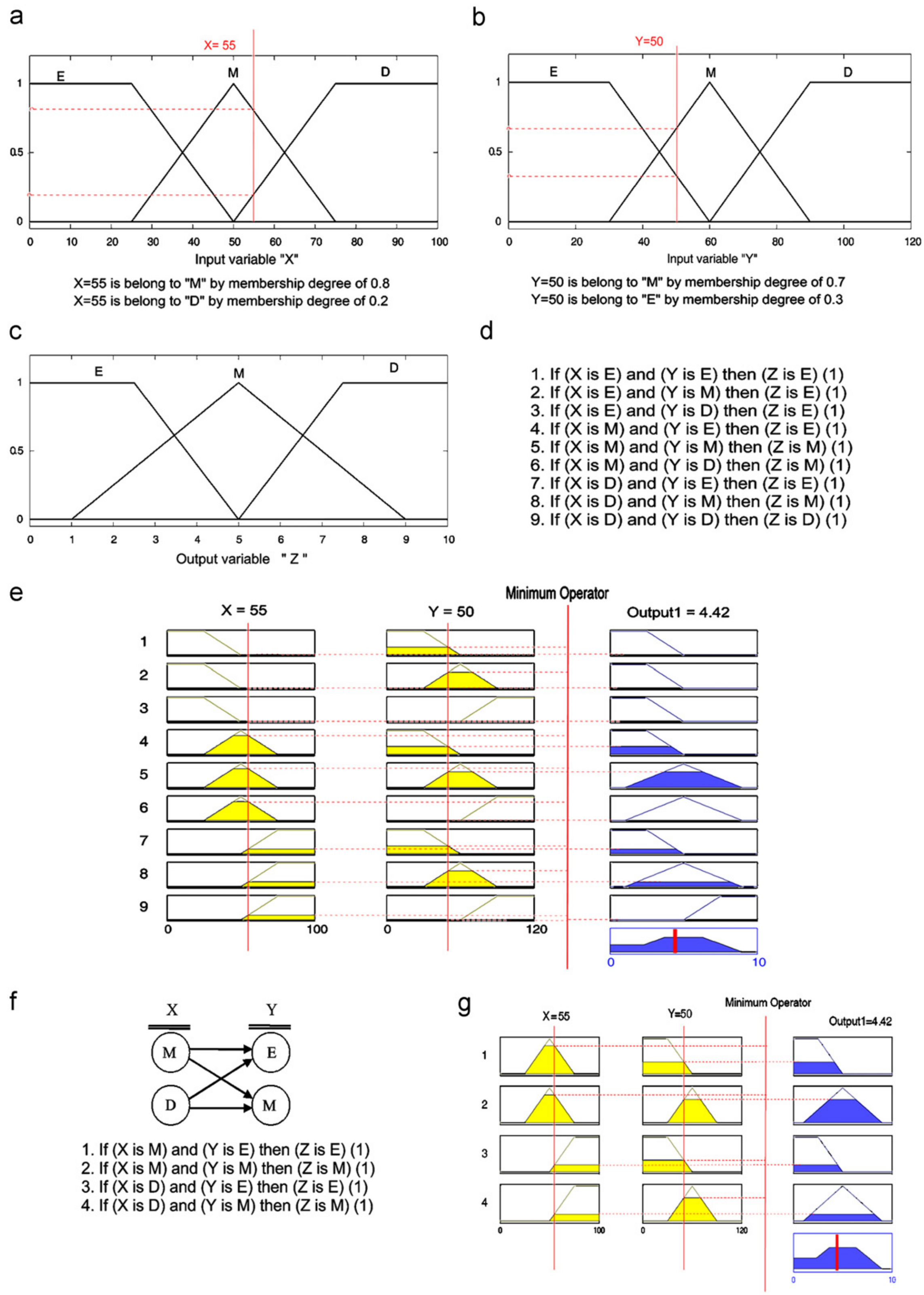


Fig. 6. An hypothetical example to explain the effective rules implementation in Mamdani FIS. (a), (b) fuzzy input variables, (c) fuzzy output variable, (d) all possible rules, (e) graphical indication of fuzzy reasoning mechanism with all its possible rules when subjected to a crisp input set, (f) graphical indication of fuzzy reasoning mechanism with relevant effective all its possible rules of the crisp input set and (g) effective rules of given input set.

corresponding effective rules would be determined and applied before running the system, to obtain the relevant final output value.

