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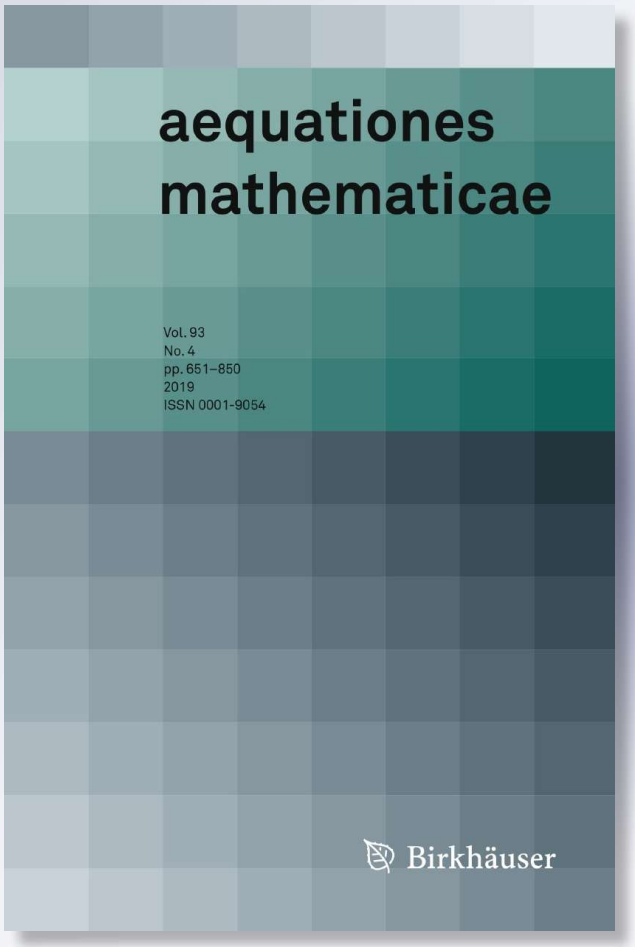

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Abstract. We introduce a generalized orthogonality relation in real normed linear spaces via norm derivatives. The relation between this concept and other types of orthogonalities such as Birkhoff–James orthogonality and orthogonality relations connected with norm derivatives is investigated. As an application, some new characterizations of smooth real normed linear spaces are obtained. Moreover, some mapping theorems with respect to this new type of orthogonality are presented.

Mathematics Subject Classification. Primary 46B20, 46C50; Secondary 39B82.

Keywords. Birkhoff–James orthogonality, Norm derivatives, Smooth normed linear spaces.

1. Introduction and preliminaries

One of the most well-known concepts in the study of the geometry of normed linear spaces is the notion of orthogonality. This concept and its connection with several geometric properties of normed linear spaces, like smoothness, has been studied extensively. Let $(X, \|\cdot\|)$ be a real normed linear space. If the norm comes from an inner product $\langle\cdot,\cdot\rangle$, there is one natural orthogonality relation on X: $x \perp y \Leftrightarrow \langle x, y\rangle = 0$. However, there is not a unique way to define the notion of orthogonality in general normed linear spaces. Birkhoff–James orthogonality is one of the most important orthogonality types, introduced by Birkhoff in [4], and developed by James in [11,12]. A vector $x \in X$ is said to be orthogonal to a vector $y \in X$ in the sense of Birkhoff–James, written as $x \perp_B y$, if

$$\|x\| \leq \|x + ty\| \quad (\forall t \in \mathbb{R}).$$

From now on we assume that the considered normed linear spaces are real and their dimensions are not less than 2. Let $(X, \|\cdot\|)$ be a real normed linear space. In 1986 norm derivatives were defined by Amir in [3] as follows: