

## Flow Investigation of natural convection in power transformer with nano-oil using lattice Boltzmann method

Alireza Rahimi<sup>1</sup>, Mohammad Saeed Saeedinik<sup>2</sup>

<sup>1</sup>Faculty of Energy, University of Kashan, Kashan, Iran. E-mail: [rahimi2@kashanu.ac.ir](mailto:rahimi2@kashanu.ac.ir)

<sup>2</sup>Faculty of Energy, University of Kashan, Kashan, Iran. E-mail: [saeedinik1377@gmail.com](mailto:saeedinik1377@gmail.com)

### Abstract

Improving the transformer oil's thermal performance is necessary since transformers are essential in power distribution networks and besides, the transformer core's temperature has an impact on how well it functions electrically. In this study, by presenting a two-dimensional model of a transformer, the natural convection heat transfer of one nano-oil sample, including Al<sub>2</sub>O<sub>3</sub> nanoparticle in base fluid of Naphthenic oil has been investigated theoretically and simulated by the lattice Boltzmann method and D2Q9 model. Modelling and solving were done for nanoparticle volume fractions of 0% to 5% and Rayleigh numbers of 10<sup>3</sup> to 10<sup>5</sup>. Flow lines and Nusselt number average are given in this range. Based on the results of this research, adding nanoparticle to the base fluid has improved the cooling operation of the transformer core. Also, increasing the volume fraction of nanoparticles and the Rayleigh number increased the natural convection heat transfer. Naphthenic-Al<sub>2</sub>O<sub>3</sub> nano-oil, at a volume fraction of 5% and Rayleigh number of 10<sup>5</sup>, has the highest Nusselt number average of 13.06 and the heat transfer compared to pure Naphthenic oil under the same conditions has improved.

**Keywords:** Heat transfer, Natural convection, Nanofluid, Transformer, Lattice Boltzmann method

### Introduction

As a result of the numerous industrial applications associated with natural convection heat transfer, as well as the significant impact of parameters such as Rayleigh number, nanoparticle volume fraction, and boundary conditions on these processes, scholars have conducted extensive investigations in this particular domain. Improving the thermal conductivity of the transformer oil is also more important because of the need for transformers in the power transmission network and how the temperature of the transformer core affects how well the device works electrically. Considering the effect of temperature on the useful life and efficiency of these systems, researchers are looking for new methods, such as the use of nanofluids, to enhance the natural heat transfer from the transformer core [1-9].

Base fluids such as water [10], oil [11] and ethylene glycol [12], which are often used as base fluids for natural convection heat transfer, have much lower thermal conductivity than nanoparticles; as a result, adding any of the nanoparticles such as Cu [13], CuO [14], Al<sub>2</sub>O<sub>3</sub> [15], TiO<sub>2</sub> [16], MWCNTs (multi walled carbon nanotubes)

[17], Fe<sub>3</sub>O<sub>4</sub> [18] and etc. could enhance the thermal conductivity of fluids. The research of Segal and Raj [8] on natural convection heat transfer by mineral oil around the transformer core shows that choosing the best value of the magnetic field intensity can reduce the heat intensity around the transformer core. Pendyala et al. [9] used computational fluid dynamics (CFD) analysis to investigate the thermal performance of nanofluids in transformers. Their study's conclusions showed that, in comparison to base fluids, nanofluids have better heat transfer properties. The study also showed how important fluid density and thermal conductivity are in improving natural heat transfer properties. Rahimi et al. [16] used the lattice Boltzmann method, entropy generation, and heat lines to study natural convection heat transfer in an L-shaped hollow chamber filled with nanofluid. It was found that when the Rayleigh number increases, the average Nusselt number and the total amount of entropy production increase in tandem. Furthermore, an increase in the average Nusselt number and a decrease in the overall entropy generation were noted in tandem with the nanofluid's solid volume fraction. According to a study conducted by Hwang et al. [19], a numerical investigation was carried out to examine the impact of aluminum oxide nanofluid on natural convection heat transfer in a rectangular chamber. Findings showed that the addition of nanoparticles and increasing the Rayleigh number resulted in an augmentation of heat transfer. In their study, Chen et al. [20] used the lattice Boltzmann method to look into the combined convection heat transfer in a chamber with an aluminum oxide nanofluid. They showed that nanoparticles can raise the Nusselt number. Karki et al. [21] conducted a study focusing on laminar natural convection and entropy generation by air, water and alumina-water nanofluid. They solved the fluid flow equations with the D2Q9 model and their results showed that heat transfer increased due to the addition of nanoparticles to the base fluid. Also, they reported the maximum increase in Nusselt number at 8% volume fraction and 10<sup>5</sup> Rayleigh number by 13.93%. Santra et al. [22], using a non-Newtonian fluid and a finite volume method, investigated the natural convection heat transfer inside a cavity filled with nanofluid containing copper nanoparticles and showed that increasing the number of nanoparticles in a certain Rayleigh range leads to a decrease in heat transfer. Sajjadi et al. [23] studied and investigated the natural convection flow in a porous cavity with sinusoidal temperature distribution using a new Boltzmann lattice method in the presence of water-copper