In practice, using this method is valuable when the numbers of possible if–then rules are so considerably more that controlling validity of each individual fuzzy rule according to the nature of the problem, and processing time of solving the system is unreliable and time consuming, respectively.

4.2. Practical implementation of the constructed fuzzy model

To demonstrate the implementation of the constructed fuzzy model for the blastability assessment system, geotechnical field data of the rock mass at a highway improvement cutting site in North Wales obtained by Latham and Lu [1] were considered. In *G* cutting site, for the highway improvement, a new route nearly 600 m long was to be created in a deep cutting (which was divided to berms of 4–6 (m) depth) to be excavated by blasting. The rock types at the site were seen to include siltstones, sandstones, tuffites, tuffs and limestones. Due to the inadequacy of previous geological data, possibly acquired at the limited exposure condition, Latham and Lu undertook additional data acquisition. These data include mapping discontinuities on various rock cuttings, taking photos of blasting results immediately after blasting, performing on-site point load tests and Schmidt Hammer tests and collecting other associated geological and blast design data. A sketch plan for the investigation, together with the positions of the scanline mapping, the point load tests and the Schmidt Hammer tests is illustrated in Fig. 7 [1]. Latham and Lu determined the blastability of this site by incorporating their own developed blastability classification system, continuous rating charts and the RES approach. The RES approach is a very useful procedure for devising a rock mass classification scheme for any rock engineering project, which can be represented by a function of the leading diagonal parameter values of an interaction matrix. In the interaction matrix, all parameters influencing the system are arranged along the leading diagonal of the matrix, and the off-diagonal positions are assigned with values, which describe the degree of the influence of one parameter on the other parameter (Fig. 8). The selection of the parameters and the definition of each parameter weight in a classification system can be made through obtaining the *C*–*E* plot in cause and effect space. The sum of each row in interaction matrix is termed the "cause" and denoted by *C*. It represents the way in which a parameter affects the rest of the system. The sum of each column in interaction matrix is termed the "effect" and denoted by *E*. It represents effect of the rest of the system has on

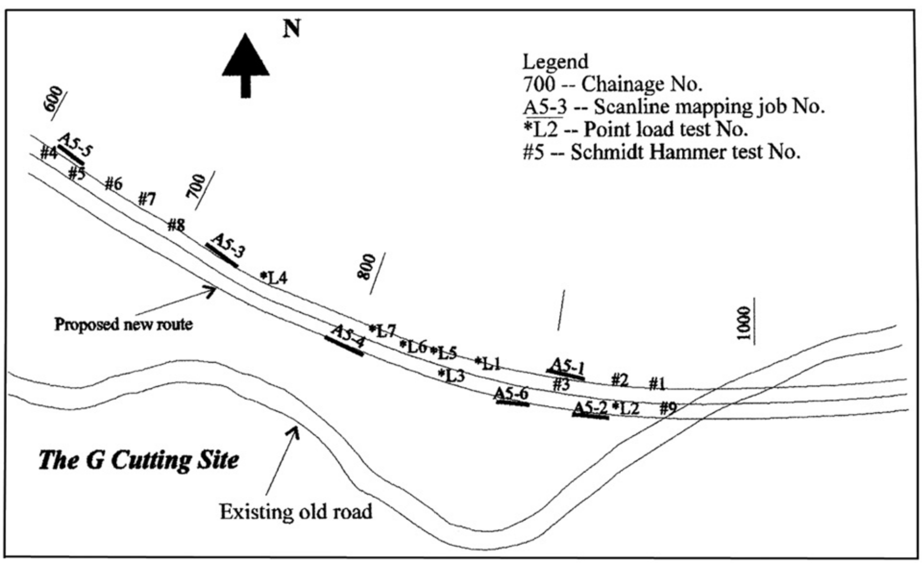


Fig. 7. A sketch plan for the geological investigation at the *G* cutting site showing locations of scanline mapping and intact rock samples [1].

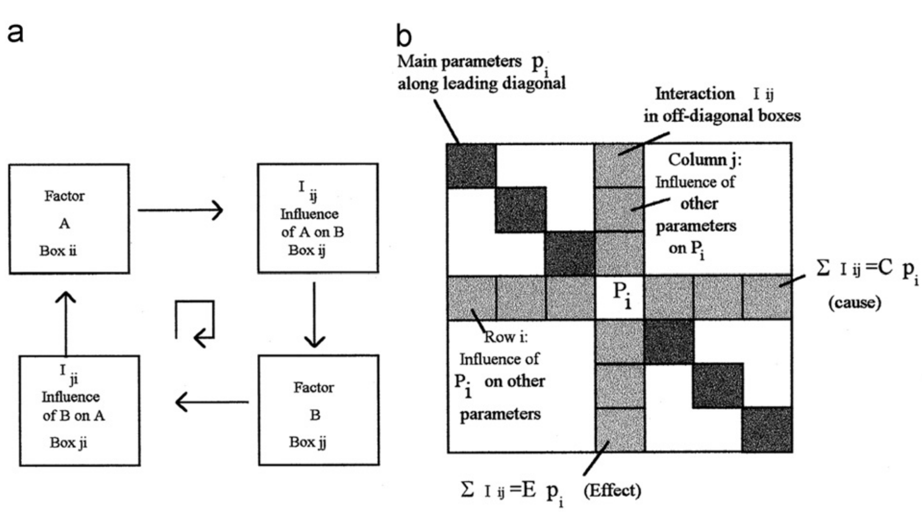


Fig. 8. Illustration of the interaction matrix in RES: (a) interaction matrix of two factors and (b) general illustration of the coding of interaction matrix and the set-up of the cause and effect coordinates [1].

that parameter. And the ordered histogram of $C+E$ can be constructed for all parameters. So those factors that contribute to most of the system, say larger than 70% of the $\sum(C+E)$ total in the ordered histogram, can be selected as the factors to be used in assessing the blastability of the rock mass. Referring to rock engineering system and interaction matrix analysis performed by Latham and Lu [1] (Fig. 9), only the eight parameters P_1, P_2, P_3, P_4, P_5, P_6, P_9 and P_{10} have been chosen as the main contributory parameters of the blastability of the rock masses at the site. For more details on selection of main contributory parameters, see Latham and Lu [1] and Hudson [41].

In this paper the 12 input variables of the original fuzzy model were reduced to 8 variables. As continuous rating charts, corresponding to each single factor, were not declared in the paper by Latham and Lu (except for P_9), the average class value of the rating chart for each factor was used in constructing fuzzy rules. So, bearing in mind the non-linearity of classification system rating, to realistically classifying of poor rock masses [42] value of 1 has been nominated for the worst mode (very difficult blasting), and rates of difficult, moderate, easy and very easy are, respectively, 0.70, 0.50, 0.25 and 0.10. Then the if–then rules were introduced to the fuzzy model. The antecedent parts of rules were build by possible combination of contributory parameters according to Table 1, and the relevant consequence of each rule was determined using Eq. (1), corresponding weights and rating of input variables.

As stated in the previous section, by increasing the number of input parameters and fuzzy rules, in a system, the reliability of controlling fuzzy rules validity decreases and the processing time increases. Therefore, the new methodology of implementing effective rules in Mamdani FIS, which was described in the previous section, was adopted to assess the blastability index.

As a hypothetical example of the followed procedure, schematic representation of the fuzzy reasoning mechanism with 16 effective rules relevant to the input parameters set of {($UCS=120$), ($UTS=6.72$), ($\rho=2.7$), ($SHV=41.2$), ($V_p=4787$), ($E=37.5$), ($MISB=2.5$), ($D=1.4$)}, is shown in Fig. 10. Initially, the numerical value of each input variable is intersected with the corresponding fuzzy set in the antecedent part of each rule. Then the minimum operator for each rule is applied and the consequent fuzzy set is truncated considering the minimum of the antecedent fuzzy set. The output fuzzy set is derived for each rule. All of the consequent fuzzy sets are combined into a single fuzzy set by means of a fuzzy operator. Finally, using the COA defuzzification method, the fuzzy output is translated into a single numerical value. The obtained final rating for this example is 0.581 in this case. The blastability class can be evaluated as easy or moderate. At this point, fuzzy set theory enables engineers to cope with such uncertainties. Following the determination of the Final Index Rating, its membership degree is obtainable by using the output variable fuzzy set (Fig. 5). The membership degree is an indication of certainty with which a rock mass belongs to a certain blastability class. As can be followed from Fig. 11 for a final index rating of 0.581, the ease of blasting is determined as "Moderate" with a membership degree of 0.92. To verify the developed model, the blastability conditions for the rock masses of G cutting site were considered. The introduced fuzzy based blastability classification system was applied to estimate the blastability classes of the site. Therefore, the relevant effective rules for each input data set of the site (six positions) were

a

P1	0.65	0.7	0.85	0.65	0.55	0.4	0.75	0.5	0.55	0.3	0.4
0.75	**P2**	0.4	0.45	0.65	0.5	0.4	0.65	0.6	0.45	0.5	0.5
0.9	0.75	**P3**	0.8	0.7	0.8	0.45	0.5	0.25	0.25	0.2	0.2
0.8	0.5	0.7	**P4**	0.6	0.6	0.55	0.25	0.25	0.3	0.45	0.15
0.45	0.65	0.7	0.45	**P5**	0.5	0.25	0.45	0.45	0.4	0.25	0.45
0.7	0.5	0.75	0.4	0.5	**P6**	0.25	0.45	0.35	0.35	0.45	0.4
0.4	0.2	0.4	0.65	0.25	0.45	**P7**	0.5	0.2	0.25	0.25	0.15
0.5	0.7	0.2	0.2	0.15	0.5	0.35	**P8**	0.4	0.25	0.2	0.3
0.25	0.3	0.25	0.5	0.35	0.25	0.3	0.4	**P9**	0.8	0.65	0.45
0.65	0.6	0.3	0.35	0.25	0.35	0.4	0.55	0.65	**P10**	0.8	0.2
0.45	0.45	0.2	0.6	0.6	0.45	0.2	0.15	0.5	0.55	**P11**	0.25
0.25	0.65	0.1	0.3	0.45	0.25	0.15	0.5	0.55	0.25	0.3	**P12**

P_1 - Strength;
P_2 - Resistance to fracturing;
P_3 - Sturdiness;
P_4 - Elasticity;
P_5 - Resistance to dynamic loading;
P_6 - Hardness of rock;
P_7 - Deformability;
P_8 - Resistance to breaking;
P_9 - *IBSD* or *PMS*;
P_{10} - Fragility of rock mass;
P_{11} - Integrity of rock mass;
P_{12} - Discontinuity plane's strength

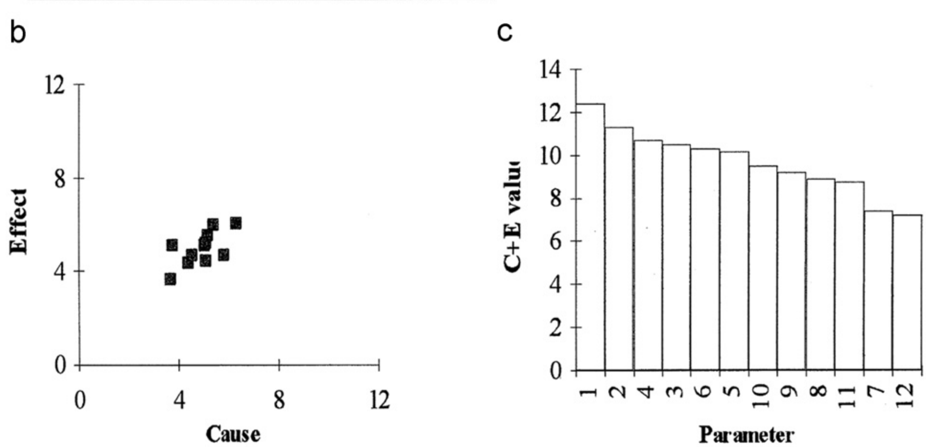
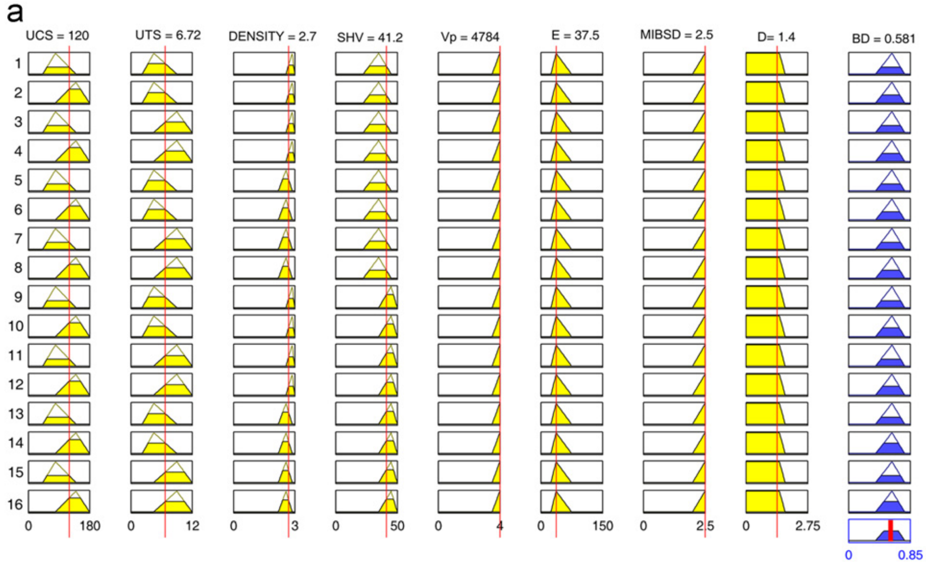


