

Thermophysical investigation of natural convection in power transformer with Nano-oil using lattice Boltzmann method

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Abstract

Considering the vital role transformers perform in power distribution networks, it is significant that transformer oil's thermal performance be enhanced. Furthermore, a transformer's core temperature affects how well it performs electrically. This study, which presents a two-dimensional model of a transformer, explores theoretically and simulates the natural convection heat transfer of a nano-oil sample containing an Fe₃O₄ nanoparticle in the base fluid of Naphthenic oil using the lattice Boltzmann method and D2Q9 model. For nanoparticle volume fractions of 0% to 5% and Rayleigh numbers of 10³ to 10⁵, modelling and solutions were carried out and, in this range, temperature contours and Nusselt number average are provided. Based on the results of this investigation, the transformer core's cooling performance has been enhanced by the addition of nanoparticles to the base fluid. Furthermore, by raising the nanoparticles' volume fraction and Rayleigh number, the natural convection heat transfer was improved. When compared to pure Naphthenic oil, the heat transfer has improved when using Naphthenic-Fe₃O₄ nano-oil, which has the greatest Nusselt number average of 13.09 at a volume fraction of 5% and Rayleigh number of 10⁵.

Keywords: Heat transfer, Natural convection, Nanofluid, Transformer, Lattice Boltzmann method

Introduction

Researchers have conducted extensive studies on the topic of natural convection heat transfer, primarily driven by the numerous industrial applications and the significant impact of certain parameters, like Rayleigh number, nanoparticle volume fraction, and boundary conditions, on these processes. Improving the thermal conductivity of the transformer oil is essential, as power transmission networks need the utilization of transformers, and the electrical performance of the transformer is affected by the temperature of its core. As temperature influences the useful life and efficiency of these systems, scientists are attempting to apply novel methods, such as the use of nanofluids, to speed up the natural heat transfer from the core of the transformer [1-9].

Base fluids like water [10], oil [11] and ethylene glycol [12], which are often used as base fluids for natural convection heat transfer, have much lower thermal conductivity than nanoparticles, so adding any of the nanoparticles such as Cu [13], CuO [14], Al₂O₃ [15], TiO₂ [16], MWCNTs (multi walled carbon nanotubes) [17],

Fe₃O₄ [18] and etc. may enhance the thermal conductivity of fluids. The research of Segal and Raj [8] on natural convection heat transfer by mineral oil around the transformer core shows that choosing the best value of the magnetic field intensity can reduce the heat intensity around the transformer core. In their study, Pendyala et al. [9] examined the thermal performance of nanofluids in transformers through computational fluid dynamics (CFD) analysis. The findings of their research demonstrated that nanofluids exhibit superior heat transfer capabilities compared to the base fluid. Furthermore, the study highlighted the significant influence of fluid density and thermal conductivity on enhancing natural heat transfer characteristics. Rahimi et al. [16] used the lattice Boltzmann method, entropy generation, and heat lines to study natural convection heat transfer in an L-shaped hollow chamber filled with nanofluid. It was found that when the Rayleigh number increases, the average Nusselt number and the total amount of entropy production increase in tandem. Furthermore, an increase in the average Nusselt number and a decrease in the overall entropy generation were noted in tandem with the nanofluid's solid volume fraction. In a numerical investigation of the natural convection heat transfer of nanofluids in an inclined chamber with a heater, Ogut [19] demonstrated that average heat transfer increased with increasing Rayleigh number and particle volume fraction but decreased with increasing heater length. The results of the numerical research of Sheikholeslami et al. [20] on the natural convection heat transfer of copper nanoparticles in water in the presence of a magnetic field show that the average Nusselt number is a function of increasing the volume fraction of nanoparticles, the number of oscillations, and the Rayleigh number. Lai and Yang [21] used the lattice Boltzmann method to look at how heat moved through a square chamber filled with a water-Al₂O₃ nanofluid by natural convection. Their findings demonstrated that an increase in the Rayleigh number and particle volume fraction corresponded with an increase in the average Nusselt number. Karki et al. [22] conducted a study focusing on laminar natural convection and entropy generation by air, water and alumina-water nanofluid. They solved the fluid flow equations with the D2Q9 model and their results showed that heat transfer increased due to the addition of nanoparticles to the base fluid. Also, they reported the maximum increase in Nusselt number at 8% volume fraction and 10⁵ Rayleigh number by 13.93%. Sajjadi et al. [23] studied and investigated the natural

convection flow in a porous cavity with sinusoidal temperature distribution using a new Boltzmann lattice method in the presence of water-copper nanofluid. They conducted this study for volume fractions 0 to 6%, Rayleigh numbers 10^3 to 10^5 and Darcy numbers 0.001 to 0.1. Their findings included the observation that the rate of heat transfer is positively correlated with the Darcy number, Rayleigh number, and volume fraction of the nanoparticles. The Rayleigh number was also introduced as the parameter with the highest sensitivity.

