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# Lattice Boltzmann simulation of velocity investigation of natural convection in power transformer with nano-oil

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## Abstract

Enhancing the thermal performance of transformer oil is imperative due to the crucial role transformers perform in power distribution networks. Additionally, the temperature of a transformer's core has an impact on its electrical performance. This paper, which presents a two-dimensional model of a transformer, investigates theoretically and simulates the natural convection heat transfer of one nano-oil sample that contains an $Fe_3O_4$ nanoparticle in the base fluid of Paraffinic oil. Modelling and solving were done for nanoparticle volume fractions of 0% to 5% and Rayleigh numbers of $10^3$ to $10^5$. Nusselt number average and charts of velocity are given in this range. According to the study's results, the transformer core's ability to cool has been enhanced by the addition of nanoparticles to the base fluid. In addition, natural convection heat transfer was enhanced by increasing the Rayleigh number and the volume fraction of nanoparticles. With the greatest Nusselt number average of 13.09 and a Rayleigh number of $10^5$, Paraffinic-$Fe_3O_4$ nano-oil had better heat transfer in comparison to pure Paraffinic oil under the same conditions.

**Keywords:** Heat transfer, Natural convection, Nanofluid, Transformer, Lattice Boltzmann method

## Introduction

Scientists have done extensive research on the topic of natural convection heat transfer, mainly motivated by the many industrial applications and the significant impact of particular parameters, such as Rayleigh number, nanoparticle volume fraction, and boundary conditions, on these processes. Enhancing the thermal behavior of the transformer oil is especially crucial since power transmission networks need transformers and because the electrical performance of the transformer is influenced by its core temperature. Due to the fact that temperature affects these systems' useful life and efficiency, researchers are attempting to use novel methods, like nanofluids, to increase the natural heat transfer from the core of the transformer [1-9].

Base fluids such as water [10], oil [11] and ethylene glycol [12], which are often used as base fluids for natural convection heat transfer, have much lower thermal conductivity than nanoparticles; therefore, adding any of the nanoparticles such as Cu [13], CuO [14], $Al_2O_3$ [15], $TiO_2$ [16], MWCNTs (multi walled carbon nanotubes) [17], $Fe_3O_4$ [18] and etc. can improve the thermal conductivity of fluids. The study by Segal and Raj [8] on the natural convection heat transfer by mineral oil surrounding the transformer core demonstrates that the heat intensity surrounding the transformer core can be decreased by selecting the optimal magnetic field intensity value. In their study, Pendyala et al. [9] examined the thermal performance of nanofluids in transformers through computational fluid dynamics (CFD) analysis. The findings of their research demonstrated that nanofluids exhibit superior heat transfer capabilities compared to the base fluid. Furthermore, the study highlighted the significant influence of fluid density and thermal conductivity on enhancing natural heat transfer characteristics. Rahimi et al. [16] investigated natural convection heat transfer in an L-shaped hollow cavity filled with nanofluid by using the lattice Boltzmann method and entropy generation and heat lines. It was found that an increase in the Rayleigh number results in a concomitant escalation in both the average Nusselt number and the overall quantity of entropy production. Additionally, it was observed that as the solid volume fraction of the nanofluid increased, there was a corresponding increase in the average Nusselt number and a decrease in the total entropy generation. A study by Hwang et al. [19] reports that the effect of aluminum oxide nanofluid on natural convection heat transfer in a rectangular chamber was investigated numerically. Results showed that increasing the Rayleigh number and adding nanoparticles enhanced heat transfer. Chen et al. [20] investigated the combined convection heat transfer in a chamber containing aluminum oxide nanofluid using the lattice Boltzmann method and showed the effect of nanoparticles on increasing the Nusselt number. Karki et al. [21] conducted a study focusing on laminar natural convection and entropy generation by air, water and alumina-water nanofluid. They solved the fluid flow equations with the D2Q9 model and their results showed that heat transfer increased due to the addition of nanoparticles to the base fluid. Also, they reported the maximum increase in Nusselt number at 8% volume fraction and $10^5$ Rayleigh number by 13.93%. A study on laminar natural convection and entropy formation by air, water, and alumina-water nanofluid was carried out by Karki et al. [21]. The addition of nanoparticles to the base fluid resulted in an increase in heat transfer, according to their solution of the fluid flow equations using the D2Q9 model. They also stated that the $10^5$ Rayleigh number increased by 13.93% and the Nusselt number increased to a high of 8% volume fraction. In order to study the natural convection heat transfer inside a cavity filled with nanofluid containing copper nanoparticles, Santra et al. [22] used a non-Newtonian fluid and the finite volume method. They found that the number of nanoparticles in a given Rayleigh range decreases the amount of heat transfer. Sajjadi et al. [23] used a new Boltzmann lattice method to look into the spontaneous convection flow in a porous cavity with a sinusoidal temperature distribution. This was done with a water-copper nanofluid present. The volume fractions 0 to 6%, Rayleigh numbers $10^3$ to $10^5$, and Darcy numbers 0.001 to 0.1 were the ranges for which they studied. Among their conclusions was the finding that the Darcy number, Rayleigh number, and volume fraction of the nanoparticles all had a positive correlation with the rate of heat transfer. As the most sensitive parameter, the Rayleigh number was also introduced.

