International Journal of Mining Science and Technology 26 (2016) 1017–1023

Contents lists available at ScienceDirect

International Journal of Mining Science and Technology

journal homepage: www.elsevier.com/locate/ijmst

Alunite processing method selection using the AHP and TOPSIS approaches under fuzzy environment

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ARTICLE INFO

Article history:
Received 4 July 2015
Received in revised form 17 September 2015
Accepted 25 April 2016
Available online 20 September 2016

Keywords:
Alunite
Mineral processing methods
Multi Attribute Decision Making
Delphi Analytical Hierarchy Process (DAHP)
Fuzzy TOPSIS

ABSTRACT

Alunite is the most important non bauxite resource for alumina. Various methods have been proposed and patented for processing alunite, but none has been performed at industrial scale and no technical, operational and economic data is available to evaluate methods. In addition, selecting the right approach for alunite beneficiation, requires introducing a wide range of criteria and careful analysis of alternatives. In this research, after studying the existing processes, 13 methods were considered and evaluated by 14 technical, economic and environmental analyzing criteria. Due to multiplicity of processing methods and attributes, in this paper, Multi Attribute Decision Making methods were employed to examine the appropriateness of choices. The Delphi Analytical Hierarchy Process (DAHP) was used for weighting selection criteria and Fuzzy TOPSIS approach was used to determine the most profitable candidates. Among 13 studied methods, Spanish, Svoronos and Hazan methods were respectively recognized to be the best choices.

1. Introduction

There is a wide range of processing methods for alunite that are created in order to access the maximum recovery of alumina and its peripheral products, using various operation units such as pre-concentration, calcination, leaching, and crystallization, utilizing different reagents including acids and bases. The only processing plant built for recovering alumina and potassium sulfate from alunite, was established in Soviet Union in the mid-1960s [1]. Some other investigations in this area were conducted in 1976 by Alumni Consortium on Utah alunite ore reserve at pilot scale [2]. Therefore, lack of technical, operational and economic data makes it hard for researchers to choose the most appropriate processing method. On the other hand, selecting the right process to recover alumina, is a complex subject, requiring complete analysis of technical, economic and environmental factors. The purpose of all these methods is to recover aluminum and potassium complexes and to remove impurities such as silica and iron. Also if the reagents used in a method, for example sulfuric acid, is able to be reused in the process stages, this is considered as an advantage of the method. Every processing method has some advantages and disadvantages. Containing fewer steps, flexibility to turn into a continuous process, requiring less reagents and lower temperature, are among advantages of a method. In a case that technical and economic data is insufficient, it is a difficult challenge to analyze advantages and disadvantages of different methods and to compare preferences. According to the characterization of alunite ore, selecting the right processing method involves various criteria analyzing with different levels of importance, such as process products (alumina and potassium sulfate), flow sheet operational complexity, operating and investing costs and environmental effects especially sulfur dioxide (SO_2) emission. Due to the lack of quantitative data on alunite processing, selection of the best approach has to be done using qualitative data. Since there are variety of factors with different impact areas, advanced multiple criteria decision making approaches have to be utilized. Otherwise, selecting the most appropriate method would not have enough accuracy and speed.

Multi criteria decision making approaches have been used in many engineering and economic research areas, for instance, they are utilized when choosing the optimum mining method, which is one of the most critical stages of mine design. The best possible mining method was selected with Monte Carlo Analytical Hierarchy Process for Jajarm bauxite mine in Iran by Ataei and associates [3]. Additionally, Karimnia and Heydar determined the most appropriate mining method for Qapiliq, a salt mine in Iran, using Fuzzy Analytical Hierarchy Process, in 2015 [4].

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http://dx.doi.org/10.1016/j.ijmst.2016.09.009
2095-2686/

But these techniques have been rarely utilized for selecting the best process, having a significant rule in technical, economic and environmental issues. In 2008, multiple criteria decision making approaches were used by Ding and associates in order to control critical roasting process on iron-containing ore in cylinder furnaces [5]. In 2009, these approaches were utilized to select Distributed Control System for controlling chemical processes in order to maximize equipment purchasing speed and minimizing investing costs [6]. The research on renewable energy resources to reduce carbon gas emission, which is part of energy policy, was conducted by Doukas and associates using Fuzzy TOPSIS, in 2009 [7]. In 2010, the location for the mineral processing plant at the Sangan iron ore mine was selected utilizing the Analytical Hierarchy Process (AHP) considering eight criteria [8]. In addition, a decision for selection of the best primary among available primary crushers was made using Multi-Criterion Decision Making (MCDM), in 2014 [9].