Fig. 9. Illustration of the interaction matrix coding results: (a) coding values, (b) the C–E plot and (c) the ordered histogram [1].

a



b

If(UCS=M) and (UTS=M) and (ρ=D) and (SHV=M) and (Vp=VD) and (E=E) and (IBSD=VD) and (D=VE)Then(BD=M)
If(UCS=D) and (UTS=M) and (ρ=D) and (SHV=M) and (Vp=VD) and (E=E) and (IBSD=VD) and (D=VE)Then(BD=M)
If(UCS=M) and (UTS=D) and (ρ=D) and (SHV=M) and (Vp=VD) and (E=E) and (IBSD=VD) and (D=VE)Then(BD=M)
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If(UCS=M) and (UTS=D) and (ρ=M) and (SHV=D) and (Vp=VD) and (E=E) and (IBSD=VD) and (D=VE)Then(BD=M)
If(UCS=D) and (UTS=D) and (ρ=M) and (SHV=D) and (Vp=VD) and (E=E) and (IBSD=VD) and (D=VE)Then(BD=M)

Fig. 10. An example calculation of fuzzy inference model: (a) graphical indication of fuzzy reasoning mechanism and (b) effective "if–then" rules of the fuzzy inference system relevant to {(UCS=120), (UTS=6.72), (ρ=2.7), (SHV=41.2), (V_p=4787), (E=37.5), ($MISB$=2.5), (D=1.4)}.

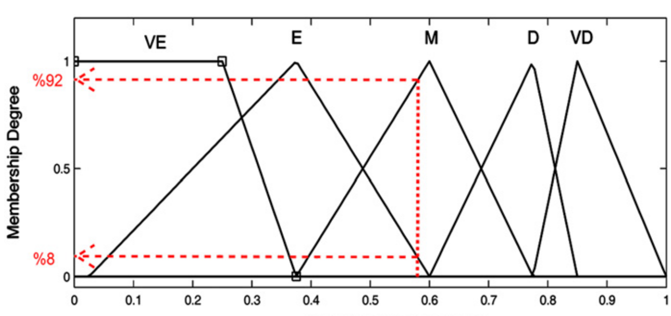

Fig. 11. A typical architecture of a fuzzy model.

constructed and implemented in calculating their final value of blastability index. Maximum and minimum numbers of effective rules for a crisp input set of 8 parameters are 2^8 and 1^8=1, respectively. The numbers of effective rules used for assessment of six input data sets of the G cutting site are shown in Table 5.

The blastability rating values and corresponding descriptions of the rock masses at the site are represented in Table 6. Based on the previously developed blastability designation rating system, the blastability rating values by considering the average class values of the rating chart and the values obtained by Latham and Lu [1] (continuous rating charts) are also included in Table 6.

Table 5
Number of fuzzy effective rules of each rock masses from G cutting site.

Site description	S1	S2	S3	S4	S5	S6
Number of effective rules	64	32	64	128	64	64

The graphical illustration given in Fig. 12 indicates that on average there is an acceptable agreement between ratings obtained from the conventional method and the fuzzy model.

As it is impossible to directly measure, the actual value of the BD of a rock mass in the field so that performance evaluation of the fuzzy models relative to conventional methods were evaluated by employing correlation coefficient R, and performance indices namely the Variance Account For (VAF) and Root Mean Square Error ($RMSE$) are given below as

$$VAF=\left(1-\frac{var(y-\hat{y})}{var(y)}\right)\times 100\% \tag{11}$$

$$RMES=\sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i-\hat{y}_i)^2} \tag{12}$$

where var=the variance, y_i =the measured value, $\hat{y}_i$=the predicted value and N=the number of samples.

Table 6
The outputs of the Blastability Index Rating Method and fuzzy model.

No. P_i	Parameter		Weight, W_i	Blastability assessment					
	Description	Unit		S1			S2		
				Value	Rating	W_i*R_i	Value	Rating	W_i*R_i
1	*Is(50)*	MPa	0.1475				4.42	0.75	0.111
	UCS	MPa		120	0.75	0.111			
2	*UTS*	MPa	0.1344	6.82	0.75	0.101	5.53	0.5	0.067
4	*E*	GPa	0.1273	48.8	0.5	0.064	45.13	0.25	0.032
3	ρ	t/m^3	0.1249	2.71	0.5	0.062	2.704	0.5	0.062
6	*SHV*		0.1225	43.6	0.75	0.092	41.2	0.75	0.092
5	V_p	m/s	0.1208	4901	1	0.121	4784	1	0.121
10	*D*		0.1131	1.486	0.5	0.057	1.397	0.1	0.011
9	*MIBS*	m	0.1095	2.18	0.75	0.082	3.1	1	0.11
Blastability designation						0.689			0.606
Ease of blastability						*M*			*M*
Blastability designation by Latham and Lu						0.664			0.647
Ease of blastability by Latham and Lu						*M*			*M*
Fuzzy model						0.612			0.602
Ease of blastability and membership degree						*M* (%98)			*M* (%100)
No. P_i	Parameter		Weight, W_i	Blastability assessment					
	Description	Unit		S3			S4		
				Value	Rating	W_i*R_i	Value	Rating	W_i*R_i
1	*Is(50)*	MPa	0.1475	6.51	0.75	0.111	4.45	0.75	0.111
	UCS	MPa							
2	*UTS*	MPa	0.1344	8.14	0.75	0.101	5.563	0.50	0.067
4	*E*	GPa	0.1273	52.39	0.25	0.032	50.07	0.50	0.064
3	ρ	t/m^3	0.1249	2.715	0.5	0.062	2.7	0.50	0.062
6	*SHV*		0.1225	46	0.75	0.092	44.85	0.75	0.092
5	V_p	m/s	0.1208	4996	1	0.121	4785	1.00	0.121
10	*D*		0.1131	1.848	0.1	0.011	2.113	0.50	0.057
9	*MIBS*	m	0.1095	2.8	1	0.11	1.31	0.50	0.055
Blastability designation						0.639			0.628
Ease of blastability						*M*			*M*
Blastability designation by Latham and Lu						0.725			0.662
Ease of blastability by Latham and Lu						*D*			*M*
Fuzzy model						0.619			0.605
Ease of blasting and membership degree						*M* (%96)			*M* (%99)
No. P_i	Parameter		Weight, W_i	Blastability assessment					
	Description	Unit		S5			S6		
				Value	Rating	W_i*R_i	Value	Rating	W_i*R_i
1	*Is(50)*	MPa	0.1475				5.05	0.75	0.111
	UCS	MPa		135	0.75	0.111			
2	*UTS*	MPa	0.1344	7.67	0.75	0.101	6.31	0.75	0.101
4	*E*	GPa	0.1273	48.47	0.25	0.032	42.9	0.25	0.032
3	ρ	t/m^3	0.1249	2.63	0.50	0.062	2.7	0.50	0.062
6	*SHV*		0.1225	46.07	0.75	0.092	39.7	0.50	0.061
5	V_p	m/s	0.1208	4908	1.00	0.121	4852	1.00	0.121
10	*D*		0.1131	1.499	0.10	0.011	1.194	0.10	0.011
9	*MIBS*	m	0.1095	1.78	0.75	0.082	2.22	0.75	0.082
Blastability designation						0.612			0.581
Ease of blastability						*M*			*M*
Blastability designation by Latham and Lu						0.669			0.631
Ease of blastability by Latham and Lu						*M*			*M*
Fuzzy model						0.599			0.596
Ease of blastability and membership degree						*M* (%100)		*M* (%99)	

These performance indices are interpreted as follows: the higher the *VAF*, the better the model performs. The lower the *RMSE*, the better the model performs. Contrary to *VAF*, *RMSE* also accounts for a bias in the model, i.e. an offset between the measured and predicted data [22].

As can be seen from Fig. 13c despite the two conventional methods being intrinsically the same except for their rating methods, they show very weak correlation with each other. Nevertheless, the proposed fuzzy model is correlated reasonably well with both conventional methods (Fig. 13a and b). Considering the *R* values and the fact that all three models are trying to describe the rock mass Blastability index, it seems that the fuzzy model behaves more properly and consistently in prediction. In Table 7, the performance indices of fuzzy inference

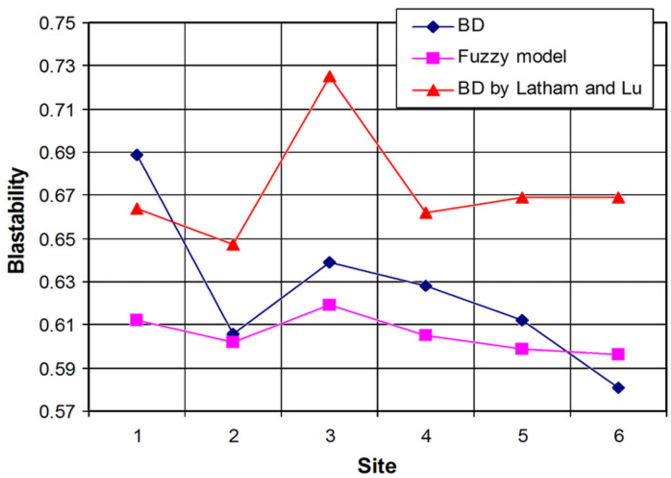

Fig. 12. Comparison of the blastability designation estimated by Mamdani FIS and conventional methods.

model relative to the two conventional methods are shown. The *VAF* index for both cases is positive but not high, and the *RMSE* index for each of the two cases are in expectable ranges (not very small nor high).

In general, according to these results despite the acceptable agreement among the three models, the constructed fuzzy model underestimates the blastability of studied rock masses in comparison with the conventional blastability classification systems. These differences can be produced as a result of the aforementioned sharp rating boundaries or the differences of rating methods.

Table 7
Performance indexes of the fuzzy system relative to the conventional methods.

Prediction models	*VFA* (%)	*RMSE* (%)
Fuzzy and *BD*	29.3	7.43
Fuzzy and *BD* by Latham and Lu	34.3	17.75