Considering the importance of transformers in the national power transmission network and also the challenges related to the cooling of the core of these transformers, it is predicted that improving the cooling of the transformer core can reduce the loss of electrical energy and also the maintenance cost of transformers.

## Problem statement

The two-dimensional model of the transformer, according to Figure 1 includes the transformer core, transformer housing and transformer oil channels. The transformer core has a high temperature $T_H$ and the oil channels have a low temperature $T_C$. The upper wall of the transformer core is assumed to have a $T_H$ temperature due to the natural convection heat transfer from the transformer core, and the dimensionless parameter A has been used to determine the dimensions of the transformer. The nano-oil used include $Fe_3O_4$ nanoparticle and Paraffinic oil.

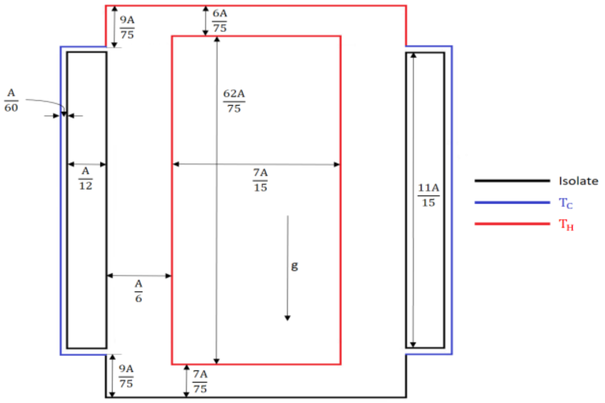



Figure 1: The transformer's two-dimensional model [24]

Table 1 shows the thermophysical properties of base oil and nanoparticles. The single-phase model's relationships and the laminar, incompressible fluid flow are the most crucial assumptions made in the numerical solution of the problem.

Table 1: Thermophysical properties of materials [25-27]

| Materials | $\beta$ ($K^{-1}$) | k (W/mK) | $C_p$ (J/kgK) | $\rho$ ($kg/m^3$) |
|---|---|---|---|---|
| $Fe_3O_4$ | 0.000206 | 80.4 | 670 | 5180 |
| Paraffinic oil | 0.000075 | 0.167 | 2200 | 783 |

Based on the geometry of the model according to Figure 1, and considering the no-slip condition, the dimensionless boundary conditions for the hot, cold and insulated walls are as follows:

Hot wall (temperature of $T_H$): $U = 0$ , $V = 0$ , $\theta = 1$

Cold wall (temperature of $T_C$): $U = 0$ , $V = 0$ , $\theta = 0$

Vertical insulation wall: $U = 0$ , $V = 0$ , $\frac{\partial\theta}{\partial X} = 0$

Horizontal insulation wall: $U = 0$ , $V = 0$ , $\frac{\partial\theta}{\partial Y} = 0$

## Governing equations

The continuity, momentum and energy equations for the laminar and steady state natural convection in two dimensional forms can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad (2)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \frac{(\rho\beta)_{nf}}{\rho_{nf}} g(T - T_c) \quad (3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

The governing equations are given in dimensionless form by (5) in order to reduce the number of variables and make the equations and results independent.

