

Lattice Boltzmann simulation of thermal investigation of natural convection in power transformer with nano-oil

Alireza Rahimi¹, Mohammad Saeed Saeedinik²

¹Faculty of Energy, University of Kashan, Kashan, Iran. E-mail: rahimi2@kashanu.ac.ir

²Faculty of Energy, University of Kashan, Kashan, Iran. E-mail: saeedinik1377@gmail.com

Abstract

The improvement of the thermal performance of transformer oil is essential due to the crucial role transformers perform in power distribution networks. Additionally, the electrical performance of a transformer is influenced by the temperature of its core. The natural convection heat transfer of a nano-oil sample, containing an Al₂O₃ nanoparticle in the base fluid of Paraffinic oil, has been studied theoretically and simulated by the lattice Boltzmann method and D2Q9 model in this paper, which presents a two-dimensional model of a transformer. For nanoparticle volume fractions of 0% to 5% and Rayleigh numbers of 10³ to 10⁵, modelling and solutions were carried out and, in this range, temperature charts and average Nusselt number are provided. In light of the findings of this study, adding nanoparticle to the base fluid has improved the cooling operation of the transformer core. Additionally, the natural convection heat transfer was enhanced by increasing the Rayleigh number and the volume fraction of nanoparticles. Paraffinic-Al₂O₃ nano-oil, at a volume fraction of 5% and Rayleigh number of 10⁵, has the highest Nusselt number average of 13.07 and under the same conditions, the heat transfer has improved as compared to pure Paraffinic oil.

Keywords: Heat transfer, Natural convection, Nanofluid, Transformer, Lattice Boltzmann method

Introduction

Because natural convection heat transfer has many industrial uses and important parameters like Rayleigh number, nanoparticle volume fraction, and boundary conditions can have a big effect on these processes, researchers have done a lot of work in this field. Furthermore, the improvement of the thermal behavior of the transformer oil is essential due to the requirement for using transformers in the power distribution network and the influence of the transformer core's temperature on the electrical performance of the equipment. Given the impact of temperature on the usable life and efficiency of these systems, scientists are attempting to employ novel techniques, such as the use of nanofluids, to enhance the natural heat transfer from the transformer core [1-9].

Base fluids like water [10], oil [11] and ethylene glycol [12], which are often used as base fluids for natural convection heat transfer, have much lower thermal conductivity than nanoparticles; thus, adding any of the nanoparticles like Cu [13], CuO [14], Al₂O₃ [15], TiO₂ [16], MWCNTs (multi walled carbon nanotubes) [17],

Fe₃O₄ [18] and etc. may enhance the thermal conductivity of fluids.

The study by Segal and Raj [8] on the natural convection heat transfer by mineral oil surrounding the transformer core demonstrates that the heat intensity surrounding the transformer core can be decreased by selecting the optimal magnetic field intensity value. Pendyala et al. [9] used computational fluid dynamics (CFD) analysis to investigate the thermal performance of nanofluids in transformers. Their study's conclusions showed that, in comparison to base fluids, nanofluids have better heat transfer properties. The study also showed how important fluid density and thermal conductivity are in improving natural heat transfer properties. Rahimi et al. [16] investigated natural convection heat transfer in an L-shaped hollow cavity filled with nanofluid by using the lattice Boltzmann method and entropy generation and heat lines. It was found that an increase in the Rayleigh number results in a concomitant escalation in both the average Nusselt number and the overall quantity of entropy production. Additionally, it was observed that as the solid volume fraction of the nanofluid increased, there was a corresponding increase in the average Nusselt number and a decrease in the total entropy generation. Ogut [19] numerically studied the natural convection heat transfer of nanofluids in an inclined chamber with a heater and showed that with the increase of the Rayleigh number and the volume fraction of particles, the average heat transfer also increased, but with the increase of the length of the heater, the heat transfer decreased. The findings of Sheikholeslami et al.'s numerical study [20] on the natural convection heat transfer of copper nanoparticles in water in the presence of a magnetic field show that increasing the nanoparticle volume fraction, oscillation frequency, and Rayleigh number all affect the average Nusselt number. Lai and Yang [21], using the lattice Boltzmann method, investigated the natural convection heat transfer inside a square chamber containing water-Al₂O₃ nanofluid and they showed that when the Rayleigh number and particle volume fraction rise, so does the average Nusselt number. Karki et al. [22] conducted a study focusing on laminar natural convection and entropy generation by air, water and alumina-water nanofluid. They solved the fluid flow equations with the D2Q9 model and their results showed that heat transfer increased due to the addition of nanoparticles to the base fluid. Also, they reported the maximum increase in Nusselt number at 8% volume fraction and 10⁵ Rayleigh number by 13.93%. Sajjadi et al. [23] used a new Boltzmann lattice method to

