Effect of a magnetic field on Schwinger mechanism in de Sitter spacetime

Ehsan Bavarsad,[1,*] Sang Pyo Kim,[2,3] Clément Stahl,[4] and She-Sheng Xue[5,6]

[1]*Department of Physics, University of Kashan, 8731753153 Kashan, Iran*
[2]*Department of Physics, Kunsan National University, Kunsan 54150, Korea*
[3]*Center for Relativistic Laser Science, Institute for Basic Science, Gwangju 61005, Korea*
[4]*Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4950, Valparaíso, Chile*
[5]*ICRANet, Piazzale della Repubblica 10, 65122 Pescara, Italy*
[6]*Dipartimento di Fisica, Universita di Roma "La Sapienza", Piazzale Aldo Moro 5, 00185 Rome, Italy*

(Received 11 September 2017; revised manuscript received 19 December 2017; published 25 January 2018)

We investigate the effect of a uniform magnetic field background on scalar QED pair production in a four-dimensional de Sitter spacetime (dS_4). We obtain a pair production rate which agrees with the known Schwinger result in the limit of Minkowski spacetime and with Hawking radiation in dS spacetime in the zero electric field limit. Our results describe how the cosmic magnetic field affects the pair production rate in cosmological setups. In addition, using the zeta function regularization scheme we calculate the induced current and examine the effect of a magnetic field on the vacuum expectation value of the current operator. We find that, in the case of a strong electromagnetic background the current responds as $E \cdot B$, while in the infrared regime, it responds as B/E, which leads to a phenomenon of infrared hyperconductivity. These results for the induced current have important applications for the cosmic magnetic field evolution.

DOI: 10.1103/PhysRevD.97.025017

I. INTRODUCTION

A fascinating effect in quantum field theory is the Schwinger effect [1]: the creation of pairs of particle and antiparticle out of the vacuum in the presence of a background electromagnetic field. While it was Sauter [2], Heisenberg, and student Euler [3] who first investigated this effect, history has remembered Schwinger who revisited their works some 20 years later [4]. Despite being a very useful tool for the theoretical understanding of quantum field theory and for the development of powerful calculation techniques in a strong field background, the Schwinger effect has so far not been detected in laboratory experiments. The production of electron-positron pairs, however, was realized in an experiment, in which high-energy gamma rays are scattered with a Coulomb potential [5]. The main reason is that the Schwinger effect is exponentially suppressed unless the electric field is close enough to a threshold electric field $E_{\text{threshold}} \simeq 1.3 \times 10^{18}$ V/m [6]. A new idea developed in recent years is aimed at detecting this effect: changing the system under study and considering the Schwinger effect in astrophysical and cosmological contexts where huge background fields could naturally be present [7]. We will investigate in this paper the Schwinger effect in four-dimensional de Sitter spacetime (dS_4) under the influence of both a constant electric field and a uniform magnetic field background.

The Schwinger effect in dS spacetime has recently become an active field of research. The seminal papers studied this effect in two-dimensional de Sitter spacetime (dS_2) [8] and in dS_4 [9]. The one-loop vacuum polarization and Schwinger effect in a two-dimensional (anti–)de Sitter spacetime was explicitly found and a thermal interpretation was proposed for the Schwinger effect in Ref. [10]. The initial motivation of [8] was to use this framework to investigate bubble nucleation in the context of the multiverse proposal. However, this toy model for pair creation turns out to have a wide range of applications, from constraining magnetogenesis scenarios [9] and investigating the ER = EPR conjecture via holographic setups [11] to pair creation around charged black holes [12–14] and baryogenesis [15].

These physical motivations led to a series of papers in which the Schwinger mechanism was investigated for various types of particles and spacetime dimensions. It was investigated whether the known equivalence between bosonic and fermionic particles with respect to the Schwinger effect holds in dS_2 [16]. Particles differentiate themselves only if one goes beyond the semiclassical limit and computes the current which, in turn, is a more physically relevant quantity to describe the Schwinger

*bavarsad@kashanu.ac.ir

mechanism in curved spacetimes. These results were generalized to dS_4 in Ref. [17]. For bosons in dS_4, the results of Ref. [9] were reinforced in Ref. [18] which considered an alternative renormalization scheme and found the same results. In Ref. [19], an alternative method was employed: the uniform asymptotic method was used to derive new results for the Schwinger effect in dS_4; see also Ref. [20]. In Ref. [21], the Schwinger mechanism in three dimensions was explored as an example of odd-dimension field theory in dS. In all these works the gravitational field and electric field were assumed to be background fields whose variations due to backreactions are negligible during the typical time scale of pair creation. This approximation can be shown to hold for some range of the parameters. However, taking a constant background field can only be seen as a toy model to understand some physical implications of pair creation, and in realistic models of inflation requiring quasi-dS, the backreaction effects both on the dS metric and on the background electric field should be taken into account. In Refs. [21,22], it was shown that both the gravitational and electromagnetic fields would be suppressed by the Schwinger effect. In Refs. [23,24], it was pointed out that the quantum-gravity originated cosmological constant term $\Lambda g^{\mu\nu}$ results in the creation of particle-antiparticle pairs and their fields whose energy-momentum tensor $T_{\mathrm{M}}^{\mu\nu}$ in turn backreacts on $\Lambda g^{\mu\nu}$, and that these are important to understand the inflationary process in the early Universe and the dark energy–matter interaction for $\Omega_\Lambda \sim \Omega_{\mathrm{M}} \sim \mathcal{O}(1)$ in the present Universe. Recently it was argued that dS was unstable due to quantum effects [25–27]. The idea is that a nontrivial Bogoliubov transformation leads, after decoherence, to a breaking of the dS invariance and therefore to a decrease of the cosmological constant.

In this article, we propose to go one step further and add a uniform magnetic field to a dS spacetime and an already present electric background. This is a common generalization of a flat spacetime in which the analytic results have been known for a long time [4], but the Schwinger effect has never been properly investigated in dS. One motivation to consider a uniform magnetic field in dS is the recent result that a uniform magnetic field is a stable configuration of dS in modified gravity theories [28]. The effect of a uniform magnetic field exhibits new properties of the Schwinger mechanism compared to a pure electric field in dS. And another possible reason to consider an electromagnetic field in the early Universe would come from the observation of blazars leading to a lower bound for the magnetic field in the intergalactic medium: $B > 6 \times 10^{-18}$ G [29]. The origin of these magnetic fields is now an open question in cosmology but two main scenarios are emerging: their origin is either after recombination or primordial; see the reviews [30–33]. In the case of a primordial origin, just as for a scalar field, the vacuum fluctuations of the gauge field are amplified to larger scales. Once inflation comes to an end, the Universe becomes conductive, causing the electric field to vanish and the magnetic field to evolve until the present epoch via flux conservation. If the primordial origin of the currently observed magnetic field is adopted, it is necessary for inflation model builders to investigate physical effects due to the presence of an electromagnetic field, i.e., the Schwinger effect, which is the main topic of this paper. One constraint from the Schwinger effect has been worked out in this scenario: the Schwinger pairs screen the parametric amplification of the magnetic field leading to an upper bound on the current magnetic field [9]. We expect that including a magnetic field in the Schwinger pair setup should give a more stringent bound on the current magnetic field. Besides, it has been found [34] that the quantum fluctuations of the vacuum of a charged scalar field in dS generates a strong magnetic field which induces an instability of the vacuum [35].

The effect of a magnetic field background on the scalar pair creation probability [36] and the number density [37] in spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) universes has been investigated. In Ref. [36], the author showed that in the presence of a pure magnetic field background, i.e., in the absence of an electric field background, the gravitational pair creation does not change in dS, whereas in a radiation-dominated universe, a pure magnetic field background minimizes the gravitational pair creation [37]. In holographic setups, the inclusion of a magnetic field in the Schwinger effect was investigated in Ref. [38]. It is, however, difficult to compare that result directly with the case of dS under consideration in this paper. Adopting the perturbative QED approach in dS, the first-order amplitude for the fermion production in a magnetic field has been analyzed in Ref. [39]; see also Refs. [40,41]. The authors [39] found that the fermion production is significant only in strong gravitational fields. This paper aims at investigating the influence of the magnetic field on the Schwinger pair creation of charged scalars in dS_4, specifically, by computing the semiclassical decay rate and analyzing the quantum vacuum expectation value of the current operator, which is equivalent to the exact one-loop approach including all one-loop diagrams.