nanofluid. They conducted this study for volume fractions 0 to 6%, Rayleigh numbers  $10^3$  to  $10^5$  and Darcy numbers 0.001 to 0.1. Their findings included the observation that the rate of heat transfer is positively correlated with the Darcy number, Rayleigh number, and volume fraction of the nanoparticles. The Rayleigh number was also introduced as the parameter with the highest sensitivity.

Considering the important role of transformers in the country's power transmission network and the challenges related to cooling their cores, it is expected that enhancing core cooling will reduce electrical energy loss and transformer maintenance costs.

### Problem statement

The transformer housing, oil channels, and core are all included in the two-dimensional model of the transformer, as shown in Figure 1. The oil channels have a low temperature ( $T_C$ ) and the transformer core has a high temperature ( $T_H$ ). The dimensionless parameter  $A$  has been utilized to calculate the transformer's dimensions. It is assumed that the upper wall of the transformer core has a  $T_H$  temperature because of the heat transfer from the core that occurs naturally.  $Al_2O_3$  nanoparticles and Naphthenic oil are the nano-oil employed.

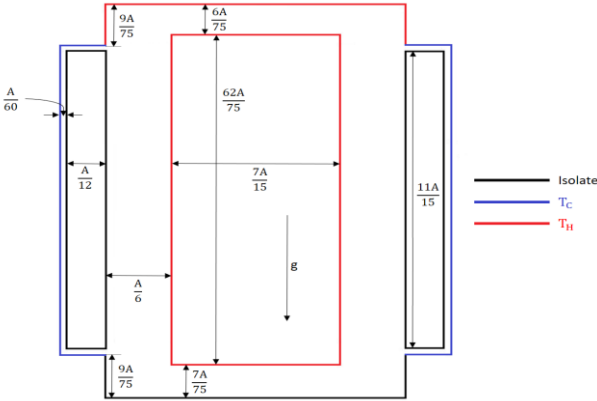


Figure 1: The transformer's two-dimensional model [24]

The thermophysical properties of nanoparticles and base oil are presented in Table 1. The most important assumptions used for the numerical solution of the problem are that the fluid flow is laminar and incompressible, and the relationships of the single-phase model are used.

Table 1: Thermophysical properties of materials [25-27]

Materials	$\beta$ ( $K^{-1}$ )	$k$ (W/mK)	$C_p$ (J/kgK)	$\rho$ ( $kg/m^3$ )
$Al_2O_3$	0.0000085	40	765	3970
Naphthenic oil	0.000075	0.146	1970	845.7

Given the no-slip condition and the model's geometry as shown in Figure 1, the dimensionless boundary conditions for the hot, cold, and insulated walls are as follows:

Hot wall (temperature of  $T_H$ ):  $U = 0, V = 0, \theta = 1$

Cold wall (temperature of  $T_C$ ):  $U = 0, V = 0, \theta = 0$

Vertical insulation wall:  $U = 0, V = 0, \frac{\partial \theta}{\partial x} = 0$

Horizontal insulation wall:  $U = 0, V = 0, \frac{\partial \theta}{\partial y} = 0$

### Governing equations

The following is a two-dimensional representation of the continuity, momentum, and energy equations for laminar and steady-state natural convection:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{(\rho\beta)_{nf}}{\rho_{nf}} g(T - T_C) \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

In order to reduce the number of variables and make the equations and results independent, the governing equations are presented in dimensionless form by the (5).

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{uL}{\alpha_{bf}}, V = \frac{vL}{\alpha_{bf}}, P = \frac{\rho L^2}{\rho_{nf} \alpha_{bf}^2}, \theta = \frac{T - T_C}{T_H - T_C} \quad (5)$$

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (6)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\nu_{nf}}{\alpha_{bf}} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (7)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\nu_{nf}}{\alpha_{bf}} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{\beta_{nf}}{\beta_{bf}} Ra Pr \theta \quad (8)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_{bf}} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (9)$$

The following gives the thermal diffusion coefficient, together with the Rayleigh and Prandtl numbers:

$$Ra = \frac{g \beta_{bf} L^3 \Delta T}{\nu_{bf} \alpha_{bf}} \quad (10)$$

$$Pr = \frac{\nu_{bf}}{\alpha_{bf}} \quad (11)$$

$$\alpha = \frac{k}{c_p \rho} \quad (12)$$

### Relationships of nanofluids

The simulations are performed in a single-phase model, and the solid particles and the base fluid are in thermal equilibrium with no-slip conditions between them. relationships of nanofluid are presented as relations 13 to 18.

$$k_{nf} = \frac{k_s + 2k_{bf} - 2\phi(k_{bf} - k_s)}{k_s + 2k_{bf} + \phi(k_{bf} - k_s)} k_{bf} \quad (13)$$

$$\rho_{nf} = \phi \rho_s + (1 - \phi) \rho_{bf} \quad (14)$$

$$(\rho c_p)_{nf} = \phi (\rho c_p)_s + (1 - \phi) (\rho c_p)_{bf} \quad (15)$$

$$(\rho \beta)_{nf} = \phi (\rho \beta)_s + (1 - \phi) (\rho \beta)_{bf} \quad (16)$$

$$\mu_{nf} = \frac{\mu_{bf}}{(1 - \phi)^{2.5}} \quad (17)$$

$$Nu_{avg} = \int_0^1 Nudy \quad (18)$$

### The lattice Boltzmann method

One of the benefits of the lattice Boltzmann approach is its simplicity and ease of application in complex domains. It can handle multi-phase and multi-component flows without the need for common surfaces between phases. The following equation can be used to determine the Boltzmann transport equation for a system in the absence of an external force. Since  $\Omega$  is a function of  $f$ ,  $f$  must be found by solving the Boltzmann equation [28].

$$\frac{\partial f}{\partial t} + c \cdot \nabla f = \Omega \quad (19)$$

The main factor in the application of the lattice Boltzmann method is the equilibrium distribution function ( $f_i^{eq}$ ). Actually, each particle in the lattice Boltzmann method is replaced by the distribution function. To solve the Boltzmann equation, the collision operator can be approximated in the form of a simple and low-error operator. The BGK model, which is a basic model for the collision operator, was presented by Bhatnagar, Gross, and Krook. The temperature and current distribution functions are shown in relations 20 and 21, respectively, using the BGK approximation [29].

$$f_i(r + c_i \Delta t, t + \Delta t) = f_i(r, t) + \frac{\Delta t}{\tau_F} [f_i^{eq}(r, t) - f_i(r, t)] + c_i \Delta t F \quad (20)$$

$$g_i(r + c_i \Delta t, t + \Delta t) = g_i(r, t) + \frac{\Delta t}{\tau_T} [g_i^{eq}(r, t) - g_i(r, t)] \quad (21)$$

In this case, the diffusion process is shown on the right side of equations 20 and 21, while the collision process is shown on the left.  $\tau$  is the relaxation coefficient. As a result, the two phases of solving Boltzmann's equation are collision and diffusion. that the lattice time step is represented by  $\Delta t$  in relations 20 and 21, and the flow field and temperature field's relaxation times, expressed as  $\tau_F$  and  $\tau_T$ , respectively, may be computed from relations 22 and 23:

$$\tau_F = 3\nu_{bf} + \frac{1}{2} \quad (22)$$

$$\tau_T = 3\alpha_{bf} + \frac{1}{2} \quad (23)$$

### Modeling and boundary conditions

The formula DnQm, where n is the problem's dimension and m is the number of velocity vectors, yields Lattice Boltzmann models. D2Q9 is used in this simulation. Qian et al. [30] provided this model. For this scenario, the Boltzmann lattice can be expressed as follows:

$$f_i(r + \Delta r, t + \Delta t) = f_i(r, t) [1 - \omega] + \omega f_i^{eq}(r, t) \quad (24)$$

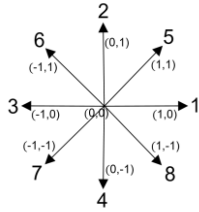


Figure 2: The D2Q9 model's velocity components

$$f_i^{eq} = \omega_i \rho(r, t) \left[ 1 + \frac{c_i \cdot u}{c_s^2} + \frac{1}{2} \frac{(c_i \cdot u)^2}{c_s^4} - \frac{1}{2} \frac{u^2}{c_s^2} \right] \quad (25)$$

$$g_i^{eq} = \omega_i T \left[ 1 + \frac{c_i \cdot u}{c_s^2} + \frac{1}{2} \frac{(c_i \cdot u)^2}{c_s^4} - \frac{1}{2} \frac{u^2}{c_s^2} \right] \quad (26)$$

$$c_s = \frac{c_i}{\sqrt{3}} \quad (27)$$

$$c_i = \frac{\Delta x}{\Delta t} i + \frac{\Delta y}{\Delta t} j \quad (28)$$

$$u = ui + vj \quad (29)$$

In equation 28,  $\Delta x$ ,  $\Delta y$ , and  $\Delta t$  represents horizontal displacement, vertical displacement, and time, respectively, and they are all assumed to be equal to 1. The density, velocity and macroscopic temperature of the fluid in the D2Q9 model are calculated from equations 30, 31 and 32, respectively.

$$\rho = \sum_0^8 f_i \quad (30)$$

$$u = \frac{1}{\rho} \sum_0^8 f_i c_i \quad (31)$$

$$T = \sum_0^8 g_i \quad (32)$$

Boundary conditions are necessary for the simulation's result computation. One of the most popular models, the bounce-back model, is applied in this study. In this model, the condition of no slip or the condition of no flow flowing over an obstruction is employed to mimic a stationary solid [31]. As an example, the transformer's bottom wall is subjected to the bounce-back boundary condition in Figure 3.

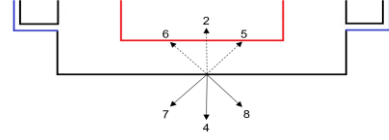


Figure 3: Boundary condition for a surface of the problem geometry

As a result, we can write for Figure 3:

$$f_{5,n} = f_{7,n} \quad , \quad f_{2,n} = f_{4,n} \quad , \quad f_{6,n} = f_{8,n} \quad (33)$$

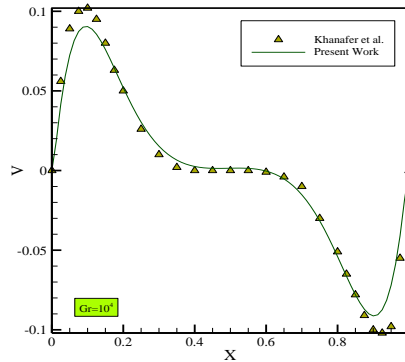
$$g_{5,n-1} = g_{5,n} \quad , \quad g_{2,n-1} = g_{2,n} \quad , \quad g_{6,n-1} = g_{6,n} \quad (34)$$

### Code validation and grid testing

Grid independence for a primary parameter of the material under study should be verified in numerical computation methods. The grid selection should be chosen so that the numerical solution's accuracy remains relatively constant after that grid. results shows that the 170\*170 lattice was chosen as the suitable lattice.