look into the spontaneous convection flow in a porous cavity with a sinusoidal temperature distribution. This was done with a water-copper nanofluid present. The volume fractions 0 to 6%, Rayleigh numbers 10^3 to 10^5 , and Darcy numbers 0.001 to 0.1 were the ranges for which they studied. Among their conclusions was the finding that the Darcy number, Rayleigh number, and volume fraction of the nanoparticles all had a positive correlation with the rate of heat transfer. As the most sensitive parameter, the Rayleigh number was also introduced.

Given the essential role of transformers in the country's power distribution network and the challenges associated with cooling their cores, it is expected that enhancing core cooling will lower electrical energy loss and transformer maintenance costs.

Problem statement

The two-dimensional model of the transformer, according to Figure 1 includes the transformer core, transformer housing and transformer oil channels. The transformer core has a high temperature T_H and the oil channels have a low temperature T_C . The upper wall of the transformer core is assumed to have a T_H temperature due to the natural convection heat transfer from the transformer core, and the dimensionless parameter A has been used to determine the dimensions of the transformer. The nano-oil used include Al_2O_3 nanoparticle and Paraffinic oil.

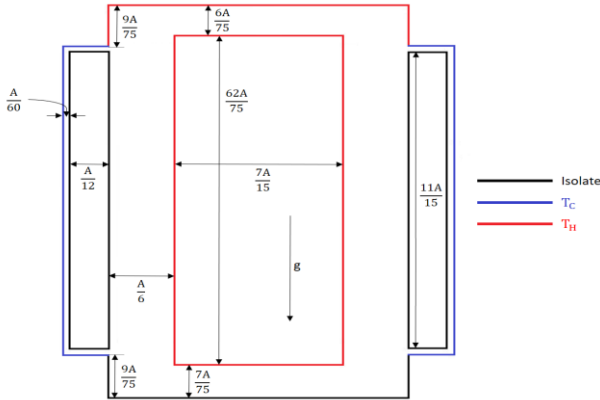


Figure 1: The transformer's two-dimensional model [24]

Table 1 shows the thermophysical properties of base oil and nanoparticle. The single-phase model's relationships and the laminar, incompressible fluid flow are the most crucial assumptions made in the numerical solution of the problem.

Table 1: Thermophysical properties of materials [25-27]

Materials	β (K^{-1})	k (W/mK)	C_p (J/kgK)	ρ (kg/m^3)
Al_2O_3	0.0000085	40	765	3970
Paraffinic oil	0.000075	0.167	2200	783

Based on the geometry of the model according to Figure 1, and considering the no-slip condition, the dimensionless boundary conditions for the hot, cold and insulated walls are as follows:

Hot wall (temperature of T_H): $U = 0, V = 0, \theta = 1$

Cold wall (temperature of T_C): $U = 0, V = 0, \theta = 0$

Vertical insulation wall: $U = 0, V = 0, \frac{\partial \theta}{\partial X} = 0$

Horizontal insulation wall: $U = 0, V = 0, \frac{\partial \theta}{\partial Y} = 0$

Governing equations

The continuity, momentum and energy equations for the laminar and steady state natural convection in two dimensional forms can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{(\rho\beta)_{nf}}{\rho_{nf}} g(T - T_C) \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

The governing equations are given in dimensionless form by (5) in order to reduce the number of variables and make the equations and results independent.

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{uL}{\alpha_{bf}}, V = \frac{vL}{\alpha_{bf}}, P = \frac{\rho L^2}{\rho_{nf} \alpha_{bf}^2}, \theta = \frac{T - T_C}{T_H - T_C} \quad (5)$$

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (6)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\nu_{nf}}{\alpha_{bf}} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (7)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\nu_{nf}}{\alpha_{bf}} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{\beta_{nf}}{\beta_{bf}} RaPr\theta \quad (8)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_{bf}} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (9)$$

The Rayleigh and Prandtl numbers and thermal diffusion coefficient are given below:

$$Ra = \frac{g\beta_{bf}L^3\Delta T}{\nu_{bf}\alpha_{bf}} \quad (10)$$

$$Pr = \frac{\nu_{bf}}{\alpha_{bf}} \quad (11)$$

$$\alpha = \frac{k}{c_p\rho} \quad (12)$$

Relationships of nanofluids

The simulations are performed in a single-phase model, and the solid particles and the base fluid are in thermal equilibrium with no-slip conditions between them. relationships of nanofluid are presented as relations 13 to 18.

$$k_{nf} = \frac{k_s + 2k_{bf} - 2\phi(k_{bf} - k_s)}{k_s + 2k_{bf} + \phi(k_{bf} - k_s)} k_{bf} \quad (13)$$

$$\rho_{nf} = \phi\rho_s + (1 - \phi)\rho_{bf} \quad (14)$$

$$(\rho c_p)_{nf} = \phi(\rho c_p)_s + (1 - \phi)(\rho c_p)_{bf} \quad (15)$$

$$(\rho\beta)_{nf} = \phi(\rho\beta)_s + (1 - \phi)(\rho\beta)_{bf} \quad (16)$$

$$\mu_{nf} = \frac{\mu_{bf}}{(1 - \phi)^{2.5}} \quad (17)$$

$$Nu_{avg} = \int_0^1 Nudy \quad (18)$$

The lattice Boltzmann method

The lattice Boltzmann method's simplicity and ease of application in complicated domains, together with its ability to handle multi-phase and multi-component flows without requiring the following of common surfaces across phases, is one of its advantages. For a system without external force, the Boltzmann transport equation is obtained according to the following equation. Ω is a function of f , and The Boltzmann equation must be solved in order to find Ω [28].

$$\frac{\partial f}{\partial t} + c \cdot \nabla f = \Omega \quad (19)$$

The equilibrium distribution function (f^{eq}) is the primary component used in the lattice Boltzmann method. In fact, the distribution function substitutes for each particle in the lattice Boltzmann method. The collision operator can be modeled as a low-error, straightforward operator to solve the Boltzmann equation. The collision operator's fundamental model, known as the BGK model, was introduced by Bhatnagar, Gross, and Krook. Using the BGK approximation, the temperature and current distribution functions are displayed in relations 20 and 21, respectively [29].

$$f_i(r + c_i \Delta t, t + \Delta t) = f_i(r, t) + \frac{\Delta t}{\tau_F} [f_i^{eq}(r, t) - f_i(r, t)] + c_i \Delta t F \quad (20)$$

$$g_i(r + c_i \Delta t, t + \Delta t) = g_i(r, t) + \frac{\Delta t}{\tau_T} [g_i^{eq}(r, t) - g_i(r, t)] \quad (21)$$