The organization of this paper is as follows. In Sec. II, the working assumptions on the gravitational and electromagnetic field backgrounds are presented. In Sec. III, we recall the main equations for charged scalars in a magnetic field as well as an electric field for the pair creation setup. In Sec. IV, we compute the pair creation rate using a semiclassical approach to the exact one-loop order. In Sec. V, we present an expression for the induced current and discuss several relevant limiting cases of different field intensities. We draw some conclusions and future lines of research in Sec. VI. Appendix A contains some mathematical aspects of this work: some useful properties of the Riemann and Hurwitz zeta functions. Eventually, in Appendix B, the computation and regularization of the current are reviewed.

II. GRAVITATIONAL AND ELECTROMAGNETIC BACKGROUNDS

To study the Schwinger effect in dS_4, we consider the action of a complex scalar field coupled to a $U(1)$ gauge field as

$$S = \int d^4x\sqrt{-g}\Big[g^{\mu\nu}(\partial_\nu - ieA_\nu)\varphi^*(\partial_\mu + ieA_\mu)\varphi - (m^2+\xi R)\varphi\varphi^* - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\Big], \quad (1)$$

where e is the gauge coupling (the charge of the particle), m is the mass of the scalar field, and ξ is a dimensionless nonminimal coupling. From now on, we consider, for simplicity, the minimal coupling case and set $\xi = 0$. We assume that the complex scalar field is a test field probing two background fields: the gravitational field and the electromagnetic field.

A. Gravitational field background

The gravitational field is described by the dS_4 metric which reads in the conformal coordinates as

$$ds^2 = \Omega^2(\tau)(d\tau^2 - dx^2 - dy^2 - dz^2),$$
$$\tau \in (-\infty, 0), \qquad \mathbf{x} = (x, y, z) \in \mathbb{R}^3, \quad (2)$$

where the scale factor $\Omega(\tau)$ and the Hubble constant H are given by

$$\Omega(\tau) = -\frac{1}{H\tau}, \qquad H = \Omega^{-2}(\tau)\frac{d\Omega(\tau)}{d\tau}. \quad (3)$$

We add that this slicing of dS covers only one half of the manifold: the Poincaré patch.

B. Electromagnetic field background

For the electromagnetic field, we consider that it is composed of a uniform electric field with a constant energy density parallel to a uniform magnetic field with a conserved flux. We choose a vector potential A_μ in the Coulomb gauge

$$A_0 = 0, \qquad \partial_i A_i = 0; \quad (4)$$

then, with respect to a comoving observer with a four-velocity vector $u^\mu = \Omega^{-1}(\tau)\delta^\mu_0$, the time and spatial components of the electric and magnetic field vectors are given by [9]

$$E_0 = 0, \qquad E_i(\tau) = \Omega^{-1}(\tau)\partial_0 A_i,$$
$$B_0 = 0, \qquad B_i(\tau) = \Omega^{-1}(\tau)\tilde{\epsilon}_{0ijk}\partial_j A_k, \quad (5)$$

where $\tilde{\epsilon}_{\mu\nu\rho\sigma}$ is the completely antisymmetric Levi-Civita symbol which is normalized as $\tilde{\epsilon}_{0123} = 1$. Having a uniform electric field with a constant energy density requires that the magnitude of the electric field vector satisfies

$$E_\mu(\tau)E^\mu(\tau) = -\Omega^{-2}(\tau)E_i(\tau)E_i(\tau) \equiv -E^2, \quad (6)$$

where E is a constant. In order to describe a uniform magnetic field with a conserved flux, the magnitude of the magnetic field vector should satisfy

$$B_\mu(\tau)B^\mu(\tau) = -\Omega^{-2}(\tau)B_i(\tau)B_i(\tau) \equiv -\Omega^{-4}(\tau)B^2, \quad (7)$$

where B is a constant. In this paper, we shall consider the case where the electric and magnetic fields are parallel to each other, namely along the z axis. It can be verified that the vector potential describing such electric and magnetic fields along the z axis, in the conformal metric (2), is given by

$$A_\mu(x) = -\frac{E}{H^2\tau}\delta^3_\mu + By\delta^1_\mu. \quad (8)$$

To discuss the Maxwell equation satisfied by the electromagnetic field configuration (8), we need to distinguish two types of currents: the induced current J of the newly created Schwinger pairs which will be calculated in Sec. V, and the classical background current $J^\mu_{\rm b}$. We assume that the electromagnetic field background $F^{\mu\nu}$ is a classical field which is produced by the background current $J^\mu_{\rm b}$ and is not affected by the pair production. In this approximation, the induced current J of the Schwinger charges is neglected. Hence, the general covariant Maxwell equation is written as

$$\nabla_\mu F^{\mu\nu} = |g|^{-\frac{1}{2}}\partial_\mu(|g|^{\frac{1}{2}}F^{\mu\nu}) = J^\nu_{\rm b}. \quad (9)$$

This leads to the Maxwell equation for the vector potential in the Coulomb gauge (4) and the geometry (2),

$$\partial_0^2 A_i - \partial_j\partial_j A_i = \Omega^2(\tau)J_{{\rm b},i}. \quad (10)$$

Substituting the vector potential (8) into the Maxwell equation (10) implies that the background current is

$$J^\mu_{\rm b} = -2EH\Omega^{-1}(\tau)\delta^\mu_3. \quad (11)$$

We comment here that in order to have a well-defined and self-consistent model, the background current (11) has to be larger than the induced current (see Sec. V below) of the pairs created by the Schwinger mechanism. Besides, this background current (11) should also be weak enough not to gravitate and significantly modify the gravitational field background (2). This implies that the electromagnetic energy density $E^2 + \Omega^{-4}(\tau)B^2$ is much smaller than the vacuum energy density $H^2M_P^2$ [34], where M_P is the Planck mass.

III. KLEIN-GORDON EQUATION

Using Eqs. (2) and (8), the Klein-Gordon equation reads from the action (1),

$$\left[\partial_0^2 + 2H\Omega(\tau)\partial_0 - (\partial_1 + ieBy)^2 - \partial_2^2 - \left(\partial_3 + \frac{ieE}{H}\Omega(\tau)\right)^2 + m^2\Omega^2(\tau)\right]\varphi(x) = 0. \quad (12)$$

The solution of the spatial part of Eq. (12) is a bit more involved than a simple Fourier transformation because of the explicit y dependence. Substituting

$$\varphi(x) = \Omega^{-1}(\tau)\tilde{\varphi}(x), \quad (13)$$

into Eq. (12) yields

$$\left[\partial_0^2 - (\partial_1 + ieBy)^2 - \partial_2^2 - \left(\partial_3 + \frac{ieE}{H}\Omega(\tau)\right)^2 + m^2\Omega^2(\tau) - 2H^2\Omega^2(\tau)\right]\tilde{\varphi}(x) = 0. \quad (14)$$

Using the ansatz

$$\tilde{\varphi}(x) = e^{\pm i\mathbf{x}\cdot\mathbf{k}_{\not{y}}}h^{\pm}(y)f^{\pm}(\tau), \quad (15)$$

where we have defined

$$\mathbf{k}_{\not{y}} \coloneqq (k_x, 0, k_z), \quad (16)$$

and $\pm$ denotes the positive- and negative-frequency solutions of Eq. (14), respectively, we decouple the spatial and time-dependent parts of Eq. (14) as

$$\frac{d^2h^{\pm}(y)}{dy^2} - (eBy \pm k_x)^2h^{\pm}(y) = -sh^{\pm}(y), \quad (17)$$

$$\frac{d^2f^{\pm}(\tau)}{d\tau^2} + \left[\left(\frac{eE}{H^2\tau} \mp k_z\right)^2 + \frac{m^2}{H^2\tau^2} - \frac{2}{\tau^2}\right]f^{\pm}(\tau) = -sf^{\pm}(\tau). \quad (18)$$

The harmonic wave function $h^{\pm}(y)$ is a Landau state given by

$$h_n(y_{\pm}) = \sqrt{\frac{\sqrt{eB}}{\sqrt{\pi}2^n n!}}\exp\left(-\frac{y_{\pm}^2}{2}\right)H_n(y_{\pm}),$$
$$y_{\pm} \coloneqq \sqrt{eB}y \pm \frac{k_x}{\sqrt{eB}}, \quad (19)$$

where H_n with $n \in \mathbb{N}$ is the Hermite polynomial and s is the Landau energy

$$s = (2n+1)eB. \quad (20)$$

The normalized wave functions (19) satisfy the orthonormality relation

$$\int_{-\infty}^{+\infty} dy h_n(y_{\pm})h_{n'}(y_{\pm}) = \delta_{n,n'}, \quad (21)$$

and the completeness relation

$$\sum_{n=0}^{\infty} h_n(y_{\pm})h_n(y'_{\pm}) = \delta(y - y'), \quad (22)$$

where $y'_{\pm}$ is given by replacing y by y' in the definition (19) of $y_{\pm}$. We note that the standard prescription in a flat spacetime applies also to our results; when one adds a magnetic field, the pair creation in the general case can be deduced from the pure electric field case ($B = 0$) by replacing the transverse momentum squared $\mathbf{k}_{\perp}^2$ by the Landau levels $(2n+1)eB$. Following Refs. [13,21], the positive- and negative-frequency solutions with the desired asymptotic forms at early times ($\tau \to -\infty$), i.e., the in-vacuum mode functions are given by the Hadamard states

$$U_{\text{in}}(x; \mathbf{k}_{\not{y}}, n) = \frac{e^{\frac{i\pi\kappa}{2}}}{\sqrt{2k}}\Omega^{-1}(\tau)e^{+i\mathbf{x}\cdot\mathbf{k}_{\not{y}}}h_n(y_+)W_{\kappa,\gamma}(e^{\frac{-i\pi}{2}}2p), \quad (23)$$

$$V_{\text{in}}(x; \mathbf{k}_{\not{y}}, n) = \frac{e^{-\frac{i\pi\kappa}{2}}}{\sqrt{2k}}\Omega^{-1}(\tau)e^{-i\mathbf{x}\cdot\mathbf{k}_{\not{y}}}h_n(y_-)W_{\kappa,-\gamma}(e^{\frac{+i\pi}{2}}2p). \quad (24)$$

Similarly, the positive- and negative-frequency solutions with the desired asymptotic forms at late times ($\tau \to 0$), i.e., the out-vacuum mode functions are given by

$$U_{\text{out}}(x; \mathbf{k}_{\not{y}}, n) = \frac{e^{\frac{i\pi\gamma}{2}}}{\sqrt{4|\gamma|k}}\Omega^{-1}(\tau)e^{+i\mathbf{x}\cdot\mathbf{k}_{\not{y}}}h_n(y_+)M_{\kappa,\gamma}(e^{\frac{-i\pi}{2}}2p), \quad (25)$$

$$V_{\text{out}}(x; \mathbf{k}_{\not{y}}, n) = \frac{e^{\frac{i\pi\gamma}{2}}}{\sqrt{4|\gamma|k}}\Omega^{-1}(\tau)e^{-i\mathbf{x}\cdot\mathbf{k}_{\not{y}}}h_n(y_-)M_{\kappa,-\gamma}(e^{\frac{+i\pi}{2}}2p). \quad (26)$$

Here, $W_{\kappa,\gamma}$ and $M_{\kappa,\gamma}$ are some hypergeometrical functions known as the Whittaker functions [42] and the parameters have been defined as

$$k = \sqrt{k_z^2 + (2n+1)eB}, \qquad r = \frac{k_z}{k}, \qquad p = -\tau k,$$
$$\mathbf{p}_{\not{z}} = -\tau \mathbf{k}_{\not{z}}, \qquad \ell = eB\tau^2, \qquad \mu = \frac{m}{H},$$
$$\lambda = \frac{eE}{H^2}, \qquad \kappa = i\lambda r, \qquad \gamma = \sqrt{\frac{9}{4} - \lambda^2 - \mu^2}. \tag{27}$$

In Secs. III and IV of this paper, we assume the *semi-classical condition*,

$$\lambda^2 + \mu^2 \gg 1, \tag{28}$$

and hence the parameter γ is purely imaginary. We adopt the sign convention $\gamma = +i|\gamma|$.

The orthonormality relations

$$\begin{aligned}
&(U_{\mathrm{in(out)}}(x; \mathbf{k}_{\not{z}}, n), U_{\mathrm{in(out)}}(x; \mathbf{k}_{\not{z}}{}', n')) \\
&= -(V_{\mathrm{in(out)}}(x; \mathbf{k}_{\not{z}}, n), V_{\mathrm{in(out)}}(x; \mathbf{k}_{\not{z}}{}', n')) \\
&= (2\pi)^2 \delta^2(\mathbf{k}_{\not{z}} - \mathbf{k}_{\not{z}}{}')\delta_{n,n'}, \\
&\quad (U_{\mathrm{in(out)}}(x; \mathbf{k}_{\not{z}}, n), V_{\mathrm{in(out)}}(x; \mathbf{k}'_{\not{z}}, n')) = 0,
\end{aligned} \tag{29}$$

can be shown to hold. Using two complete sets of orthonormal mode functions, we expand the scalar field operator. In terms of the in-mode functions we can express the field operator as

$$\begin{aligned}
\varphi(x) = \sum_{n=0}^{\infty} \int \frac{d^2 k_{\not{z}}}{(2\pi)^2} [&U_{\mathrm{in}}(x; \mathbf{k}_{\not{z}}, n) a_{\mathrm{in}}(\mathbf{k}_{\not{z}}, n) \\
&+ V_{\mathrm{in}}(x; \mathbf{k}_{\not{z}}, n) b^{\dagger}_{\mathrm{in}}(\mathbf{k}_{\not{z}}, n)],
\end{aligned} \tag{30}$$

where the operator a_{in} annihilates a particle and the operator $b^{\dagger}_{\mathrm{in}}$ creates an antiparticle in the state with the momentum $\mathbf{k}_{\not{z}}$ and the Landau level n. The quantization is implemented by imposing the commutation relations

$$\begin{aligned}
[a_{\mathrm{in}}(\mathbf{k}_{\not{z}}, n), a^{\dagger}_{\mathrm{in}}(\mathbf{k}'_{\not{z}}, n')] &= [b_{\mathrm{in}}(\mathbf{k}_{\not{z}}, n), b^{\dagger}_{\mathrm{in}}(\mathbf{k}'_{\not{z}}, n')] \\
&= (2\pi)^2 \delta^2(\mathbf{k}_{\not{z}} - \mathbf{k}'_{\not{z}})\delta_{n,n'},
\end{aligned} \tag{31}$$

and the in-vacuum state is defined as

$$a_{\mathrm{in}}(\mathbf{k}_{\not{z}}, n)|\mathrm{in}\rangle = 0, \quad \forall\ \mathbf{k}_{\not{z}}, n. \tag{32}$$

We can expand the scalar field operator in terms of the out-mode functions and we similarly define the out-annihilation a_{out} and creation $b^{\dagger}_{\mathrm{out}}$ operators as

$$\begin{aligned}
\varphi(x) = \sum_{n=0}^{\infty} \int \frac{d^2 k_{\not{z}}}{(2\pi)^2} [&U_{\mathrm{out}}(x; \mathbf{k}_{\not{z}}, n) a_{\mathrm{out}}(\mathbf{k}_{\not{z}}, n) \\
&+ V_{\mathrm{out}}(x; \mathbf{k}_{\not{z}}, n) b^{\dagger}_{\mathrm{out}}(\mathbf{k}_{\not{z}}, n)],
\end{aligned} \tag{33}$$

where the quantization commutation relations are given by

$$\begin{aligned}
[a_{\mathrm{out}}(\mathbf{k}_{\not{z}}, n), a^{\dagger}_{\mathrm{out}}(\mathbf{k}'_{\not{z}}, n')] &= [b_{\mathrm{out}}(\mathbf{k}_{\not{z}}, n), b^{\dagger}_{\mathrm{out}}(\mathbf{k}'_{\not{z}}, n')] \\
&= (2\pi)^2 \delta^2(\mathbf{k}_{\not{z}} - \mathbf{k}'_{\not{z}})\delta_{n,n'},
\end{aligned} \tag{34}$$

and the out-vacuum state is defined as

$$a_{\mathrm{out}}(\mathbf{k}_{\not{z}}, n)|\mathrm{out}\rangle = 0, \quad \forall\ \mathbf{k}_{\not{z}}, n. \tag{35}$$

The canonical momentum $\pi(x)$ conjugated to the scalar field $\varphi(x)$ is defined through the Lagrangian. It reads from Eq. (1),

$$\pi(x) = \frac{\partial \mathcal{L}}{\partial(\partial_0 \varphi)} = \Omega^2(\tau)\partial_0 \varphi^*. \tag{36}$$

Then, using the explicit form of the scalar field operator $\varphi(x)$ and the canonical momentum $\pi(x)$ in terms of the mode functions, one can verify that the canonical equal-time commutation relation correctly holds

$$[\varphi(\tau, \mathbf{x}), \pi(\tau, \mathbf{x}')] = i\delta^3(\mathbf{x} - \mathbf{x}'). \tag{37}$$