In this research, after preliminary considerations, evaluating criteria were determined and different processing methods were listed. These criteria were weighted using paired comparisons in Delphi Analytical Hierarchy Process (DAHP) and then different processing methods were considered utilizing Fuzzy TOPSIS.

Finally, sensitivity was analyzed on changing criteria weights and the best methods were introduced.

2. Methods and patterns

2.1. Stages review

The evaluation process in this research is shown in Fig. 1. After determination of criteria and analyzing different alunite processing methods, "choices" were introduced. By creating a form, shown in Table 1, and asking experts and specialists to fill it out, the weights of each criterion were calculated using DAHP. In the next step, Expert Choice software was utilized to determine weights and also incompatibility factor. Then, by using Fuzzy TOPSIS, choices were scored with regard to the criteria, and were finally sorted. The explanation of main stages is provided in subsections.

2.2. Evaluation criteria

Fig. 2 shows a general view of alunite processing methods and peripheral products. Calcination and leaching are among the most important operating units to separate alumina from tailings. The critical difference of these methods is in the possible combinations of these units. Generally, alunite calcination is carried out in four ways; simple calcination, oxidizing, reducing, and calcination in the presence of some salts, such as potassium chloride and sodium chloride. Alunite calcination at high temperatures decreases solubility of alumina. To overcome this problem, calcination is carried out at lower temperatures with potassium chloride and sodium chloride. Therefore, alumina is leached without undesired solution of silica, in lower-cost circumstances i.e. pressure, temperature, and soluble concentrates [10].

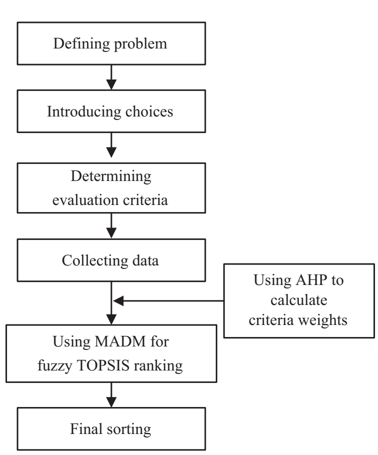

Fig. 1. Research procedure stages.

Next unit is leaching that can be performed either in acidic form or in alkaline form. In cases of alkaline leaching, calcination unit can be omitted in certain processes, for example Nasyrov and Hazen [11,12], which will cause changes in future units as well. But in processes containing acidic leaching, calcination unit is necessary before leaching. Leaching can be performed in one or two steps, depending on process type. In case of alkaline leaching, silica complexes of the ore turn into insoluble aluminosilicate and leave the circuit as tailing. As a result, the quantity of alumina and base decrease and recovery of alumina falls. Obviously, it is recommended to remove silica from the solution in alkaline leaching processes [11]. Acidic leaching on low-grade ore types faces some problems, for example, alumina contamination with iron oxide impurities existing in alunite; therefore, producing iron-free aluminum complexes is the aim in most methods [10–17].

According to preliminary researches on worldwide alunite processing methods, 13 different methods with certain technical and environmental circumstances were selected and mentioned in Table 2, by the researchers' names. Almost all alunite processing methods, obey these 13 methods, except in operational details. Concerning alunite processing methods, the most appropriate criteria can be categorized into 3 main groups, technical, economic and environmental criteria, with some sub criteria in each group (see Table 1). In Fig. 3, the relation between criteria is shown in a hierarchical structure in order to choose proper methods.

2.3. Decision making approaches

2.3.1. Delphi Analytical Hierarchy Process (DAHP)

Analytical hierarchy process is one of the most comprehensive systems for multiple criteria decision making [18]. The method was first used by Saaty in 1980 [19]. AHP is a tool to combine qualitative and quantitative factors in selecting a process and to determine preferences in an unpredictable issue [20].