Here, τ is the relaxation coefficient and the left side of equations 20 and 21 shows the collision process, while the right side shows the diffusion process. Therefore, Boltzmann's equation can be solved in two phases: collision and diffusion. that in relations 20 and 21, Δt is the time step of the lattice and τ_F and τ_T respectively express the relaxation time of the flow field and temperature field, which can be calculated from relations 22 and 23:

$$\tau_F = 3\nu_{bf} + \frac{1}{2} \quad (22)$$

$$\tau_T = 3\alpha_{bf} + \frac{1}{2} \quad (23)$$

Modeling and boundary conditions

The formula DnQm, where n is the problem's dimension and m is the number of velocity vectors, yields Lattice Boltzmann models. D2Q9 is used in this simulation. Qian et al. [30] provided this model. For this scenario, the Boltzmann lattice can be expressed as follows:

$$f_i(r + \Delta r, t + \Delta t) = f_i(r, t) [1 - \omega] + \omega f_i^{eq}(r, t) \quad (24)$$

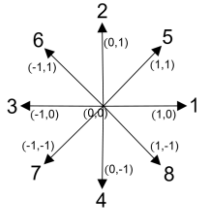


Figure 2: The D2Q9 model's velocity components

$$f_i^{eq} = \omega_i \rho(r, t) \left[1 + \frac{c_i \cdot u}{c_s^2} + \frac{1}{2} \frac{(c_i \cdot u)^2}{c_s^4} - \frac{1}{2} \frac{u^2}{c_s^2} \right] \quad (25)$$

$$g_i^{eq} = \omega_i T \left[1 + \frac{c_i \cdot u}{c_s^2} + \frac{1}{2} \frac{(c_i \cdot u)^2}{c_s^4} - \frac{1}{2} \frac{u^2}{c_s^2} \right] \quad (26)$$

$$c_s = \frac{c_i}{\sqrt{3}} \quad (27)$$

$$c_i = \frac{\Delta x}{\Delta t} i + \frac{\Delta y}{\Delta t} j \quad (28)$$

$$u = ui + vj \quad (29)$$

In equation 28, Δx , Δy , and Δt represents horizontal displacement, vertical displacement, and time, respectively, and they are all assumed to be equal to 1.

The density, velocity and macroscopic temperature of the fluid in the D2Q9 model are calculated from equations 30, 31 and 32, respectively.

$$\rho = \sum_0^8 f_i \quad (30)$$

$$u = \frac{1}{\rho} \sum_0^8 f_i c_i \quad (31)$$

$$T = \sum_0^8 g_i \quad (32)$$

Boundary conditions are necessary for the simulation's result computation. One of the most popular models, the bounce-back model, is applied in this study. In this model, the condition of no slip or the condition of no flow flowing over an obstruction is employed to mimic a stationary solid [31]. As an example, the transformer's bottom wall is subjected to the bounce-back boundary condition in Figure 3.

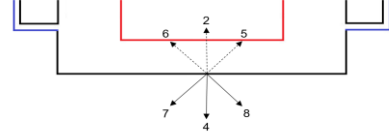


Figure 3: Boundary condition for a surface of the problem geometry

Thus, for Figure 3, we can write:

$$f_{5,n} = f_{7,n} \quad , \quad f_{2,n} = f_{4,n} \quad , \quad f_{6,n} = f_{8,n} \quad (33)$$

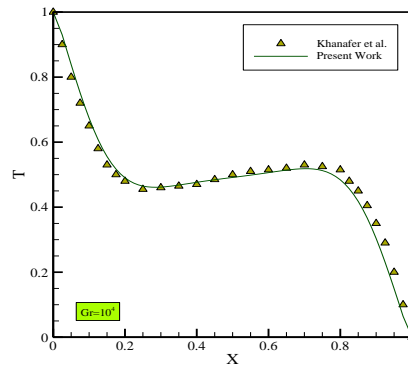
$$g_{5,n-1} = g_{5,n} \quad , \quad g_{2,n-1} = g_{2,n} \quad , \quad g_{6,n-1} = g_{6,n} \quad (34)$$

Grid testing and code validation

In numerical calculation methods, grid independence should be checked for a main parameter of the studied material. The grid should be chosen in such a way that, after that grid, the accuracy of the numerical solution does not change much. The 170*170 lattice is selected as the appropriate lattice.