IV. SCHWINGER EFFECT

The usual quantity describing the Schwinger effect is the pair creation or decay rate which is derived from the Bogoliubov coefficients [43,44],

$$\mathcal{A}(\mathbf{k}_{\not{z}}, n; \mathbf{k}'_{\not{z}}, n') = (U_{\mathrm{out}}(x; \mathbf{k}_{\not{z}}, n), U_{\mathrm{in}}(x; \mathbf{k}'_{\not{z}}, n')), \tag{38}$$

$$\mathcal{B}(\mathbf{k}_{\not{z}}, n; \mathbf{k}'_{\not{z}}, n') = -(U_{\mathrm{out}}(x; \mathbf{k}_{\not{z}}, n), V_{\mathrm{in}}(x; \mathbf{k}'_{\not{z}}, n')). \tag{39}$$

Substituting the explicit form of the mode functions (23)–(26) into Eqs. (38) and (39) leads to

$$\begin{aligned}
\mathcal{A}(\mathbf{k}_{\not{z}} n; \mathbf{k}'_{\not{z}} n') &= (2\pi)^2 \delta^2(\mathbf{k}_{\not{z}} - \mathbf{k}'_{\not{z}})\delta_{n,n'}\alpha, \\
\alpha &= \frac{(2|\gamma|)^{\frac{1}{2}}\Gamma(2\gamma)}{\Gamma(\frac{1}{2} + \kappa + \gamma)} e^{\frac{i\pi}{2}(\kappa - \gamma)},
\end{aligned} \tag{40}$$

$$\begin{aligned}
\mathcal{B}(\mathbf{k}_{\not{z}} n; \mathbf{k}_{\not{z}}{}' n') &= (2\pi)^2 \delta^2(\mathbf{k}_{\not{z}} + \mathbf{k}'_{\not{z}})\delta_{n,n'}\beta, \\
\beta &= -i\frac{(2|\gamma|)^{\frac{1}{2}}\Gamma(-2\gamma)}{\Gamma(\frac{1}{2} + \kappa - \gamma)} e^{\frac{i\pi}{2}(\kappa + \gamma)},
\end{aligned} \tag{41}$$

where the coefficients satisfy the bosonic relation $|\alpha|^2 - |\beta|^2 = 1$. A Bogoliubov transformation relates the out-operator a_{out} to the in-operator a_{in} as

$$\begin{aligned}
a_{\mathrm{out}}(\mathbf{k}_{\not{z}}, n) = \sum_{n'=0}^{\infty} \int \frac{d^2 k'_{\not{z}}}{(2\pi)^2} [&\mathcal{A}^*(\mathbf{k}_{\not{z}}, n; \mathbf{k}'_{\not{z}}, n') a_{\mathrm{in}}(\mathbf{k}'_{\not{z}}, n') \\
&- \mathcal{B}^*(\mathbf{k}_{\not{z}}, n; \mathbf{k}'_{\not{z}}, n') b^{\dagger}_{\mathrm{in}}(\mathbf{k}'_{\not{z}}, n')].
\end{aligned} \tag{42}$$

Using the out-operator $a_{\rm out}(\mathbf{k}_{\not y}, n)$, we can calculate the expected number of created pairs with the comoving momentum $\mathbf{k}_{\not y}$ and the Landau level n carried by the in-vacuum state

$$\frac{1}{L_x L_z}\langle {\rm in}|a^\dagger_{\rm out}(\mathbf{k}_{\not y}, n) a_{\rm out}(\mathbf{k}_{\not y}, n)|{\rm in}\rangle = |\beta(k_z, n)|^2, \quad (43)$$

where we have used Eqs. (41) and (42) and, for convenience, the three-volume of dS_4 is normalized into a box with dimensions $V = L_x L_y L_z$. Then the decay rate Γ, i.e., the number of created pairs N per unit of the physical four-volume of the dS_4 is given by

$$\Gamma := \frac{N}{\sqrt{|g|}TV} = \frac{1}{\Omega^4(\tau)TL_y}\sum_{n=0}^{\infty}\int \frac{dk_z}{(2\pi)}\frac{dk_x}{(2\pi)}|\beta(k_z, n)|^2, \quad (44)$$

where T is the time interval of the pair creation. The Bogoliubov coefficient β is independent of the momentum component k_x which determines the position of the center of the Gaussian wave packet on the y axis by the relation $y = k_x/(eB)$. Consequently, the integral gives [45]

$$\int \frac{dk_x}{(2\pi)} = \frac{eBL_y}{(2\pi)}. \quad (45)$$

To perform the k_z integral on the right-hand side of Eq. (44), we adopt the semiclassical method used in Refs. [8,9]: most of the particles are created around the time

$$\tau \sim -\frac{|\gamma|}{k_z}. \quad (46)$$

Imposing the relation (46) and transforming the k_z integral into a τ integral, we then obtain

$$\Gamma = \frac{H^4 \ell |\gamma|}{4\pi^2}\sum_{n=0}^{\infty}\frac{e^{2\pi|\kappa|} + e^{-2\pi|\gamma|}}{e^{2\pi|\gamma|} - e^{-2\pi|\gamma|}}, \quad (47)$$

where

$$|\kappa| = \frac{\lambda|\gamma|}{\sqrt{|\gamma|^2 + (2n+1)\ell}}. \quad (48)$$

A physical magnetic field in a spatially flat FLRW universe with a cosmological scale factor $\Omega(\tau)$ dilutes as $B\Omega^{-2}(\tau)$ where B behaves as a magnetic field in the comoving spacetime [46,47]. This preserves the flux conservation for the physical magnetic field. Recalling that $\ell = eB\tau^2$, consequently, the decay rate Γ depends on the time τ due to the dilution of the physical magnetic field. We may write Eq. (47) in another form

$$\Gamma = \left(\frac{eB\Omega^{-2}}{2\pi}\right)\left(\frac{H^2|\gamma|}{2\pi}\right)\sum_{n=0}^{\infty}\left[\frac{e^{2\pi|\kappa|} - 1}{e^{2\pi|\gamma|} - e^{-2\pi|\gamma|}} + \frac{1}{e^{2\pi|\gamma|} - 1}\right]. \quad (49)$$

The first term in the square brackets in Eq. (49) is the pair creation rate from the electromagnetic field while the second term is the dS radiation with a new temperature $T = m/(2\pi|\gamma|)$ weighted by the density of states for the electromagnetic field.

A few comments are in order. First, there is a term independent of the Landau levels, whose sum apparently gives a diverging factor. We tackle this issue by using the Riemann zeta function prescription as in Ref. [48]. We also use the $n = 0$ term which gives a constant factor

$$\sum_{n=0}^{\infty} = 1 + \zeta(0) = \frac{1}{2}, \quad (50)$$

where Eq. (A3) has been used. Thus, the pair production from the zeta regularization technique leads to a finite result

$$\Gamma = \left(\frac{eB\Omega^{-2}}{2\pi}\right)\left(\frac{H^2|\gamma|}{2\pi}\right)\left(\frac{1}{e^{4\pi|\gamma|} - 1}\right)\left[\frac{1}{2} + \sum_{n=0}^{\infty} e^{2\pi(|\kappa|+|\gamma|)}\right]. \quad (51)$$

Second, in the regime of the weak magnetic field [$\ell \ll \min(1, \mu, \lambda)$] and the strong electric field [$\lambda \gg \max(1, \mu, \ell)$], Eq. (51) leads to

$$\Gamma = \frac{1}{2}\left(\frac{eB\Omega^{-2}}{2\pi}\right)\left(\frac{eE}{2\pi}\right)e^{\frac{-\pi m^2}{|eE|}}. \quad (52)$$

Third, in the limit of zero electric field $E = 0$, the first term in the square brackets of Eq. (49) vanishes and the second term is the dS radiation with the Gibbons-Hawking temperature [49]

$$\Gamma = \frac{1}{2}\left(\frac{eB\Omega^{-2}}{2\pi}\right)\left(\frac{H^2|\gamma|}{2\pi}\right)\frac{1}{e^{2\pi|\gamma|} - 1}. \quad (53)$$

The factor $1/2$ comes from the spin multiplicity for spinless bosons while it is 1 for spin-1/2 fermions. The radiation in the pure dS_4 without electromagnetic fields consists of massive particles $m \geq 3H/2$ and the leading term of $H^2|\gamma|$ is Hm for the density of states [50]. Thus, the presence of a cosmic magnetic field enhances the dS radiation through the density of states by a factor of $eB\Omega^{-2}$. The density of states eB becomes H^2 when there is no magnetic field. Finally, in the Minkowski spacetime limit $H = 0$, Eq. (47) gives the Schwinger formula in scalar QED [4]

$$\Gamma = \frac{1}{2}\left(\frac{eB}{2\pi}\right)\left(\frac{eE}{2\pi}\right)\frac{e^{\frac{-\pi m^2}{|eE|}}}{\sinh(\frac{\pi B}{E})}. \quad (54)$$