The main advantage of this approach is the ability to solve problems with complex structures on a paired comparison basis that cannot be solved with usual mathematical methods. With Fuzzy Delphi, in addition to the option of saving time and money, experts' opinions are known. Changing the weights of criteria according to experts' views is a basic feature in this approach [20,21].

2.3.2. TOPSIS

TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) can be used both as a weighting approach and as a MADM approach. TOPSIS is based on the fact that ideal choice is the highest priority for all criteria whereas negative ideal is a choice with lowest priority for all criteria. The principle of this approach is that the best choice should have the most similarity to the ideal and the most dissimilarity to the negative ideal, of a geometrical aspect. In this approach, it is assumed that all criteria have equal increasing or decreasing profit [22]. Considering all possible solutions simultaneously, simple calculation process and simple programming are amongst features of this approach.

2.3.3. Fuzzy TOPSIS

In this section some important explanations and definitions are presented. It is usually challenging for decision makers to rank the operation on choices for criteria. The advantage of fuzzy theory is

Table 1
Sample of forms filled out by experts.

Alunite processing criteria			Importance of criteria						
			VVL	VL	L	F	M	VM	VVM
1	Technical criteria	Alumina recovery (AR)							
2		Alumina quality (AQ)							
3		Potash recovery (PR)							
4		Potash quality (PQ)							
5		Quality of silica (QS)							
6		Complexity of process (CP)							
7		Process controllability (PC)							
8	Economic criteria	Investing costs (IC)							
9		Operating costs (OC)							
10		Cost of reagents (CR)							
11		Including peripheral products (IP)							
12	Environmental criteria	Emission of pollutant gas (EPG)							
13		Production of solid waste (PSW)							
14		Production of harmful wastewater (PHW)							

Note: VVL = Very very little, VL = Very little, L = Little, F = Fair, M = Much, VM = Very much, VVM = Very very much.

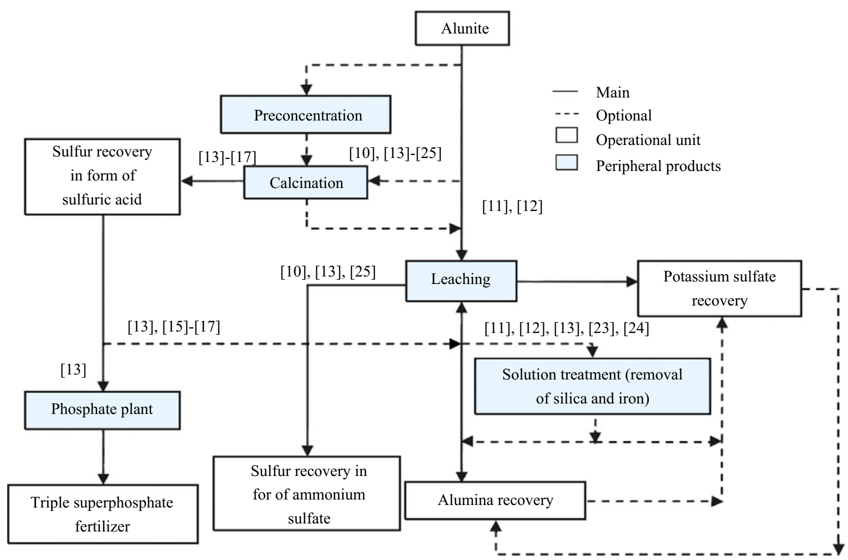

Fig. 2. General flow sheet of alunite processing methods.

Table 2
Alunite processing choices.

Choice	Process name	Choice	Process name
C_1	Mc Cullough [16]	C_8	UG 1st [13]
C_2	Spanish [17]	C_9	UG 3rd [13]
C_3	Haff [14]	C_{10}	Nasirov 1 [23]
C_4	Loest [15]	C_{11}	Nasirov 2 [12]
C_5	Kalunite [13]	C_{12}	Hazen [11]
C_6	Svoronos [10]	C_{13}	Stevens [24]
C_7	Tanaka [25]		

the determination of relative criterion importance using fuzzy numbers instead of specified numbers [22].