Considering the importance of transformers in the electricity transmission network of the country and the challenges related to their core cooling, it is predicted that if the addition of nanoparticles to the base oil improves the cooling performance of the transformer core, the loss of electrical energy and maintenance costs can be reduced, reduced in the transformer.

Problem statement

The transformer housing, oil channels, and core are all included in the two-dimensional model of the transformer, as shown in Figure 1. The oil channels have a low temperature (T_C) and the transformer core has a high temperature (T_H). The dimensionless parameter A has been utilized to calculate the transformer's dimensions. It is assumed that the upper wall of the transformer core has a T_H temperature because of the heat transfer from the core that occurs naturally. Fe_3O_4 nanoparticles and Naphthenic oil are the nano-oil employed.

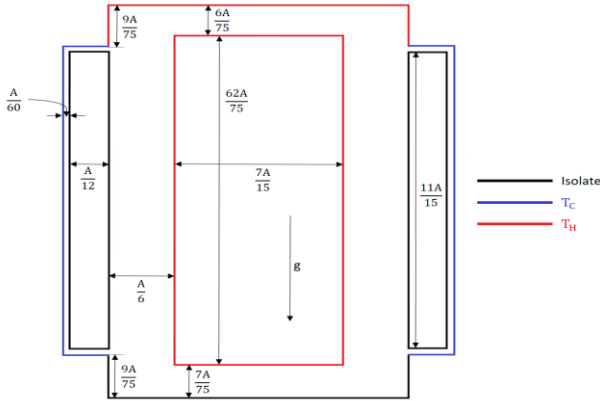


Figure 1: Two-dimensional model of the transformer [24]

The thermophysical properties of nanoparticles and base oil are presented in Table 1. The most important assumptions used for the numerical solution of the problem are that the fluid flow is laminar and incompressible, and the relationships of the single-phase model are used.

Table 1: Thermophysical properties of materials [25-27]

Materials	β (K^{-1})	k (W/mK)	C_p (J/kgK)	ρ (kg/m^3)
Fe_3O_4	0.000206	80.4	670	5180
Naphthenic oil	0.000075	0.146	1970	845.7

Given the no-slip condition and the model's geometry as shown in Figure 1, the dimensionless boundary

conditions for the hot, cold, and insulated walls are as follows:

Hot wall (temperature of T_H): $U = 0, V = 0, \theta = 1$

Cold wall (temperature of T_C): $U = 0, V = 0, \theta = 0$

Wall of vertical insulation: $U = 0, V = 0, \frac{\partial \theta}{\partial X} = 0$

Wall of horizontal insulation: $U = 0, V = 0, \frac{\partial \theta}{\partial Y} = 0$

Governing equations

The following is a two-dimensional representation of the continuity, momentum, and energy equations for laminar and steady-state natural convection:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{(\rho\beta)_{nf}}{\rho_{nf}} g(T - T_C) \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

In order to reduce the number of variables and make the equations and results independent, the governing equations are presented in dimensionless form by the (5).

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{uL}{\alpha_{bf}}, V = \frac{vL}{\alpha_{bf}}, P = \frac{\rho L^2}{\rho_{nf} \alpha_{bf}^2}, \theta = \frac{T - T_C}{T_H - T_C} \quad (5)$$

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (6)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\nu_{nf}}{\alpha_{bf}} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (7)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\nu_{nf}}{\alpha_{bf}} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{\beta_{nf}}{\beta_{bf}} RaPr\theta \quad (8)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_{bf}} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (9)$$

The following gives the thermal diffusion coefficient, together with the Rayleigh and Prandtl numbers:

$$Ra = \frac{g\beta_{bf}L^3\Delta T}{\nu_{bf}\alpha_{bf}} \quad (10)$$

$$Pr = \frac{\nu_{bf}}{\alpha_{bf}} \quad (11)$$

$$\alpha = \frac{k}{c_p\rho} \quad (12)$$

Relationships of nanofluids

The simulations are run in a single-phase model where there is no slip condition between the base fluid and the solid particles and they are in thermal equilibrium. Relationships 13 through 18 are displayed for nanofluid relationships.