In order to verify the accuracy of the numerical solution and validate the results, the findings of two studies by Khanafer et al. [32] and Oztop and Abu-nada [13] were compared with the results of the current study. The heat transfer of natural convection within two studies of the two-dimensional square enclosure has been examined in the numerical validation of this work.

As can be seen in Figure 4, a comparison was made between the Nusselt numbers derived from the current study and those obtained from Oztop and Abu-Nada, as well as between the temperature figure obtained in the current investigation and those reported by Khanafer et al. [32].



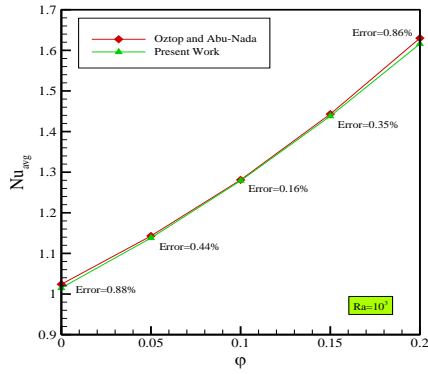


Figure 4: Comparison between the present results and the temperature results by Khanafar et al. [32] and average Nusselt number by Oztop and Abu-Nada [13]

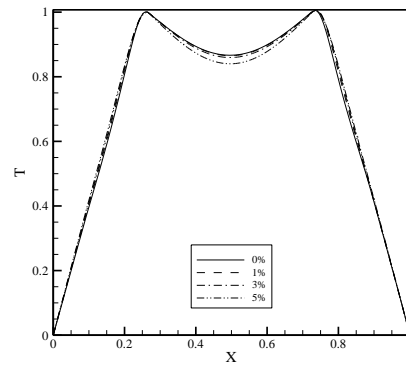


Figure 7: Paraffinic- Al_2O_3 nano-oil temperature for different nanoparticle volume fractions $Ra=10^5$

Temperature distribution

The temperature distribution for Paraffinic- Al_2O_3 nano-oil in different volume fractions and Rayleigh numbers 10^3 to 10^5 is presented in Figures 5 to 7. The noteworthy point in Figures 5 to 7 is that the temperature behavior of the nano-oil at Rayleigh number 10^5 is different from 10^3 and 10^4 .

In Rayleigh numbers 10^3 and 10^4 , in the upper part of the transformer core, the slope of the parabola decreases with the increase of the nanoparticle volume fraction, and this is the case in Rayleigh number 10^5 , where its slope increases with the increase of the nanoparticle volume fraction.

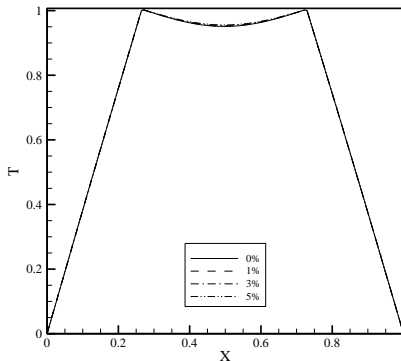


Figure 5: Paraffinic- Al_2O_3 nano-oil temperature for different nanoparticle volume fractions $Ra=10^3$

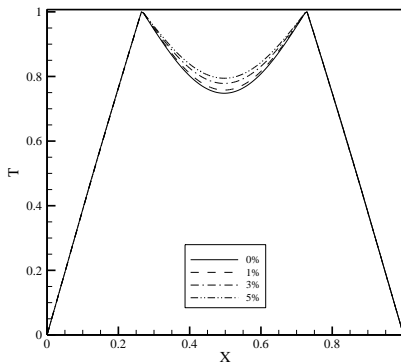


Figure 6: Paraffinic- Al_2O_3 nano-oil temperature for different nanoparticle volume fractions $Ra=10^4$

Influence of Rayleigh number and nano-oil on Nu_{avg}

A heat transfer's ratio from convection to conduction is called its Nusselt number. Therefore, when the Nusselt number increases, so does the nano-oil's natural convection's capacity for heat transfer.