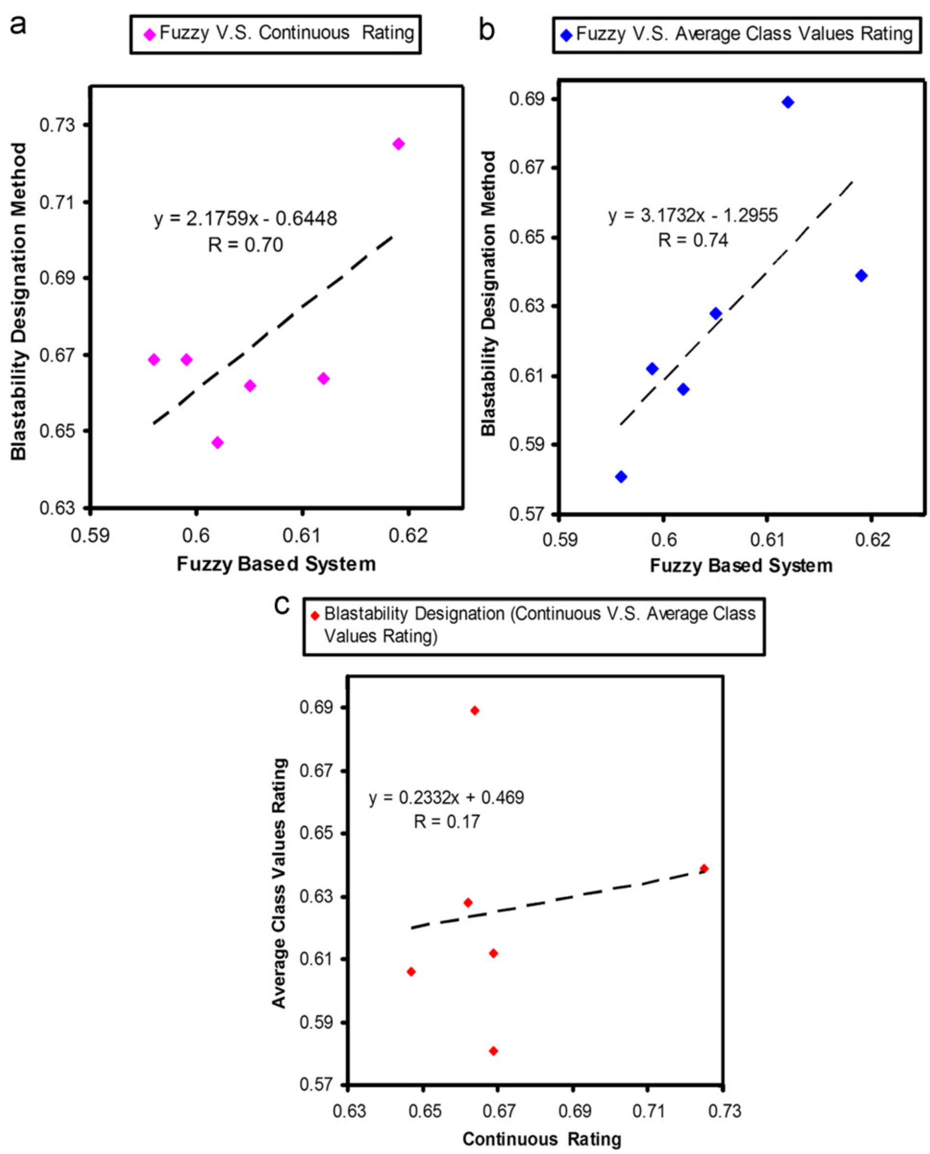

Fig. 13. Relations between ratings obtained from fuzzy based and previously developed practical blastability classification systems.

5. Conclusion

The rock engineering classification system is a very useful procedure for devising a rock mass classification scheme for a given engineering project that is to take place within the rock mass.

As emphasized in this paper, these classification systems may possess some uncertainties or fuzziness in their practical applications. These limitations especially related to sharp transitions between two adjacent blastability classes and the subjective uncertainties on data, which are close to the range boundaries of rock classes. It is well known from previous studies in the literature that such uncertainties can be best dealt by using the fuzzy set theory.

In this paper a fuzzy logic based blastability designation predictor model is developed. The suggested approach takes into account the important intact rock and rock mass uncertainty in estimation of the Blastability Designation value, which is an input parameter of the EBT model developed by Latham and Lu for Blasted Bock Size Distribution prediction.

Employing the regular Mamdani FIS for evaluating rock mass blastability was needed to construct nearly 400,000 and control their possibility, which does not seem to be practical and easily manageable. So an equivalent fuzzy system adoptable to each input parameters set, resulting in a small number of rules, was developed to efficiently solve the big Mamdani FIS. This method provides the possibility of overcoming the difficulties encountered in dealing with a large number of fuzzy rules.

It is shown that blastability values obtained from both the blastability designation fuzzy inference system and the quantitative blastability chart are in acceptable agreement with each other. However, the fuzzy model underestimates the blastability of the studied rock masses (G cutting site) in comparison with the conventional methods. This difference could be due to the aforementioned uncertainties existing in conventional methods. It can be said that fuzzy set based classification eliminates the biased evaluation assignment of the rating values, which is common in conventional blastability classification systems and may lead the engineers to a wrong decision.

The fuzzy set theory helps the blast engineers to judge the obtained final ratings by means of constructed membership degree functions (indicate the degree with which a rock mass belongs to a certain blastability class), as an advantage to the conventional classification systems. It seems that fuzzy set theory could be used as an effective tool in decision-making processes where limited data with some uncertainties are available.

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