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{uL}{\alpha_{bf}}, \quad V = \frac{vL}{\alpha_{bf}}, \quad P = \frac{pL^2}{\rho_{nf}\alpha_{bf}^2}, \quad \theta = \frac{T - T_c}{T_H - T_c} \quad (5)$$

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (6)$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\nu_{nf}}{\alpha_{bf}}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \quad (7)$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\nu_{nf}}{\alpha_{bf}}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{\beta_{nf}}{\beta_{bf}} RaPr\theta \quad (8)$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_{bf}}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \quad (9)$$

The Rayleigh and Prandtl numbers and thermal diffusion coefficient are given below:

$$Ra = \frac{g\beta_{bf}L^3\Delta T}{\nu_{bf}\alpha_{bf}} \quad (10)$$

$$Pr = \frac{\nu_{bf}}{\alpha_{bf}} \quad (11)$$

$$\alpha = \frac{k}{c_p\rho} \quad (12)$$

## Relationships of nanofluids

The simulations are run in a single-phase model where there is no slip condition between the base fluid and the solid particles and they are in thermal equilibrium. Relationships 13 through 18 are displayed for nanofluid relationships.

$$k_{nf} = \frac{k_s + 2k_{bf} - 2\varphi(k_{bf} - k_s)}{k_s + 2k_{bf} + \varphi(k_{bf} - k_s)} k_{bf} \quad (13)$$

$$\rho_{nf} = \varphi\rho_s + (1 - \varphi)\rho_{bf} \quad (14)$$

$$(\rho c_p)_{nf} = \varphi(\rho c_p)_s + (1 - \varphi)(\rho c_p)_{bf} \quad (15)$$

$$(\rho\beta)_{nf} = \varphi(\rho\beta)_s + (1 - \varphi)(\rho\beta)_{bf} \quad (16)$$

$$\mu_{nf} = \frac{\mu_{bf}}{(1 - \varphi)^{2.5}} \quad (17)$$

$$Nu_{avg} = \int_0^1 Nu dy \quad (18)$$

## The lattice Boltzmann method

The lattice Boltzmann method's simplicity and ease of application in complicated domains, together with its ability to handle multi-phase and multi-component flows without requiring the following of common surfaces across phases, is one of its advantages. For a system without external force, the Boltzmann transport equation is obtained according to the following equation. $\Omega$ is a function of f, and The Boltzmann equation must be solved in order to find $\Omega$ [28].

$$\frac{\partial f}{\partial t} + c.\nabla f = \Omega \tag{19}$$

The equilibrium distribution function ($f^{eq}$) is the primary component used in the lattice Boltzmann method. In fact, the distribution function substitutes for each particle in the lattice Boltzmann method. The collision operator can be modeled as a low-error, straightforward operator to solve the Boltzmann equation. The collision operator's fundamental model, known as the BGK model, was introduced by Bhatnagar, Gross, and Krook. Using the BGK approximation, the temperature and current distribution functions are displayed in relations 20 and 21, respectively [29].

$$f_i(r + c_i\Delta t, t + \Delta t) = f_i(r, t) + \frac{\Delta t}{\tau_F}[f^{eq}{}_i(r, t) - f_i(r, t)] + c_i\Delta t F \tag{20}$$

$$g_i(r + c_i\Delta t, t + \Delta t) = g_i(r, t) + \frac{\Delta t}{\tau_T}\left[g^{eq}{}_i(r, t) - g_i(r, t)\right] \tag{21}$$