V. INDUCED CURRENT

Semiclassically, the conductive current $J_{\rm sem}$ of the newly created Schwinger pairs having a charge e, a number density $\mathcal{N}$, and a velocity v due to the background electric field is defined as $J_{\rm sem} = 2e\mathcal{N}v$. The number density of the semiclassical Schwinger pairs at the time τ reads

$$\mathcal{N}(\tau) = \Omega^{-3}(\tau)\int_{-\infty}^{\tau}\Omega^4(\tau')\Gamma(\tau')d\tau' \sim \frac{\Gamma(\tau)}{H}, \quad (55)$$

where Γ is given by Eq. (47). The current $J_{\rm sem}$ is valid under the semiclassical condition, which is given by Eq. (28). In this section we investigate the in-vacuum expectation value of the current operator which is referred to as the induced current, without assuming the constraint (28) on the parameters. Hence, γ can be real or purely imaginary depending on the value of involved parameters, λ and μ.

The current operator is defined by

$$j^\mu(x) = \frac{ie}{2}g^{\mu\nu}(\{(\partial_\nu + ieA_\nu)\varphi, \varphi^*\} - \{(\partial_\nu - ieA_\nu)\varphi^*, \varphi\}), \quad (56)$$

and can be shown to be conserved $\nabla_\mu j^\mu = 0$ [43]. In order to compute the expectation value of the current operator, we will use the in-vacuum state since it is a Hadamard state [8,51]. Substituting the scalar field operator (30) into the current expression (56) and using Eqs. (31) and (32), we find that the only nonvanishing component of the current is the one parallel to the electric field which is given by

$$\langle{\rm in}|j^3(x)|{\rm in}\rangle = \frac{eH^2}{4\pi^2}\Omega^{-2}(\tau)\sum_{n=0}^{\infty} \times \int_{-\infty}^{+\infty}\frac{dp_z}{p}(rp+\lambda)e^{-\pi\lambda r}|W_{i\lambda r,\gamma}(-2ip)|^2 \times \int_{-\infty}^{+\infty}dp_x h_n^2(y_+). \quad (57)$$

Using the orthonormality relation (21) the p_x integral is performed

$$\int_{-\infty}^{+\infty}dp_x h_n^2(y_+) = -eB\tau. \quad (58)$$

If we parametrize the induced current as

$$J = \Omega(\tau)\langle{\rm in}|j^3(x)|{\rm in}\rangle, \quad (59)$$

then Eq. (57) is simplified to

$$J = \frac{eH^3\ell}{4\pi^2}\sum_{n=0}^{\infty}\int_{-\infty}^{+\infty}\frac{dp_z}{p}(rp+\lambda)e^{-\pi\lambda r}|W_{i\lambda r,\gamma}(-2ip)|^2. \quad (60)$$

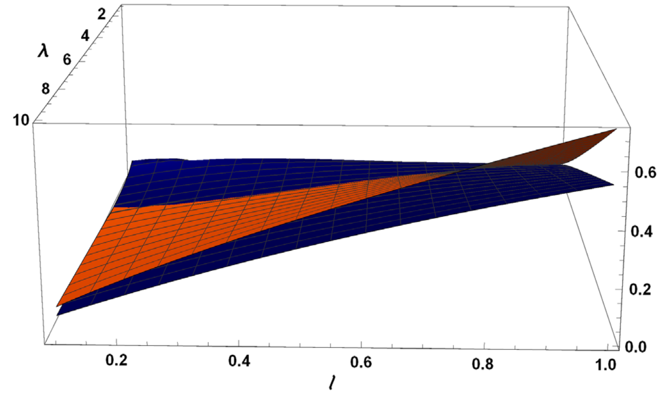

FIG. 1. The normalized induced current J/eH^3 (upper surface) and semiclassical current $J_{\rm sem}/eH^3$ (lower surface) are plotted as functions of λ and ℓ, in the lowest Landau state $n = 0$ with $\mu = 1$.

The remaining integral in the induced current expression (60) deals with the Whittaker functions. In the absence of the magnetic field background, the translational symmetry helps us perform the integral using the Mellin-Barnes representation of the Whittaker functions; see Refs. [8,9]. However, even in this case the exact expression for the induced current is very complicated and one has to look at limiting regimes to better grasp the physics of the results. In the regime of $\lambda \gg 1$ the semiclassical condition (28) is satisfied, and the induced current (60) is comparable to the semiclassical current $J_{\rm sem} = 2e\mathcal{N}v$. Considering ultrarelativistic particles with velocity $v \sim 1$, Fig. 1 shows that the induced current J approaches the semiclassical current $J_{\rm sem}$ for the strong electric field regime $\lambda \gg \max(1, \mu, \ell)$. In Figs. 2 and 3 we plot the induced current expression (60) as a function of the electric and magnetic fields, respectively. The figures illustrate that the induced current of a massive scalar field responds to the strong electromagnetic field as $J \propto B \cdot E$; for additional numerical investigations see Ref. [52]. As a matter of consistency,

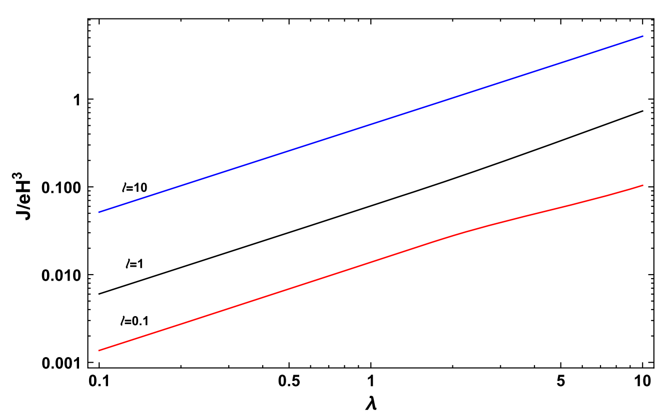

FIG. 2. For different values of ℓ, the normalized induced current J/eH^3 is plotted as a function of λ, in the lowest Landau state $n = 0$ with $\mu = 1$.

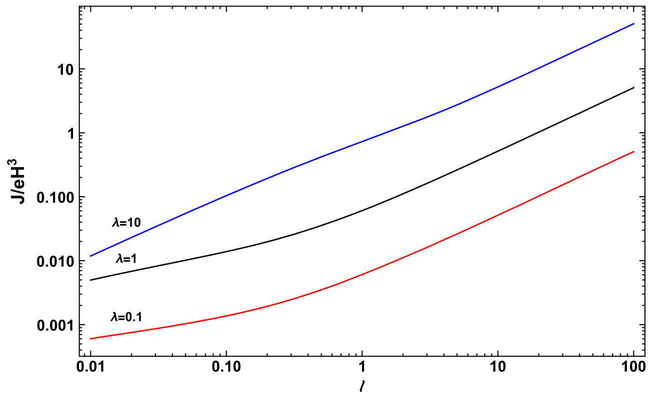

FIG. 3. For different values of λ, the normalized induced current J/eH^3 is plotted as a function of ℓ, in the lowest Landau state $n = 0$ with $\mu = 1$.

we will now analytically investigate the limiting behavior of the induced current (60) to show that it agrees with the numerical investigations.