Definition 1. The fuzzy set $\tilde{a}$ in topic X by $\mu_{\tilde{a}(x)}$ function that is related to any elements of x in X, is a real number in the range $[0,1]$. The value of $\mu_{\tilde{a}(x)}$ function, is known as x membership rank in $\tilde{a}$ [22].

In this research, triangular fuzzy numbers are used. Triangular fuzzy number $\tilde{a}$ can be defined with three points (a_1, a_2, a_3). Mathematical forms are given [22]:

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x-a_1}{a_2-a_1}, & a_1 < x \leqslant a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 < x \leqslant a_3 \\ 0, & x > a_3 \end{cases} \quad (1)$$

Definition 2. If $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers, the distance between them can be determined using vector method [22]:

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]} \quad (2)$$

Problems can be explained by following sets [22]:

(a) A set of j possible choices: $A = \{A_1, \ldots, A_j\}$;
(b) A set of n criteria: $C = \{C_1, \ldots, C_j\}$;
(c) A set of operational rankings A_j $(j = 1, 2, 3, \ldots, j)$ relative to the criterion C_i $(i = 1, 2, 3, \ldots, n)$;

$$X = \{\tilde{x}_{ij}, \quad i = 1, 2, 3, \ldots, n, \quad j = 1, 2, 3, \ldots, J\} \quad (3)$$

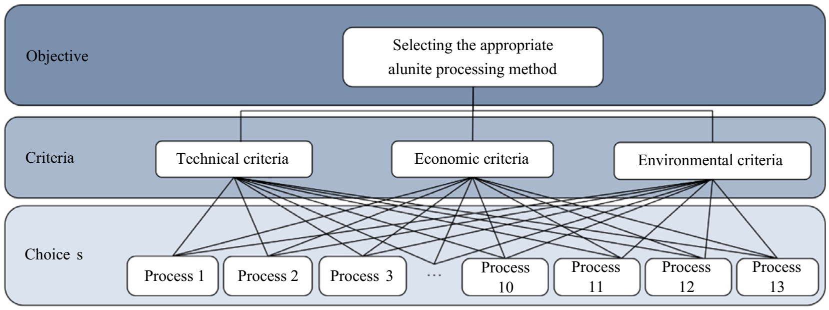

Fig. 3. Hierarchy structure for selecting the best method.

(d) A set of weight importance of each criterion w_i $(i = 1, 2, 3, \ldots, n)$.

As mentioned earlier, problem matrix can be shown as Eq. (4) [10]:

$$\widetilde{X} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \ldots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \ldots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{J1} & \tilde{x}_{J2} & \cdots & \tilde{x}_{Jn} \end{bmatrix} \tag{4}$$

Definition 3. By considering difference between criterion values, weight normalized decision making matrix is created:

$$\widetilde{V} = [\tilde{v}_{ij}]_{n\times J}, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, J \tag{5}$$

where $\tilde{v}_{ij} = \tilde{x}_{ij}(.)w_i$.

According to the fuzzy theory mentioned above, Fuzzy TOPSIS steps can be summarized [23,24]:

Step 1. Selecting word rankings $\tilde{x}_{ij}, i = 1, 2, 3, \ldots, n, j = 1, 2, 3 \ldots, J$ for choices with regard to each criterion. Fuzzy word rating $(\tilde{x}_{ij})$ keeps properties which mean ranks of normalized triangular fuzzy numbers in the range $[0, 1]$; thus, there is no need to normalize.

Step 2. Calculating weight normalized fuzzy decision making matrix. Weight normalized values $\tilde{v}_{ij}$ are calculated using Eq. (5).

Step 3. Determining positive ideal solutions (A^*) and negative ideal solutions (A^-). Fuzzy positive ideal solution $(FPIS, A^*)$ and fuzzy negative ideal solution $(FPIS, A^-)$ are shown:

$$A^* = \{\tilde{v}_1^*, \ldots, \tilde{v}_1^*\} = \{(\max_j v_{ij}|i \in I'), (\min_j v_{ij}|i \in I'')\}$$
$$i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, J \tag{6}$$

$$A^- = \{\tilde{v}_1^-, \ldots, \tilde{v}_1^-\} = \{(\min_j v_{ij}|i \in I'), (\max_j v_{ij}|i \in I'')\}$$
$$i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, J \tag{7}$$

where I' is profit criterion and I'' is cost criterion.

$$D_j^* = \sum_{j=1}^{n} d(\tilde{v}_{ij}, v_i^*) \quad j = 1, 2, 3, \ldots J \tag{8}$$

$$D_j^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, v_i^-) \quad j = 1, 2, 3, \ldots J \tag{9}$$

Step 4. Calculating the distance from each choice to A^* and A^-.

Step 5. Calculating similarity to ideal solution.

$$CC_j = \frac{D_j^-}{D_j^* + D_j^-} \quad j = 1, 2, \ldots, J \tag{10}$$

Step 6. Ranking on the basis of preference; selecting a choice with maximum CC_j^* or sorting choices on a basis of descending CC_j^*.

3. Discussion

If decision making matrix containing fuzzy data, is explained with words, word expressions have to be changed to fuzzy numbers using an appropriate scale. In this research, Chen and Hwang numerical estimation system in 1992 has been used to exchange words into equal fuzzy numbers. There are ordinary word expressions in this system with scale 1, containing only 2 word expressions, and scale 8, containing 13 word expressions.

Since words are unable to be shown mathematically, each word expression is demonstrated with a fuzzy number that shows its definition. The structure of this method is to choose a scale that puts all word expressions in a row of decision making matrix and uses fuzzy numbers to express their values [25,26]. Word expressions were changed into fuzzy preferences, using Table 3. Fuzzy preferences used in this research are shown in Table 4. Fuzzy preferences for 13 studied processes are available in Table 5.

Having word expressions changed into fuzzy preferences is the first step in Fuzzy TOPSIS. After achieving fuzzy preferences, criteria weights were calculated using Expert Choice software. Fig. 4 shows descending order of criterion weights for evaluating alunite processes. The second step is to create fuzzy weighting decision making table that is demonstrated in Table 6 with values. As shown in Table 4, fuzzy preferences are normalized to triangular positive numbers in the range $[0, 1]$. Then fuzzy positive ideal solution $(FPIS, A^*)$ and fuzzy negative ideal solution $(FPIS, A^-)$ can be defined as $\tilde{v}_i^* = (1, 1, 1)$ and $\tilde{v}_i^- = (0, 0, 0)$ for profit criterion and as $\tilde{v}_i^* = (0, 0, 0)$ and $\tilde{v}_i^- = (1, 1, 1)$ for cost criterion, in third step. In forth step, the distance from each choice to A^* and A^- is calculated using Eqs. (8) and (9). Finding similarities to the ideal solution using Eq. (10) is carried out in fifth step.

Table 3
Fuzzy equivalents of word expressions.

Word expressions	Fuzzy ranks
Very very much	(0.8, 1, 1)
Very much	(0.1, 0.2, 0.3)
Much	(0.2, 0.35, 0.3)
Fair	(0.4, 0.5, 0.6)
Little	(0.5, 0.65, 0.8)
Very little	(0.7, 0.8, 0.9)
Very very little	(0, 0, 0.2)

Table 4
Parts of fuzzy preferences table for introduced processes.

Choices	C_1	...	C_5	...	C_7	...	C_{12}	C_{13}
1	(0.4, 0.5, 0.6)	...	(0.2, 0.35, 0.5)	...	(0.1, 0.2, 0.3)	...	(0.2, 0.35, 0.5)	(0.4, 0.5, 0.6)
2	(0.5, 0.65, 0.8)	...	(0.5, 0.65, 0.8)	...	(0.7, 0.8, 0.9)	...	(0.8, 1, 1)	(0.4, 0.5, 0.6)
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
6	(0.7, 0.8, 0.9)	...	(0.8, 1, 1)	...	(0.5, 0.65, 0.8)	...	(0.1, 0.2, 0.3)	(0.4, 0.5, 0.6)
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
13	(0.5, 0.65, 0.8)	...	(0.2, 0.35, 0.5)	...	(0.7, 0.8, 0.9)	...	(0.5, 0.65, 0.8)	(0.4, 0.5, 0.6)

Table 5
Parts of decision making fuzzy weights table.