$$k_{nf} = \frac{k_s + 2k_{bf} - 2\phi(k_{bf} - k_s)}{k_s + 2k_{bf} + \phi(k_{bf} - k_s)} k_{bf} \quad (13)$$

$$\rho_{nf} = \phi\rho_s + (1 - \phi)\rho_{bf} \quad (14)$$

$$(\rho c_p)_{nf} = \phi(\rho c_p)_s + (1 - \phi)(\rho c_p)_{bf} \quad (15)$$

$$(\rho\beta)_{nf} = \phi(\rho\beta)_s + (1 - \phi)(\rho\beta)_{bf} \quad (16)$$

$$\mu_{nf} = \frac{\mu_{bf}}{(1 - \phi)^{2.5}} \quad (17)$$

$$Nu_{avg} = \int_0^1 Nudy \quad (18)$$

The lattice Boltzmann method

One of the benefits of the lattice Boltzmann approach is its simplicity and ease of application in complex domains. It can handle multi-phase and multi-component flows without the need for common surfaces between phases. The following equation can be used to determine the

Boltzmann transport equation for a system in the absence of an external force. Since Ω is a function of f , f must be found by solving the Boltzmann equation [28].

$$\frac{\partial f}{\partial t} + c \cdot \nabla f = \Omega \quad (19)$$

The main factor in the application of the lattice Boltzmann method is the equilibrium distribution function (f^{eq}). Actually, each particle in the lattice Boltzmann method is replaced by the distribution function. To solve the Boltzmann equation, the collision operator can be approximated in the form of a simple and low-error operator. The BGK model, which is a basic model for the collision operator, was presented by Bhatnagar, Gross, and Krook. The temperature and current distribution functions are shown in relations 20 and 21, respectively, using the BGK approximation [29].

$$f_i(r + c_i \Delta t, t + \Delta t) = f_i(r, t) + \frac{\Delta t}{\tau_F} [f_i^{eq}(r, t) - f_i(r, t)] + c_i \Delta t F \quad (20)$$

$$g_i(r + c_i \Delta t, t + \Delta t) = g_i(r, t) + \frac{\Delta t}{\tau_T} [g_i^{eq}(r, t) - g_i(r, t)] \quad (21)$$

In this case, the diffusion process is shown on the right side of equations 20 and 21, while the collision process is shown on the left. τ is the relaxation coefficient. As a result, the two phases of solving Boltzmann's equation are collision and diffusion. that the lattice time step is represented by Δt in relations 20 and 21, and the flow field and temperature field's relaxation times, expressed as τ_F and τ_T , respectively, may be computed from relations 22 and 23:

$$\tau_F = 3\nu_{bf} + \frac{1}{2} \quad (22)$$

$$\tau_T = 3\alpha_{bf} + \frac{1}{2} \quad (23)$$

Modeling and boundary conditions

Lattice Boltzmann models are given by the formula $DnQm$, where n is the dimension of the problem and m is the number of velocity vectors. In this simulation, $D2Q9$ is utilized. This model was given by Qian et al. [30]. The Boltzmann lattice for this model can be written as follows:

$$f_i(r + \Delta r, t + \Delta t) = f_i(r, t) [1 - \omega] + \omega f_i^{eq}(r, t) \quad (24)$$

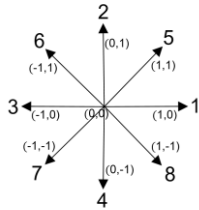


Figure 2: Velocity components of the D2Q9 model

$$f_i^{eq} = \omega_i \rho(r, t) \left[1 + \frac{c_i \cdot u}{c_s^2} + \frac{1}{2} \frac{(c_i \cdot u)^2}{c_s^4} - \frac{1}{2} \frac{u^2}{c_s^2} \right] \quad (25)$$

$$g_i^{eq} = \omega_i T \left[1 + \frac{c_i \cdot u}{c_s^2} + \frac{1}{2} \frac{(c_i \cdot u)^2}{c_s^4} - \frac{1}{2} \frac{u^2}{c_s^2} \right] \quad (26)$$

$$c_s = \frac{c_i}{\sqrt{3}} \quad (27)$$

$$c_i = \frac{\Delta x}{\Delta t} i + \frac{\Delta y}{\Delta t} j \quad (28)$$

$$u = ui + vj \quad (29)$$

Equation 28 assumes that the values of Δx , Δy , and Δt , which stand for horizontal displacement, vertical displacement, and time, respectively, are equal to 1. Equations 30, 31, and 32 can be used to calculate the

fluid's density, velocity, and macroscopic temperature in the D2Q9 model.