A. Weak magnetic field regime

In the weak magnetic field regime the relation $\ell \ll \min(1, \mu, \lambda)$ is satisfied. Taking the limit $\ell \to 0$ in the momentum p gives $p \sim |p_z|$; see the definition of p in Eq. (27). Then the induced current expression (60) is simplified to

$$J \simeq \frac{eH^3\ell}{4\pi^2}\sum_{n=0}^{\infty}\sum_{r=\pm1}\int_0^\infty \frac{dp_z}{p_z}(rp_z+\lambda)e^{-\pi\lambda r}|W_{i\lambda r,\gamma}(-2ip_z)|^2. \tag{61}$$

The integrand on the right-hand side of Eq. (61) is independent of the Landau states. Hence, similarly to the prescription used in Sec. IV, using the zeta function representation (50), the current expression (61) is regularized to

$$J \simeq \frac{eH^3\ell}{8\pi^2}\sum_{r=\pm1}\int_0^\infty \frac{dp_z}{p_z}(rp_z+\lambda)e^{-\pi\lambda r}|W_{i\lambda r,\gamma}(-2ip_z)|^2. \tag{62}$$

The computation and adiabatic regularization of the current (62) are reviewed in Appendix B and the final result can be read from Eq. (B14). We then obtain

$$J_{\text{reg}} = \left(\frac{eH^3}{4\pi^2}\right)\frac{\ell\gamma\sinh(2\pi\lambda)}{\sin(2\pi\gamma)}. \tag{63}$$

We comment here that our result is unlike the case of a pure electric field in dS_4 [9], where in order to renormalize the current an adiabatic expansion up to order two has been performed to remove the quadratic divergence; here the zeroth adiabatic order is enough as in the dS_2 case [8]. The reason is that we deal here with an effective integration in $1 + 1$ dimensions, and the integration over momentum in the directions orthogonal to the magnetic field is replaced by a discrete sum over quantized Landau levels, which is regularized and renormalized by using the Riemann zeta function technique; see Appendix A.

1. Strong electric field regime

In the strong electric field regime the relation $\lambda \gg \max(1, \mu, \ell)$ is satisfied. Taking the limit $\lambda \to \infty$ in the regularized induced current (63) with μ and ℓ fixed, to the leading order, gives rise to

$$J_{\text{reg}} \simeq \frac{e}{H}\left(\frac{eB\Omega^{-2}}{2\pi}\right)\left(\frac{eE}{2\pi}\right)e^{\frac{-\pi m^2}{|eE|}}. \tag{64}$$

In this regime the decay rate is given by Eq. (52) and the semiclassical current follows from Eq. (55). Then, one can verify that the induced current (64) agrees with the semiclassical current for particles with velocity $v \sim 1$.

We remark that the limit $eE/H^2 \gg 1$ of Eq. (60) presented in Eq. (64) nicely recovers the Schwinger formula, which comes from the fact that the background current (11) vanishes in the Minkowski spacetime limit, when $H \to 0$ and $\Omega(\tau) \to 1$. Therefore, this model could be relevant for modeling our Universe deep in the inflationary regime when a de Sitter spacetime (2) can be assumed or close enough to the Minkowski spacetime limit so that the background current (11) stays weak.

2. Weak electric field and heavy scalar field regime

In this regime the relations $\lambda \ll 1$ and $\mu \gg 1$ are satisfied. Taking the limits $\lambda \to 0$ and $\mu \to \infty$ in the regularized induced current expression (63) with ℓ fixed, to the leading order, gives rise to

$$J_{\text{reg}} \simeq \frac{4\pi em}{H^2}\left(\frac{eB\Omega^{-2}}{2\pi}\right)\left(\frac{eE}{2\pi}\right)e^{\frac{-2\pi m}{H}}. \tag{65}$$

In this regime the decay rate from Eq. (53) and the induced current (65) agree with the semiclassical current J_{sem} for particles with velocity $v \sim (4\pi eE)/H^2$.

3. Infrared regime

In this regime the relations $\ell \ll \mu \ll \lambda \ll 1$ are satisfied. Hence the semiclassical current cannot be compared to the induced current in this regime. Taking the limits $\lambda \to 0$ and $\mu \to 0$ in the induced current expression (63), we then find

$$J_{\text{reg}} \simeq \frac{9eH^3}{8\pi^2}\left(\frac{\ell\lambda}{\lambda^2+\mu^2}\right), \tag{66}$$

or in terms of dimensionful variables

$$J_{\rm reg} \simeq \frac{9eH^3}{2}\left(\frac{eB\Omega^{-2}}{2\pi}\right)\left(\frac{eE}{2\pi}\right)\left(\frac{1}{(eE)^2+(mH)^2}\right). \quad (67)$$

In this regime for an interval of $\mu \lesssim \lambda \lesssim 1$ a decreasing electric field gives rise to an increasing current and consequently hyperconductivity. This infrared phenomenon was first discovered in Ref. [8] and dubbed infrared hyperconductivity (IRHC) for the case of a scalar field coupled to a constant, purely electric field background in dS_2. In Ref. [9], using an alternative approach to that in Ref. [19], the authors computed the current due to a pure electric field in dS_4 and found IRHC. In Ref. [9], the second-order adiabatic expansion led to a term of the form $\log(m/H)$ in the regularized induced current expression. Therefore, it was not possible to discuss IRHC for the case of a massless minimally coupled scalar field. However, we note here that the inclusion of the magnetic field and the change of the renormalization prescription allow to explore IRHC in the massless limit. We find indeed that the induced current responds as $J \sim B/E$ and increases unboundedly in the case of a massless minimally coupled scalar field. For a massive scalar field, an upper bound on the induced current occurs at $\lambda = \mu$ and is given by

$$J_{\rm reg} \simeq \frac{9eH^2}{8\pi m}\left(\frac{eB\Omega^{-2}}{2\pi}\right). \quad (68)$$

The exact nature of IRHC remains a mystery but it has been reported in various works in the past. It is unexpected as for a decreasing cause (the electric field background) the consequence (the induced current due to the creation of Schwinger pairs) increases. It might be a signal for the need for backreaction and the breaking of the working assumptions of the toy model used to derive it or it could have a deeper physical meaning that is yet to be understood. In any case, if it is confirmed within the next few years, it has to be taken into account and will give constraints on inflation scenarios.

B. Strong magnetic field regime

In the strong magnetic field regime the relation $\ell \gg \max(1,\mu,\lambda)$ is satisfied. In this regime, in order to examine the limiting behavior of the induced current, it is convenient to rewrite Eq. (60) in the form

$$J = \frac{eH^3\ell}{4\pi^2}\sum_{n=0}^{\infty}\int_{-1}^{+1}\frac{dr}{(1-r^2)}(rp(r)+\lambda) \times e^{-\pi\lambda r}|W_{i\lambda r,\gamma}(-2ip(r))|^2, \quad (69)$$

where the momentum p as a function of r is given by

$$p(r) = \sqrt{\frac{(1+2n)\ell}{1-r^2}}. \quad (70)$$

In the limit $\ell \to \infty$ and as a consequence $p(r) \to \infty$, the Whittaker function approximates

$$|W_{i\lambda r,\gamma}(-2ip(r))|^2 \sim e^{\pi\lambda r}. \quad (71)$$

Substituting the asymptotic form (71) into Eq. (69) and using the prescription (50), we obtain

$$J \simeq \frac{eH^3\ell\lambda}{8\pi^2}\int_{-1}^{+1}\frac{dr}{(1-r^2)}. \quad (72)$$

In order to regularize the integral in Eq. (72), we use the prescription

$$\int_{-1}^{+1}\frac{dr}{(1-r^2)} = \sum_{n=0}^{\infty}\int_{-1}^{+1} dr r^{2n} = \sum_{n=0}^{\infty}\frac{1}{n+\frac{1}{2}}, \quad (73)$$

and using the definition of the Hurwitz zeta function given by Eq. (A4), we represent the summation as

$$\sum_{n=0}^{\infty}\frac{1}{n+\frac{1}{2}} = -\frac{\partial^2}{\partial a\partial s}\zeta\left(s=0, a=\frac{1}{2}\right). \quad (74)$$

Finally, with Eqs. (72)–(74) and (A12), we obtain the regularized induced current in the strong magnetic field regime

$$J_{\rm reg} \simeq (\gamma_{\rm Euler}+\ln(4))\frac{eH^3\ell\lambda}{8\pi^2} \sim \frac{e}{H}\left(\frac{eB\Omega^{-2}}{2\pi}\right)\left(\frac{eE}{2\pi}\right), \quad (75)$$

where $\gamma_{\rm Euler} = 0.577\cdots$ is Euler's constant. This result shows the new contribution of the magnetic field in the strong magnetic field regime. As for the strong electric field regimes, the induced current presents a linear behavior in the magnetic field. As expected, it is the pair production due to the electromagnetic field which dominates its gravitational counterpart, in this regime.

VI. CONCLUSION

We have investigated for the first time the effect of a uniform magnetic field on Schwinger pair production and the induced current due to a constant electric field in dS_4. On the one hand, in Minkowski spacetime, a strong constant electric field can create pairs of charged particles from the vacuum at the cost of electrostatic energy. This is known as the Schwinger effect. A pure magnetic field does not produce pairs of charged particles since the virtual pair from the vacuum immediately annihilate each other. On the other hand, dS can emit radiation of all species of particles. This is known as Gibbons-Hawking radiation. These two effects have been considered together in the past. In this case, two important results are that the Gibbons-Hawking radiation enhances the pair production [10] and the superhorizon behavior of the field leads to a phenomenon of infrared hyperconductivity for the induced current [8,9,19,21].