To validate the results and ensure the correctness of the numerical solution in this research, the results obtained in the present research have been validated with the results of two Research by Khanafer et al. [32] and Oztop and Abu-nada [13]. In the numerical validation of this study, the heat transfer of natural convection within two studies of the two-dimensional square enclosure has been investigated.

As shown in Figure 4, a comparison was performed between the velocity figure obtained in the current investigation and those reported by Khanafer et al. [32] and also the comparison between the Nusselt numbers derived from the current study and those obtained from Oztop and Abu-Nada [13].



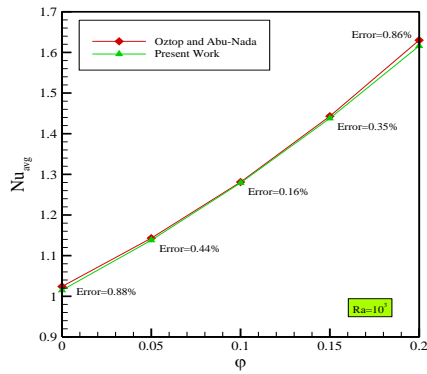


Figure 4: Comparison between the present results and the vertical velocity results by Khanafer et al. [32] and average Nusselt number by Oztop and Abu-Nada [13]

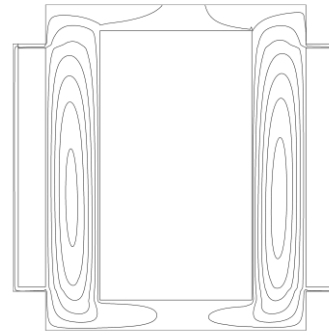


Figure 7: Flow lines for Naphthenic- $\text{Al}_2\text{O}_3$  nano-oil for  $\phi=2\%$  and  $\text{Ra}=10^5$

### Flow structure

Figures 5 to 7 shows the flow lines for a 2% volume fraction of Naphthenic- $\text{Al}_2\text{O}_3$  nano-oil with Rayleigh numbers  $10^3$  to  $10^5$ . Figures 5 to 7 shows the oval-shaped flow of nano-oil rotating within the transformer chamber.

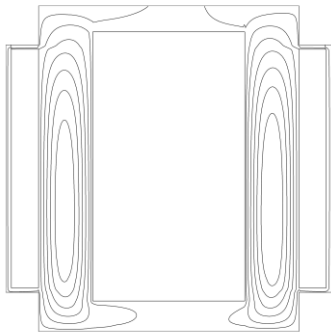


Figure 5: Flow lines for Naphthenic- $\text{Al}_2\text{O}_3$  nano-oil for  $\phi=2\%$  and  $\text{Ra}=10^3$

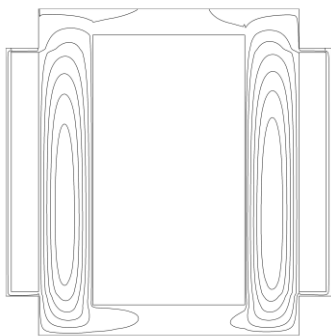


Figure 6: Flow lines for Naphthenic- $\text{Al}_2\text{O}_3$  nano-oil for  $\phi=2\%$  and  $\text{Ra}=10^4$

In the upper and lower levels of the transformer core, there is less nano-oil flow, and this flow is more observable around the core and between the core and the side channels. It should be noted that the velocity of the nano-oil will increase with decreasing distance between the flow lines.