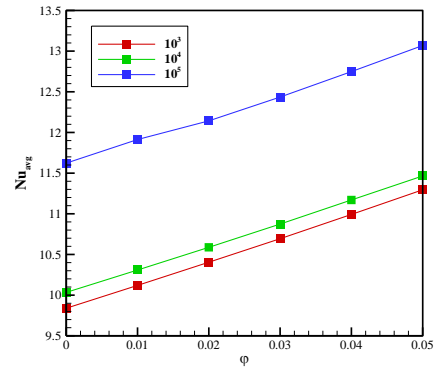


Figure 8: The average Nusselt number for nano-oil for a range of Rayleigh numbers and volume fractions of nanoparticles

Figure 8 show that improving the Rayleigh number and adding nanoparticles into the oil base fluid enhances heat transfer efficiency. Across all Rayleigh number ranges, an increase in the volume fraction of nanoparticles has been shown to increase the average Nusselt number. Enhancing heat transfer can lead to higher transformer oil thermal performance, which in turn enhances transformer core cooling performance. This is one of the main benefits of increased heat transfer.

Conclusions

The two-dimensional natural convection in a power transformer with one nano-oil, including an Al_2O_3 nanoparticle in Paraffinic oil, is analyzed. The two-dimensional lattice Boltzmann method is used for this research. The influences of different Rayleigh numbers and nanoparticle volume fractions on heat transfer performance are presented. Based on the results of this research:

- 1) Adding nanoparticles to the oil base fluid has increased the natural convection heat transfer in transformer.
- 2) The average Nusselt number increases with increasing Rayleigh number.

3) The average Nusselt number increases with increasing nanoparticle volume fraction.

Nomenclature

A Dimensionless transformer length parameter
 C_p Specific heat capacity [J/kgK]
 c_i Velocity of virtual particles on lattice [m/s]
 c_s Velocity of sound on lattice [m/s]
 F External force [N]
 f_i Distribution function of density
 f_i^{eq} Equilibrium distribution function of density
 g_i Distribution function of temperature
 g_i^{eq} Equilibrium distribution function of temperature
 g Gravitational acceleration [m/s²]
 h Convection heat transfer coefficient [W/m²K]
 k Thermal conductivity coefficient [W/mK]
 L Length [m]
 n The number of grids in the x direction
 Nu_{avg} Average Nusselt number
 Nu Local Nusselt number
 P Dimensionless pressure
 p Pressure [N/m²]
 Pr Prandtl number
 r Location vector [m]
 Ra Rayleigh number
 T Temperature [K]
 t Time [s]
 U, V Dimensionless velocity components
 u, v Velocity components [m/s]
 x, y Cartesian coordinates

Greek symbols

α Coefficient of heat diffusion [m²/s]
 β Thermal expansion coefficient [K⁻¹]
 θ Dimensionless temperature
 μ Dynamic viscosity [Ns/m²]
 ν Kinematic viscosity [m²/s]
 ρ Density [kg/m³]
 τ Relaxation factor
 φ Nanoparticle volume fraction
 Ω Collision operator
 ω Weighting factor

Subscripts

bf Base fluid
c Cold wall
F Flow
h Hot wall
nf Nanofluid
s Nanoparticle
T Temperature