In this paper, we have added one more ingredient to this setup: we included a uniform magnetic field parallel to the electric field in dS_4. The results of this paper recover the Schwinger effect and the induced current in the absence of a magnetic field, which has been systematically investigated in Ref. [9]. The effect of a uniform magnetic field on the Schwinger effect and the induced current with or without an electric field in dS_4 have been extensively studied.

First, the Schwinger effect is enhanced due to the density of states proportional to the magnetic field. Even in the absence of the electric field, the pair production rate is a product of the Gibbons-Hawking radiation and the magnetic field. This means that a strong magnetic field indeed assists the pair production in dS; see the result in Eq. (53). This is in contrast to the Schwinger effect due to parallel electric and magnetic fields in Minkowski spacetime, in which the density of states is proportional to both the electric field and the magnetic field and vanishes when the electric field is absent because a pure magnetic field is stable against spontaneous pair production.

Second, infrared hyperconductivity has been observed in the regime $\mu \ll \lambda \ll 1$, for weak magnetic fields; see the result in Eq. (67). This indicates that in dS (i) $\mu = m/H \ll 1$ (i.e., the Compton wavelength m^{-1} of the charge is much bigger than the Hubble radius H^{-1}), (ii) $\lambda = eE/H^2 \ll 1$ (i.e., the electric field E is much smaller than the scalar curvature $R = 12H^2$), and (iii) $\mu \ll \lambda$ or $eE/H \gg m$ (i.e., the electric potential energy across the Hubble radius H^{-1} is much larger than the mass of charge). This is in contrast to the regime $eE/m \gg m$ for the Schwinger effect for a pure electric field in flat spacetime, i.e., the electric potential energy across one Compton wavelength of the charge is much larger than the mass of the charge. The upper bound for the induced current in the magnetic field and electric field is given by $eB\Omega^{-2}H^2/m$ modulo a constant of order one, while in the pure electric field, the induced current has an upper bound given by eH^4/m, independently of the electric field.

Finally, in the limit of a magnetic field stronger than the mass of the charges, the electric field and scalar curvature of the dS, the induced current is proportional to the pseudoscalar of the Maxwell theory [see the result in Eq. (75)] which corresponds to the chiral magnetic effect for spin-$1/2$ fermions [53]. The chiral magnetic effect for fermions in dS, which is likely to hold for spinor QED considering the analogy with scalar QED, would be physically interesting but is beyond the scope of this paper and will be addressed in a future study.

Our new results open the road to systematically constrain primordial magnetogenesis scenarios. The inclusion of a magnetic field leads to an increase of the creation of Schwinger pairs which themselves screen the magnetogenesis amplification. We will present this new bound in a future paper [54].

Going further, an extension of the setups already known to investigate the Schwinger effect in dS would be to consider anisotropic inflationary spacetime where a constant electric field could be naturally sustained. Links to axion inflation and possibly a mechanism of baryogenesis with the help of the Schwinger effect could also be exhibited.

ACKNOWLEDGMENTS

S. P. K. would like to thank Remo Ruffini at ICRANet, where this work was initiated and also W.-Y. Pauchy Hwang for the warm hospitality at National Taiwan University. The work of S. P. K. was supported by IBS (Institute for Basic Science) under IBS-R012-D1 and also by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (NRF-2015R1D1A1A 01060626). E. B. and S.-S. X. would like to thank S. P. K. for the warm hospitality at Kunsan National University. E. B. is supported by the University of Kashan Grant No. 317203/3.

APPENDIX A: RIEMANN AND HURWITZ ZETA FUNCTIONS

In this appendix some useful properties of the Riemann and Hurwitz zeta functions are reviewed; for more properties see, e.g., Ref. [42].

The Riemann zeta function is a function of the complex variable s that analytically continue the Dirichlet series

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \tag{A1}$$

for when $\Re(s) > 1$. Another representation of it is

$$\zeta(s) = \frac{1}{1 - 2^{-s}} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^s}. \tag{A2}$$

Via the analytic continuation of Eq. (A1), it is possible to assign a finite result to the divergent series

$$\sum_{n=1}^{\infty} 1 = \zeta(0) = -\frac{1}{2}. \tag{A3}$$

Similarly, the Hurwitz zeta function is defined by the series expansion

$$\zeta(s, a) = \sum_{n=0}^{\infty} \frac{1}{(n+a)^s}, \qquad \Re(s) > 1, a \neq 0, -1, -2, \ldots \tag{A4}$$

The Riemann zeta function is nothing but a special case

$$\zeta(s, 1) = \zeta(s). \tag{A5}$$

The following special values of the Hurwitz zeta function are relevant here:

$$\zeta\left(s,\frac{1}{2}\right) = (2^s - 1)\zeta(s), \qquad s \neq 1. \tag{A6}$$

$$\zeta(0,a) = \frac{1}{2} - a. \tag{A7}$$

The Hurwitz zeta function satisfies the symmetry of second derivatives and its derivative in the second argument is a shift

$$\frac{\partial^2}{\partial s\partial a}\zeta(s,a) = \frac{\partial^2}{\partial a\partial s}\zeta(s,a), \tag{A8}$$

$$\frac{\partial}{\partial a}\zeta(s,a) = -s\zeta(s+1,a), \quad s\neq 0,1; \quad \Re(a) > 0. \tag{A9}$$

One of its limiting behaviors reads

$$\lim_{s\to 1}\left[\zeta(s,a) - \frac{1}{s-1}\right] = -\psi(a), \tag{A10}$$

where $\psi(a)$ is the digamma function which has the special value

$$\psi\left(\frac{1}{2}\right) = -\gamma_E - \ln(4). \tag{A11}$$

With Eqs. (A8)–(A11), one can verify the useful mathematical formula

$$\frac{\partial^2}{\partial a\partial s}\zeta\left(s=0, a=\frac{1}{2}\right) = -\gamma_{\mathrm{Euler}} - \ln(4). \tag{A12}$$

APPENDIX B: ADIABATIC REGULARIZATION OF THE CURRENT

In order to compute the one-dimensional momentum integral on the right-hand side of Eq. (62), we adopt the integration procedure that has been introduced in Refs. [8,9]. The Mellin-Barnes representation of the Whittaker function is given by [42]

$$W_{\kappa,\gamma}(z) = e^{\frac{-z}{2}}\int_{-i\infty}^{+i\infty}\frac{ds}{2\pi i}\frac{\Gamma(\frac{1}{2}+\gamma+s)\Gamma(\frac{1}{2}-\gamma+s)\Gamma(-\kappa-s)}{\Gamma(\frac{1}{2}+\gamma-\kappa)\Gamma(\frac{1}{2}-\gamma-\kappa)}z^{-s},$$
$$|ph(z)| < \frac{3\pi}{2}, \frac{1}{2}\pm\gamma-\kappa\neq 0,-1,-2,\ldots, \tag{B1}$$

where the contour of integration separates the poles of $\Gamma(\frac{1}{2}+\gamma+s)\Gamma(\frac{1}{2}-\gamma+s)$ from those of $\Gamma(-\kappa-s)$. Substituting the integral representation (B1) into Eq. (62) and choosing the contour of integration similar to Ref. [9], leads to the final result

$$J = \frac{eH^3\ell}{4\pi^2}\left(\frac{\gamma\sinh(2\pi\lambda)}{\sin(2\pi\gamma)} + \lambda\right). \tag{B2}$$

In order to regularize the current (B2) we apply the adiabatic subtraction method. Starting from Eq. (18), for positive-frequency modes it can be rewritten as

$$\frac{d^2 f_{\mathrm{A}}(\tau)}{d\tau^2} + \omega^2(\tau) f_{\mathrm{A}}(\tau) = 0, \tag{B3}$$

where ω reads

$$\omega(\tau) = +\left(k^2 - \frac{2\lambda k_z}{\tau} + \frac{\lambda^2+\mu^2}{\tau^2} - \frac{2}{\tau^2}\right)^{\frac{1}{2}}. \tag{B4}$$