Choices	C_1	...	C_5	...	C_7	...	C_{12}	C_{13}
1	(0.03, 0.04, 0.05)	...	(0, 0.1, 0.1)	...	(0, 0.01, 0.02)	...	(0.02, 0.04, 0.05)	(0.02, 0.02, 0.03)
2	(0.04, 0.06, 0.07)	...	(0.01, 0.01, 0.01)	...	(0.05, 0.06, 0.07)	...	(0.08, 0.1, 0.1)	(0.02, 0.02, 0.03)
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
6	(0.06, 0.07, 0.08)	...	(0.01, 0.02, 0.02)	...	(0.04, 0.05, 0.06)	...	(0.01, 0.02, 0.03)	(0.02, 0.02, 0.03)
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
13	(0.04, 0.06, 0.07)	...	(0, 0.01, 0.01)	...	(0.05, 0.06, 0.07)	...	(0.05, 0.07, 0.08)	(0.02, 0.02, 0.03)

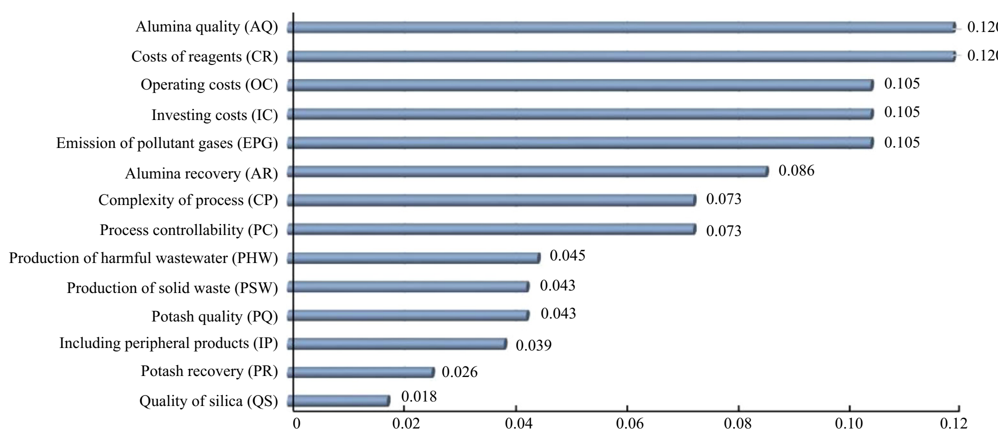

Fig. 4. Descending order of alunite processing evaluation criteria in Expert Choice software.

Table 6
Fuzzy TOPSIS results.

C	D_j^*	D_j^-	CC_j
Choice 1	7.859	0.234	0.0289
Choice 2	7.678	0.410	0.0507
Choice 3	7.894	0.245	0.0302
Choice 4	7.813	0.278	0.0344
Choice 5	7.758	0.332	0.0411
Choice 6	7.709	0.378	0.0468
Choice 7	7.794	0.295	0.0365
Choice 8	7.788	0.301	0.0372
Choice 9	7.779	0.309	0.0382
Choice 10	7.756	0.332	0.0411
Choice 11	7.759	0.330	0.0407
Choice 12	7.711	0.337	0.0466
Choice 13	7.714	0.374	0.0463

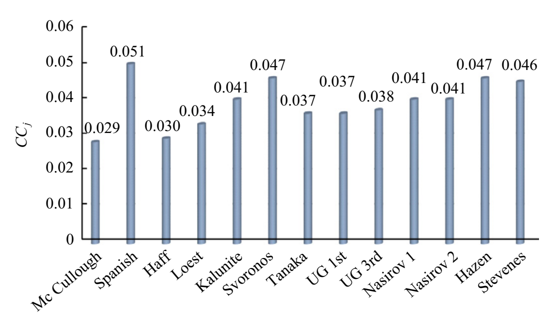

Fig. 5. Ranking histogram of alunite processing method using DAHP, Fuzzy TOPSIS.

The results of Fuzzy TOPSIS analyses are shown in Table 6 and Fig. 5. According to CC_j values, choices descending order is Spanish (choice 2), Svoronos (choice 6) and Hazen (choice 12).