$$\rho = \sum_0^8 f_i \quad (30)$$

$$u = \frac{1}{\rho} \sum_0^8 f_i c_i \quad (31)$$

$$T = \sum_0^8 g_i \quad (32)$$

Boundary conditions are necessary for the simulation's result computation. One of the most popular models, the bounce-back model, is applied in this study. In this model, the condition of no slip or the condition of no flow flowing over an obstruction is employed to mimic a stationary solid [31]. As an example, the transformer's bottom wall is subjected to the bounce-back boundary condition in Figure 3.

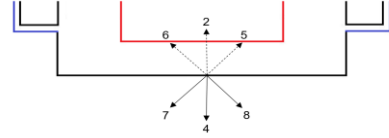


Figure 3: Boundary conditions for a surface geometry problem

As a result, we can write for Figure 3:

$$f_{5,n} = f_{7,n} \quad , \quad f_{2,n} = f_{4,n} \quad , \quad f_{6,n} = f_{8,n} \quad (33)$$

$$g_{5,n-1} = g_{5,n} \quad , \quad g_{2,n-1} = g_{2,n} \quad , \quad g_{6,n-1} = g_{6,n} \quad (34)$$

Code validation and grid testing

Grid independence for a primary parameter of the material under study should be verified in numerical computation methods. The grid selection should be chosen so that the numerical solution's accuracy remains relatively constant after that grid. Grid independence shows that the 170*170 lattice was chosen as the suitable lattice.

To validate the results and ensure the correctness of the numerical solution in this research, the results obtained in the present research have been validated with the results of two Research by Khanafer et al. [32] and Oztop and Abu-nada [13]. In the numerical validation of this study, the heat transfer of natural convection within two studies of the two-dimensional square enclosure has been investigated.

As shown in Figure 4 and Figure 5, a comparison was performed between the temperature figure obtained in the current investigation and that reported by Khanafer et al. [32] and also the comparison between the Nusselt numbers derived from the current study and those obtained from Oztop and Abu-Nada [13].

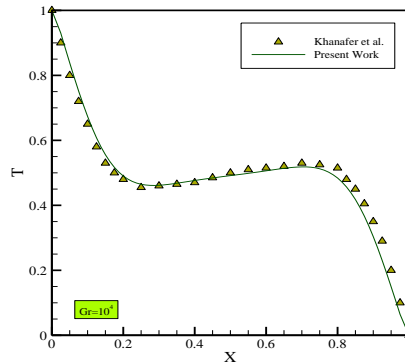


Figure 4: Comparison between the present results and the temperature results by Khanafer et al. [32]

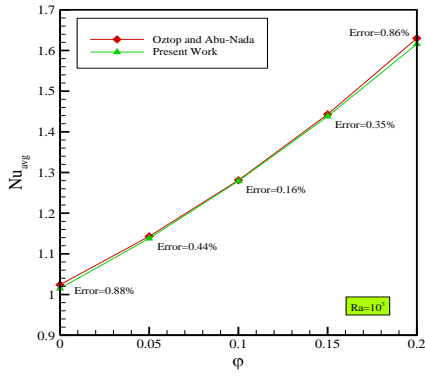


Figure 5: Comparison between the present results and average Nusselt number by Oztop and Abu-Nada [13]

Temperature contours

Figures 6 to 8 presents the temperature line for Naphthenic- Fe_3O_4 nano-oil in volume fraction 2% and Rayleigh numbers 10^3 to 10^5 . Figures 6 to 8 shows that the area surrounding the transformer core and its upper surface is where the nano-oil is at its maximum temperature. Because of natural convection and density, the temperature of the upper surface of the core of the transformer is higher than the temperature of the lower surface, indicating the anticipated rotating flow of oil in the transformer. Additionally, the temperature of the nano-oil drops as it moves closer to the channels around the core.