### Influence of Rayleigh number and nano-oil on $\text{Nu}_{\text{avg}}$

A heat transfer's ratio from convection to conduction is called its Nusselt number. Therefore, when the Nusselt number increases, so does the nano-oil's natural convection's capacity for heat transfer.

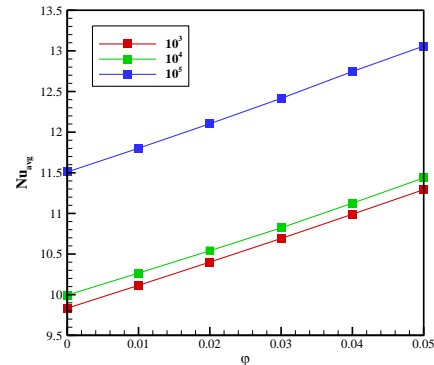


Figure 8: The average Nusselt number for nano-oil for a range of Rayleigh numbers and volume fractions of nanoparticles

Figure 8 show that increasing the Rayleigh number and adding nanoparticles into the oil base fluid enhances heat transfer efficiency. Across all Rayleigh number ranges, an increase in the volume fraction of nanoparticles has been shown to increase the average Nusselt number. Enhancing heat transfer can lead to higher transformer oil thermal performance, which in turn enhances transformer core cooling performance. This is one of the main benefits of increased heat transfer.

### Conclusions

The two-dimensional natural convection in a power transformer with one nano-oil, including an  $\text{Al}_2\text{O}_3$  nanoparticle in Naphthenic oil, is analyzed. The two-dimensional lattice Boltzmann method is used for this research. The influences of different Rayleigh numbers and nanoparticle volume fractions on fluid flow and nano-

oil velocity are presented. Based on the results of this research:

1) In the upper and lower levels of the transformer core, there is less nano-oil flow, and this flow is more observable around the core and between the core and the side channels.

2) The natural convection heat transfer in transformers has increased with the addition of nanoparticles to the oil base fluid.

3) As the Rayleigh number increases, so does the average Nusselt number.

4) As the volume fraction of nanoparticles increases, the average Nusselt number also increases.

### Nomenclature

$A$	Dimensionless transformer length parameter
$C_p$	Specific heat capacity [J/kgK]
$c_i$	Velocity of virtual particles on lattice [m/s]
$c_s$	Velocity of sound on lattice [m/s]
$F$	External force [N]
$f_i$	Distribution function of density
$f_i^{eq}$	Equilibrium distribution function of density
$g_i$	Distribution function of temperature
$g_i^{eq}$	Equilibrium distribution function of temperature
$g$	Gravitational acceleration [m/s <sup>2</sup> ]
$h$	Convection heat transfer coefficient [W/m <sup>2</sup> K]
$k$	Thermal conductivity coefficient [W/mK]
$L$	Length [m]
$n$	The number of grids in the x direction
$Nu_{avg}$	Average Nusselt number
$Nu$	Local Nusselt number
$P$	Dimensionless pressure
$p$	Pressure [N/m <sup>2</sup> ]
$Pr$	Prandtl number
$r$	Location vector [m]
$Ra$	Rayleigh number
$T$	Temperature [K]
$t$	Time [s]
$U, V$	Dimensionless velocity components
$u, v$	Velocity components [m/s]
$x, y$	Cartesian coordinates

### Greek symbols

$\alpha$	Coefficient of heat diffusion [m <sup>2</sup> /s]
$\beta$	Thermal expansion coefficient [K <sup>-1</sup> ]
$\theta$	Dimensionless temperature
$\mu$	Dynamic viscosity [Ns/m <sup>2</sup> ]
$\nu$	Kinematic viscosity [m <sup>2</sup> /s]
$\rho$	Density [kg/m <sup>3</sup> ]
$\tau$	Relaxation factor
$\varphi$	Nanoparticle volume fraction
$\Omega$	Collision operator
$\omega$	Weighting factor

### Subscripts

bf	Base fluid
c	Cold wall
F	Flow
h	Hot wall
nf	Nanofluid
s	Nanoparticle
T	Temperature

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