Here, $\tau$ is the relaxation coefficient and the left side of equations 20 and 21 shows the collision process, while the right side shows the diffusion process. Therefore, Boltzmann's equation can be solved in two phases: collision and diffusion. that in relations 20 and 21, $\Delta t$ is the time step of the lattice and $\tau_F$ and $\tau_T$ respectively express the relaxation time of the flow field and temperature field, which can be calculated from relations 22 and 23:

$$\tau_F = 3\nu_{bf} + \frac{1}{2} \tag{22}$$

$$\tau_T = 3\alpha_{bf} + \frac{1}{2} \tag{23}$$

## Modeling and boundary conditions

Lattice Boltzmann models are given by the formula DnQm, where n is the dimension of the problem and m is the number of velocity vectors. In this simulation, D2Q9 is utilized. This model was given by Qian et al. [30]. The Boltzmann lattice for this model can be written as follows:

$$f_i(r + \Delta r, t + \Delta t) = f_i(r, t)\,[1 - \omega] + \omega f_i^{eq}(r, t) \tag{24}$$

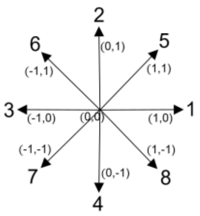



Figure 2: Velocity components of the D2Q9 model

$$f_i^{eq} = \omega_i \rho(r, t)\left[1 + \frac{c_i.u}{c_s^2} + \frac{1}{2}\frac{(c_i.u)^2}{c_s^4} - \frac{1}{2}\frac{u^2}{c_s^2}\right] \tag{25}$$

$$g_i^{eq} = \omega_i T\left[1 + \frac{c_i.u}{c_s^2} + \frac{1}{2}\frac{(c_i.u)^2}{c_s^4} - \frac{1}{2}\frac{u^2}{c_s^2}\right] \tag{26}$$

$$c_s = \frac{c_i}{\sqrt{3}} \tag{27}$$

$$c_i = \frac{\Delta x}{\Delta t} i + \frac{\Delta y}{\Delta t} j \tag{28}$$

$$u = ui + vj \tag{29}$$

Equation 28 assumes that the values of $\Delta x$, $\Delta y$, and $\Delta t$, which stand for horizontal displacement, vertical displacement, and time, respectively, are equal to 1. Equations 30, 31, and 32 can be used to calculate the fluid's density, velocity, and macroscopic temperature in the D2Q9 model.

$$\rho = \sum_0^8 f_i \tag{30}$$

$$u = \frac{1}{\rho}\sum_0^8 f_i c_i \tag{31}$$

$$T = \sum_0^8 g_i \tag{32}$$

Boundary conditions are necessary for the simulation's result computation. One of the most popular models, the bounce-back model, is applied in this study. In this model, the condition of no slip or the condition of no flow flowing over an obstruction is employed to mimic a stationary solid [31]. As an example, the transformer's bottom wall is subjected to the bounce-back boundary condition in Figure 3.

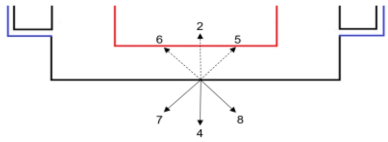



Figure 3: Boundary conditions for a surface geometry problem

Thus, for Figure 3, we can write:

$$f_{5,n} = f_{7,n} \quad , \quad f_{2,n} = f_{4,n} \quad , \quad f_{6,n} = f_{8,n} \tag{33}$$

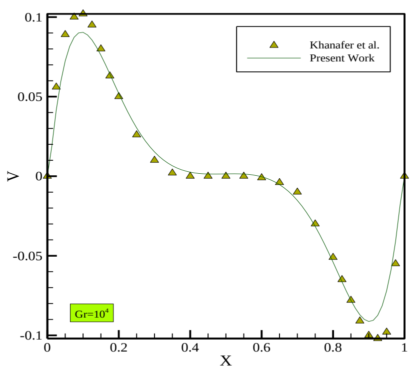
$$g_{5,n-1} = g_{5,n} \quad , \quad g_{2,n-1} = g_{2,n} \quad , \quad g_{6,n-1} = g_{6,n} \tag{34}$$

## Grid testing and code validation

In numerical calculation methods, grid independence should be checked for a main parameter of the studied material. The grid should be chosen in such a way that, after that grid, the accuracy of the numerical solution does not change much. The 170*170 lattice is selected as the appropriate lattice.

In order to verify the accuracy of the numerical solution and validate the results, the findings of two studies by Khanafer et al. [32] and Oztop and Abu-nada [13] were compared with the results of the current study. The heat transfer of natural convection within two studies of the two-dimensional square enclosure has been examined in the numerical validation of this work.

As can be seen in Figure 4, a comparison was made between the Nusselt numbers derived from the current study and those obtained from Oztop and Abu-Nada [13], as well as between the velocity figure obtained in the current investigation and those reported by Khanafer et al. [32].



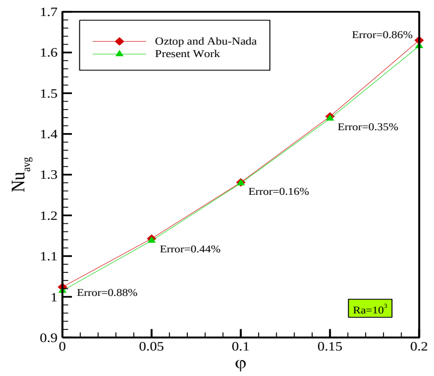



Figure 4: Comparison between the present results and the vertical velocity results by Khanafer et al. [32] and average Nusselt number by Oztop and Abu-Nada [13]

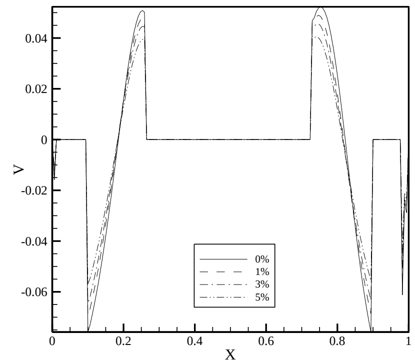



Figure 7: Paraffinic-$Fe_3O_4$ nano-oil's vertical velocity for $Ra=10^5$ and nanoparticle volume fractions

**Vertical velocity relative to the central axis**

The vertical velocity of the nano-oil relative to the horizontal axis passing through the middle of the transformer is presented in Figures 5 to 7, for the Paraffinic-$Fe_3O_4$ nano-oil in the volume fraction of 2% and the range of Rayleigh numbers from $10^3$ to $10^5$. According to these Figures 5 to 7, the velocity is increasing from the wall of the oil channels to the wall of the transformer core. The reason for this velocity change is the existence of natural convection heat transfer. It is worth noting that the velocity profile is symmetrical concerning the center of the transformer geometry in Rayleigh numbers $10^3$ to $10^5$.

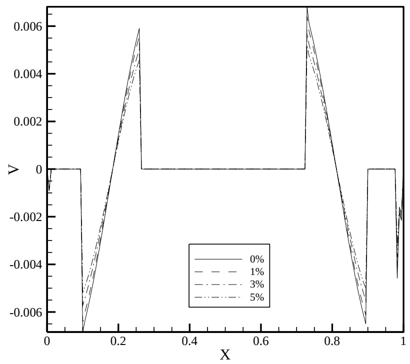



Figure 5: Paraffinic-$Fe_3O_4$ nano-oil's vertical velocity for $Ra=10^3$ and nanoparticle volume fractions

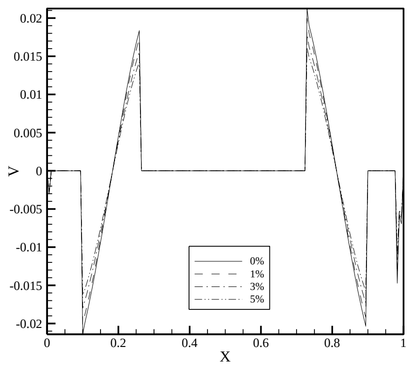



Figure 6: Paraffinic-$Fe_3O_4$ nano-oil's vertical velocity for $Ra=10^4$ and nanoparticle volume fractions

According to Figures 5 to 7, adding nanoparticles to the base fluid decreases the vertical velocity. Also, increasing the concentration of nanoparticles decreases the velocity more. The effect of increasing the nanoparticle volume fraction on decreasing the vertical velocity of nano-oil is due to the increase in the viscosity of nano-oil due to the increase in nanoparticle volume fraction (according to 33). Another point is the movement of nano-oil upwards and towards the upper surface of the transformer core, which indicates heat transfer from the core to the transformer housing.