4. Sensitivity analysis results

Sensitivity analysis is done with the purpose of analyzing alunite processing methods when changing criteria weights in a specific range that defines different scenarios. The principle of sensitivity analysis is to exchange the weight of a criterion with other criteria. Due to the numerous criteria, 19 scenario, that seemed to give better results, where chosen. The results of this analysis are demonstrated in Table 7 and Fig. 6. As shown in Table 7, the main scenario introduces the main result of the study. In scenario number 19, Normal Weighting system was used in order to achieve the preference of processes, in a case that all criteria have equal weights $w = w_1 = w_2 = \ldots = w_{14}$. In this analysis, the similarity to ideal solution is calculated for each criterion.

Table 7
Parts of sensitivity analysis numeric results.

Scenarios	Weights					CC_j values				
	w_1	w_2	...	w_{13}	w_{14}	C_1	C_2	...	C_{12}	C_{13}
Main	0.086	0.120	...	0.043	0.045	0.029	0.051	...	0.047	0.046
1	0.12	0.086	...	0.043	0.045	0.029	0.051	...	0.046	0.046
2	0.043	0.120	...	0.043	0.045	0.029	0.051	...	0.047	0.047
3	0.073	0.120	...	0.043	0.045	0.029	0.051	...	0.047	0.046
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
17	0.086	0.120	...	0.043	0.045	0.029	0.049	...	0.048	0.046
18	0.086	0.120	...	0.043	0.045	0.026	0.045	...	0.043	0.041
19	0.071	0.071	...	0.071	0.071	0.030	0.051	...	0.044	0.045

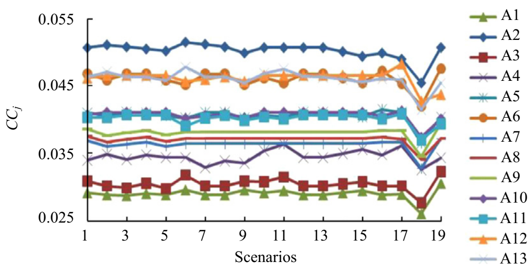

Fig. 6. Sensitivity analysis.

According to Table 6 and Fig. 6, the choice number 2 always takes first rank and choices number 4, 3 and 1 take the lowest rank which means they are respectively in the 11th, 12th and 13th rank. The choices number 7, 8 and 9 showed no special reaction to different scenarios and were almost stable; these three choices experience the maximum CC_j in scenario number 4, whereas the minimum CC_j occurred in scenario number 18. The choice number 11 obtained the minimum CC_j value in scenarios number 6 and 18. The choice number 6, achieved its highest value, which is the overall second rank, in normal weighting scenario (equal weight for all criteria).

Choice number 5, in the best case, took the 5th rank, when CR criteria was exchanged with EPG in scenario number 16. Choice number 10 achieved almost the same CC_j value as choice 11. When the weights changed in IC, IP and CR criteria in 17th scenario, choice 12 sharply exceeded choices 6 and 13 and was placed in the second rank with a small reduction in CC_j value relative to choice 2, which owned the first rank constantly. Choice 13, was placed in the second rank in scenarios number 2, 6, 10 and 11. Decision makers can use these weighting scenarios in the process of defining priorities.

5. Conclusions

Process selecting is to evaluate methods on a basis of technical, economic and environmental features and capacities in order to select limited number of methods for complementary studies. The aim of this research is to compare different alunite processing methods and to select the best case for recovering alumina from alunite ore. If there is any uncertainty in measurements, decision making process will be difficult, therefore, utilizing fuzzy preferences in explaining inaccuracy of factors can simplify complex decision making process. In other words, using word preferences is very effective in expressing uncertain states. In this research, TOPSIS is used to select the right process. Weighting criteria is carried out utilizing DAHP on a paired comparison basis. Sensitivity analysis on Fuzzy TOPSIS results proved that choices number 2, 6 and 12 that are respectively, Spanish, Svoronos and Hazen process, take the best ranks in most of our defined scenarios. In conclusion, by using fuzzy multiple criteria decision making models, these three processes are introduced as the most appropriate methods for alunite ore dressing.

Acknowledgments

Authors sincerely thank reviewers for their valuable comments that resulted in the current revised paper. Authors would also like to thank Sanaz Naderi (mining engineer) who provided supreme assistance during the research and article preparation procedure.

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