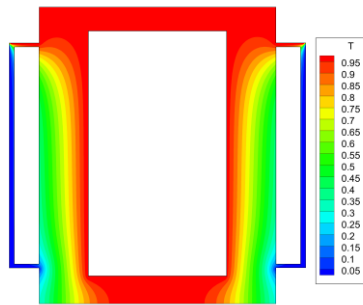


Figure 6: Temperature contour for Naphthenic- Fe_3O_4 nano-oil for $\phi=2\%$ and $Ra=10^3$

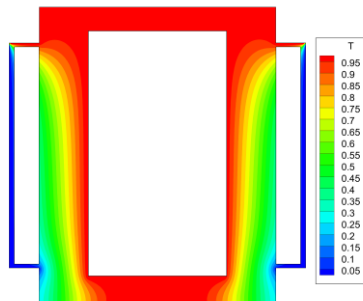


Figure 7: Temperature contour for Naphthenic- Fe_3O_4 nano-oil for $\phi=2\%$ and $Ra=10^4$

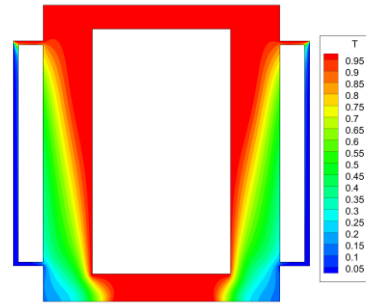


Figure 8: Temperature contour for Naphthenic- Fe_3O_4 nano-oil for $\phi=2\%$ and $Ra=10^5$

Influence of Rayleigh number and nano-oil on Nu_{avg}

The Nusselt number is the ratio of convection heat transfer to conduction heat transfer. Therefore, with the increase in the Nusselt number, the heat transfer performance of the natural convection of the nano-oil improves.

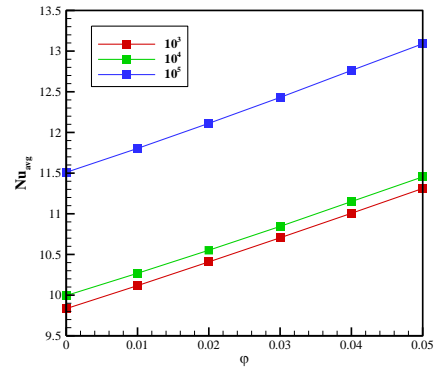


Figure 9: Average Nusselt number for nano-oil in different Rayleigh numbers and nanoparticle volume fractions

Figure 9 show that adding nanoparticles to the oil base fluid and increasing the Rayleigh number improves the heat transfer performance. The average Nusselt number has been observed to increase with an increase in the volume fraction of nanoparticles across all ranges of Rayleigh number. One of the most important advantages of increasing heat transfer is improving the thermal performance of transformer oil, as a result of which the cooling operation of the transformer core is performed better.

Conclusions

The two-dimensional natural convection in a power transformer with one nano-oil, including an Fe_3O_4 nanoparticle in Naphthenic oil, is analyzed. The two-dimensional lattice Boltzmann method is used for this research. The influences of different Rayleigh numbers and nanoparticle volume fractions on heat transfer performance are presented. Based on the results of this research:

- 1) Adding nanoparticles to the oil base fluid has increased the natural convection heat transfer in transformer.
- 2) The average Nusselt number increases with increasing Rayleigh number.

3) The average Nusselt number increases with increasing nanoparticle volume fraction.

4) The area surrounding the transformer core and its upper surface is where the nano-oil is at its maximum temperature.

Nomenclature

A Dimensionless transformer length parameter

C_p Specific heat capacity [J/kgK]

c_i Velocity of virtual particles on lattice [m/s]

c_s Velocity of sound on lattice [m/s]

F External force [N]

f_i Distribution function of density

f_i^{eq} Equilibrium distribution function of density

g_i Distribution function of temperature

g_i^{eq} Equilibrium distribution function of temperature

g Gravitational acceleration [m/s²]

h Convection heat transfer coefficient [W/m²K]

k Thermal conductivity coefficient [W/mK]

L Length [m]

n The number of grids in the x direction

Nu_{avg} Average Nusselt number

Nu Local Nusselt number

P Dimensionless pressure

p Pressure [N/m²]

Pr Prandtl number

r Location vector [m]

Ra Rayleigh number

T Temperature [K]

t Time [s]

U, V Dimensionless velocity components

u, v Velocity components [m/s]

x, y Cartesian coordinates

Greek symbols

α Coefficient of heat diffusion [m²/s]

β Thermal expansion coefficient [K⁻¹]

θ Dimensionless temperature

μ Dynamic viscosity [Ns/m²]

ν Kinematic viscosity [m²/s]

ρ Density [kg/m³]

τ Relaxation factor

ϕ Nanoparticle volume fraction

Ω Collision operator

ω Weighting factor

Subscripts

bf Base fluid

c Cold wall

F Flow

h Hot wall

nf Nanofluid

s Nanoparticle

T Temperature

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