A WKB-type solution for the mode equation (B3) is

$$f_{\mathrm{A}}(\tau) = (2W(\tau))^{\frac{-1}{2}}\exp\left(-i\int^{\tau} W(\tau')d\tau'\right), \tag{B5}$$

provided that the function $W(\tau)$ satisfies the equation

$$W^2(\tau) = \omega^2(\tau) + \frac{3\dot{W}^2}{4W^2} - \frac{\ddot{W}}{2W}, \tag{B6}$$

where the dot indicates a derivative with respect to the conformal time τ. For the zeroth-order adiabatic expansion of $W(\tau)$, the derivative terms on the right-hand side of Eq. (B6) are neglected, and we then obtain

$$W^{(0)}(\tau) = \omega_0(\tau). \tag{B7}$$

Since the last term in Eq. (B4) is of second adiabatic order

$$\frac{2}{\tau^2} = 2\frac{\dot{\Omega}^2}{\Omega^2}, \tag{B8}$$

we then have

$$\omega_0(\tau) = +\left(k^2 - \frac{2\lambda k_z}{\tau} + \frac{\lambda^2+\mu^2}{\tau^2}\right)^{\frac{1}{2}}. \tag{B9}$$

With Eqs. (13), (15), (B5), (B7), and (B9) the zeroth adiabatic order for the positive- and negative-frequency mode functions are

$$U_{\mathrm{A}}(x;\mathbf{k}_{\not{y}},n) = \Omega^{-1}(\tau)e^{+i\mathbf{x}\cdot\mathbf{k}_{\not{y}}}h(y_+)(2\omega_0(\tau))^{\frac{-1}{2}} \times\exp\left(-i\int^{\tau}\omega_0(\tau')d\tau'\right), \tag{B10}$$

$$V_{\mathrm{A}}(x;-\mathbf{k}_{\not{y}},n) = \Omega^{-1}(\tau)e^{+i\mathbf{x}\cdot\mathbf{k}_{\not{y}}}h(y_+)(2\omega_0(\tau))^{\frac{-1}{2}} \times\exp\left(+i\int^{\tau}\omega_0(\tau')d\tau'\right). \tag{B11}$$

This adiabatic complete set of orthonormal mode functions can be used to construct the Fock space. Then, the zeroth adiabatic order expansion of the vacuum expectation value of the current operator is given by

$$j_{\mathrm{A}} = e\Omega^{-2}(\tau)\sum_{n=0}^{\infty}\int\frac{d^2k_{\not{y}}}{(2\pi)^2}\left(k_z-\frac{\lambda}{\tau}\right)\left(|U_{\mathrm{A}}|^2+|V_{\mathrm{A}}|^2\right). \tag{B12}$$

After some algebra and using Eq. (50), it can be shown that

$$J_{\mathrm{A}} = \Omega(\tau)j_A = \frac{eH^3\ell\lambda}{4\pi^2}. \tag{B13}$$

The adiabatic regularization scheme consists in subtracting the counterterm (B13) from the original expression (B2),

$$J_{\mathrm{reg}} = J - J_{\mathrm{A}}. \tag{B14}$$

[1] F. Gelis and N. Tanji, Prog. Part. Nucl. Phys. **87**, 1 (2016).
[2] F. Sauter, Z. Phys. **69**, 742 (1931).
[3] W. Heisenberg and H. Euler, Z. Phys. **98**, 714 (1936).
[4] J. S. Schwinger, Phys. Rev. **82**, 664 (1951).
[5] D. L. Burke *et al.*, Phys. Rev. Lett. **79**, 1626 (1997).
[6] A. Di Piazza, C. Muller, K. Z. Hatsagortsyan, and C. H. Keitel, Rev. Mod. Phys. **84**, 1177 (2012).
[7] R. Ruffini, G. Vereshchagin, and S. S. Xue, Phys. Rep. **487**, 1 (2010).
[8] M. B. Fröb, J. Garriga, S. Kanno, M. Sasaki, J. Soda, T. Tanaka, and A. Vilenkin, J. Cosmol. Astropart. Phys. 04 (2014) 009.
[9] T. Kobayashi and N. Afshordi, J. High Energy Phys. 10 (2014) 166.
[10] R. G. Cai and S. P. Kim, J. High Energy Phys. 09 (2014) 072.
[11] W. Fischler, P. H. Nguyen, J. F. Pedraza, and W. Tangarife, Phys. Rev. D **91**, 086015 (2015).
[12] C. M. Chen, S. P. Kim, I-Chieh Lin, J. R. Sun, and M. F. Wu, Phys. Rev. D **85**, 124041 (2012).
[13] S. P. Kim, Int. J. Mod. Phys. D **25**, 1645005 (2016).
[14] C. M. Chen, S. P. Kim, J. R. Sun, and F. Y. Tang, Phys. Rev. D **95**, 044043 (2017).
[15] C. Stahl, E. Strobel, and S. S. Xue, arXiv:1602.09090.
[16] C. Stahl, E. Strobel, and S. S. Xue, Phys. Rev. D **93**, 025004 (2016).
[17] T. Hayashinaka, T. Fujita, and J. Yokoyama, J. Cosmol. Astropart. Phys. 07 (2016) 010.
[18] T. Hayashinaka and J. Yokoyama, J. Cosmol. Astropart. Phys. 07 (2016) 012.
[19] J. J. Geng, B. F. Li, J. Soda, A. Wang, Q. Wu, and T. Zhu, arXiv:1706.02833.
[20] R. Sharma and S. Singh, Phys. Rev. D **96**, 025012 (2017).
[21] E. Bavarsad, C. Stahl, and S. S. Xue, Phys. Rev. D **94**, 104011 (2016).
[22] C. Stahl and S. S. Xue, Phys. Lett. B **760**, 288 (2016).
[23] S. S. Xue, Nucl. Phys. **B897**, 326 (2015).
[24] S. S. Xue, Int. J. Mod. Phys. A **30**, 1545003 (2015).
[25] T. Markkanen and A. Rajantie, J. High Energy Phys. 01 (2017) 133.
[26] T. Markkanen, J. Cosmol. Astropart. Phys. 11 (2016) 026.
[27] T. Markkanen, arXiv:1703.06898.
[28] S. Mukohyama, Phys. Rev. D **94**, 121302 (2016).
[29] I. Vovk, A. M. Taylor, D. Semikoz, and A. Neronov, Astrophys. J. **747**, L14 (2012).
[30] L. M. Widrow, Rev. Mod. Phys. **74**, 775 (2002).
[31] A. Kandus, K. E. Kunze, and C. G. Tsagas, Phys. Rep. **505**, 1 (2011).
[32] R. Durrer and A. Neronov, Astron. Astrophys. Rev. **21**, 62 (2013).
[33] K. Subramanian, Rep. Prog. Phys. **79**, 076901 (2016).
[34] S. Kawati and A. Kokado, Phys. Rev. D **39**, 3612 (1989).
[35] S. Kawati and A. Kokado, Phys. Rev. D **39**, 2959 (1989).
[36] S. Moradi, Mod. Phys. Lett. A **24**, 1129 (2009).
[37] S. Haouat and R. Chekireb, Eur. Phys. J. C **72**, 2034 (2012).
[38] M. Ghodrati, Phys. Rev. D **92**, 065015 (2015).
[39] C. Crucean and M. A. Băloi, Phys. Rev. D **93**, 044070 (2016).
[40] N. Nicolaevici and A. Farkas, Phys. Rev. D **95**, 048501 (2017).
[41] C. Crucean and M. A. Băloi, Phys. Rev. D **95**, 048502 (2017).
[42] F. W. J. Olver, D. W. Lozier, R. F. Boisvert, and C. W. Clark, *NIST Handbook of Mathematical Functions* (Cambridge University Press, Cambridge, England, 2010).
[43] L. Parker and D. Toms, *Quantum Field Theory in Curved Spacetime: Quantized Fields and Gravity* (Cambridge University Press, Cambridge, England, 2009).
[44] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1984).
[45] A. Kuznetsov and N. Mikheev, Springer Tracts Mod. Phys. **197**, 1 (2004).
[46] C. G. Tsagas, J. Phys. Conf. Ser. **68**, 012051 (2007).
[47] M. Giovannini, Int. J. Mod. Phys. D **13**, 391 (2004).
[48] E. Elizalde, S. D. Odintsov, A. Romeo, A. A. Bytsenko, and S. Zerbini, *Zeta Regularization Techniques with Applications* (World Scientific, Singapore, 1994).

[49] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977).

[50] S. P. Kim, arXiv:1008.0577.

[51] J. Garriga, Phys. Rev. D **49**, 6343 (1994).

[52] E. Bavarsad, S. P. Kim, C. Stahl, and S. S. Xue, Eur. Phys. J. Web Conf. **168**, 03002 (2018).

[53] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D **78**, 074033 (2008).

[54] E. Bavarsad, S. P. Kim, C. Stahl, and S. S. Xue, Constraints on primordial magnetogenesis from Schwinger pair creation (to be published).