**Influence of Rayleigh number and nano-oil on $Nu_{avg}$**

The Nusselt number is the ratio of convection heat transfer to conduction heat transfer. Therefore, with the increase in the Nusselt number, the heat transfer performance of the natural convection of the nano-oil improves.

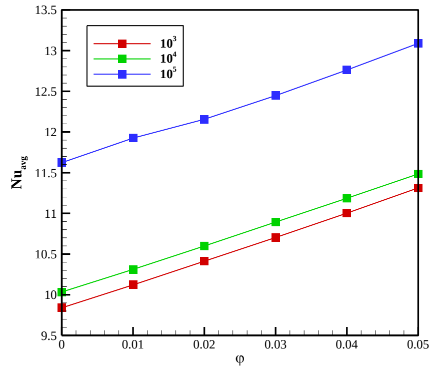



Figure 8: Average Nusselt number for nano-oil in different Rayleigh numbers and nanoparticle volume fractions

Figure 8 show that improving the Rayleigh number and adding nanoparticles into the oil base fluid enhances heat transfer efficiency. Across all Rayleigh number ranges, an increase in the volume fraction of nanoparticles has been shown to increase the average Nusselt number. Enhancing heat transfer can lead to higher transformer oil thermal performance, which in turn enhances transformer core cooling performance. This is one of the main benefits of increased heat transfer.

**Conclusions**

Analysis is done on the two-dimensional natural convection in a power transformer that has one nano-oil, which includes an $Fe_3O_4$ nanoparticle in Paraffinic oil. This study uses the two-dimensional lattice Boltzmann method. The effects of varying Rayleigh numbers and volume fractions of nanoparticles on fluid flow and nano-oil velocity are discussed. In light of this study's results:

1) The velocity will be reduced when nanoparticles are added to the base oil. Additionally, increasing the concentration of nanoparticles causes a further decrease in velocity.

2) The average Nusselt number increases with increasing Rayleigh number.

3) The average Nusselt number increases with increasing nanoparticle volume fraction.

4) Adding nanoparticles to the oil base fluid has increased the natural convection heat transfer in transformer.

**Nomenclature**

$A$ Dimensionless transformer length parameter
$C_p$ Specific heat capacity [J/kgK]
$c_i$ Velocity of virtual particles on lattice [m/s]
$c_s$ Velocity of sound on lattice [m/s]
$F$ External force [N]
$f_i$ Distribution function of density
$f_i^{eq}$ Equilibrium distribution function of density
$g_i$ Distribution function of temperature
$g_i^{eq}$ Equilibrium distribution function of temperature
$g$ Gravitational acceleration [$m/s^2$]
$h$ Convection heat transfer coefficient [$W/m^2K$]
$k$ Thermal conductivity coefficient [W/mK]
$L$ Length [m]
$n$ The number of grids in the x direction
$Nu_{avg}$ Average Nusselt number
$Nu$ Local Nusselt number
$P$ Dimensionless pressure
$p$ Pressure [$N/m^2$]
$Pr$ Prandtl number
$r$ Location vector [m]
$Ra$ Rayleigh number
$T$ Temperature [K]
$t$ Time [s]
$U,V$ Dimensionless velocity components
$u,v$ Velocity components [m/s]
$x,y$ Cartesian coordinates

**Greek symbols**

$\alpha$ Coefficient of heat diffusion [$m^2/s$]
$\beta$ Thermal expansion coefficient [$K^{-1}$]
$\theta$ Dimensionless temperature
$\mu$ Dynamic viscosity [$Ns/m^2$]
$\nu$ Kinematic viscosity [$m^2/s$]
$\rho$ Density [$kg/m^3$]
$\tau$ Relaxation factor
$\varphi$ Nanoparticle volume fraction
$\Omega$ Collision operator
$\omega$ Weighting factor

**Subscripts**

bf Base fluid
c Cold wall
F Flow
h Hot wall
nf Nanofluid
s Nanoparticle
T Temperature

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