References

- [1] Taheri, A. A., Abdali, A., Taghilou, M., Alhelou, H. H., & Mazlumi, K. (2021). Investigation of mineral oil-based nanofluids effect on oil temperature reduction and loading capacity increment of distribution transformers. *Energy Reports*, 7, 4325-4334.
- [2] Giwa, S. O., Sharifpur, M., & Meyer, J. P. (2020). Experimental study of thermo-convection performance of hybrid nanofluids of Al₂O₃-MWCNT/water in a differentially heated square cavity. *International Journal of Heat and Mass Transfer*, 148, 119072.
- [3] Khan, A. I., & Arasu, A. V. (2019). A review of influence of nanoparticle synthesis and geometrical parameters on thermophysical properties and stability of nanofluids. *Thermal Science and Engineering Progress*, 11, 334-364.
- [4] Ma, Y., Mohebbi, R., Rashidi, M. M., Yang, Z., & Sheremet, M. A. (2019). Numerical study of MHD nanofluid natural convection in a baffled U-shaped enclosure. *International Journal of Heat and Mass Transfer*, 130, 123-134.
- [5] Heris, S. Z., Nassan, T. H., Noie, S. H., Sardarabadi, H., & Sardarabadi, M. (2013). Laminar convective heat transfer of Al₂O₃/water nanofluid through square cross-sectional duct. *International Journal of Heat and Fluid Flow*, 44, 375-382.
- [6] Sheri, S. R., & Thumma, T. (2018). Numerical study of heat transfer enhancement in MHD free convection flow over vertical plate utilizing nanofluids. *Ain Shams Engineering Journal*, 9(4), 1169-1180.
- [7] Choi, S. U., & Eastman, J. A. (1995). *Enhancing thermal conductivity of fluids with nanoparticles* (No. ANL/MSD/CP-84938; CONF-951135-29). Argonne National Lab. (ANL), Argonne, IL (United States).
- [8] Segal, V., & Raj, K. (1998). An investigation of power transformer cooling with magnetic fluids.
- [9] Pendyala, R., Ilyas, S. U., Lim, L. R., & Marneni, N. (2016). CFD Analysis of Heat Transfer Performance of Nanofluids in Distributor Transformer. *Procedia Engineering*, 148, 1162-1169.
- [10] Raizah, Z. A., Aly, A. M., & Ahmed, S. E. (2021). Natural convection flow of a nanofluid-filled V-shaped cavity saturated with a heterogeneous porous medium: Incompressible smoothed particle hydrodynamics analysis. *Ain Shams Engineering Journal*, 12(2), 2033-2046.
- [11] Kia, S., Khanmohammadi, S., & Jahangiri, A. (2023). Experimental and numerical investigation on heat transfer and pressure drop of SiO₂ and Al₂O₃ oil-based nanofluid characteristics through the different helical tubes under constant heat fluxes. *International Journal of Thermal Sciences*, 185, 108082.
- [12] Karakaş, A., Harikrishnan, S., & Öztop, H. F. (2022). Preparation of EG/water mixture-based nanofluids using metal-oxide nanocomposite and measurement of their thermophysical properties. *Thermal Science and Engineering Progress*, 36, 101538.
- [13] Öztop, H. F., & Abu-Nada, E. (2008). Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. *International Journal of Heat and Fluid Flow*, 29(5), 1326-1336.
- [14] Huang, W., & Marefati, M. (2020). Energy, exergy, environmental and economic comparison of various solar thermal systems using water and Therminol Oil B base fluids, and CuO and Al₂O₃ nanofluids. *Energy Reports*, 6, 2919-2947.
- [15] SÁCHICA, D., TREVIÑO, C., & MARTÍNEZ-SUÁSTEGUI, L. (2020). Numerical study of magnetohydrodynamic mixed convection and entropy generation of Al₂O₃-water nanofluid in a channel with two facing cavities with discrete heating. *International Journal of Heat and Fluid Flow*, 86, 108713.

- [16] Rahimi, A., Kasaeipour, A., Malekshah, E. H., & Amiri, A. (2018). Natural convection analysis employing entropy generation and heatline visualization in a hollow L-shaped cavity filled with nanofluid using lattice Boltzmann method-experimental thermo-physical properties. *Physica E: Low-Dimensional Systems and Nanostructures*, 97, 82-97.
- [17] Mansir, I. B., Singh, P. K., Abed, A. M., Ameen, H. F. M., Khan, S. A., Usmani, A. Y., ... & Wae-hayee, M. (2023). Investigating the effects of employing a cooling radiator on MHD natural convection by injecting MWCNTs into water. *Ain Shams Engineering Journal*, 102216.
- [18] Ghaffarpassand, O. (2016). Numerical study of MHD natural convection inside a sinusoidally heated lid-driven cavity filled with Fe₃O₄-water nanofluid in the presence of Joule heating. *Applied Mathematical Modelling*, 40(21-22), 9165-9182.
- [19] Büyüç Ögüt, E. (2009). Natural convection of water-based nanofluids in an inclined enclosure with a heat source. *International Journal of Thermal Sciences*, 48(11), 2063–2073.
- [20] Sheikholeslami, M., Gorji-Bandpy, M., Ganji, D. D., Soleimani, S., & Seyyedi, S. M. (2012). Natural convection of nanofluids in an enclosure between a circular and a sinusoidal cylinder in the presence of magnetic field. *International Communications in Heat and Mass Transfer*, 39(9), 1435–1443.
- [21] Lai, F. H., & Yang, Y. T. (2011). Lattice Boltzmann simulation of natural convection heat transfer of Al₂O₃/water nanofluids in a square enclosure. *International Journal of Thermal Sciences*, 50(10), 1930–1941.
- [22] Karki, P., Perumal, D. A., & Yadav, A. K. (2022). Comparative studies on air, water and nanofluids based Rayleigh–Benard natural convection using lattice Boltzmann method: CFD and exergy analysis. *Journal of Thermal Analysis and Calorimetry*, 147(2), 1487-1503.
- [23] Sajjadi, H., Delouei, A. A., Mohebbi, R., Izadi, M., & Succi, S. (2021). Natural convection heat transfer in a porous cavity with sinusoidal temperature distribution using Cu/water nanofluid: Double MRT lattice Boltzmann method. *Commun. Comput. Phys.*, 29(1), 292-318.
- [24] Campos, A. R. T., Mariscal, I. C., & Hernandez, S. G. (2012). Simulation of a distribution transformer. *WSEAS Transactions on Fluid Mechanics*, 7(3), 106–115.
- [25] Ahmed, S. E., Hussein, A. K., Mansour, M. A., Raizah, Z. A., & Zhang, X. (2018). Mhd Mixed Convection in Trapezoidal Enclosures Filled With Micropolar Nanofluids. *Nanoscience and Technology: An International Journal*, 9(4), 343–372.
- [26] Al Kalbani, K. S., Alam, M. S., & Rahman, M. M. (2016). Finite element analysis of unsteady natural convective heat transfer and fluid flow of nanofluids inside a tilted square enclosure in the presence of oriented magnetic field. *American Journal of Heat and Mass Transfer*, 3(3), 186–224.
- [27] Pahlavanpour, B., Wolmarans, C. P. (2019). *Cooling ability of insulating liquids*. Paper presented in the 4th transformer conference.
- [28] Mohamad, A. A. (2011). Lattice Boltzmann method: Fundamentals and Engineering Applications with Computer Codes.
- [29] Bhatnagar, P. L., Gross, E. P., & Krook, M. (1954). A model for collision processes in gases. I. Small amplitude processes in charged and neutral one-component systems. *Physical review*, 94(3), 511.
- [30] Qian, Y. H., D’Humières, D., & Lallemand, P. (1992). Lattice BGK models for Navier-stokes equation. *Epl*, 17(6), 479–484.
- [31] Sukop, M. C., & Thorne, D. T. (2006). Lattice Boltzmann Modeling: An Introduction for Geoscientists and Engineers.
- [32] Khanafer, K., Vafai, K., & Lightstone, M. (2003). Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. *International Journal of Heat and Mass Transfer*, 46(19), 3639–3653.