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ON THE LIMITED p-SCHUR PROPERTY OF SOME OPERATOR SPACES

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Abstract. We introduce and study the notion of limited p-Schur property ($1 \leq p \leq \infty$) of Banach spaces. Also, we establish some necessary and sufficient conditions under which some operator spaces have the limited p-Schur property. In particular, we prove that if X and Y are two Banach spaces such that X contains no copy of ℓ_1 and Y has the limited p-Schur property, then $K(X,Y)$ (the space of all compact operators from X into Y) has the limited p-Schur property.

1. Introduction

A non-empty subset K of a Banach space X is said to be limited (resp Dunford-Pettis (DP)), if for every $weak^*$-null (resp. weakly null) sequence (x_n^*) in the dual space X^* of X converges uniformly on K, that is,

$$\lim_{n\to\infty} \sup_{x\in K} |\langle x, x_n^*\rangle| = 0$$

where $\langle x, x^*\rangle$ denotes the duality between $x \in X$ and $x^* \in X^*$. In particular, a sequence $(x_n) \subset X$ is limited if and only if $\langle x_n, x_n^*\rangle \to 0$, for all $weak^*$-null sequences (x_n^*) in X^*.

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A subset K of a Banach space X is a limited set if and only if for any Banach space Y, every pointwise convergent sequence $(T_n) \subset L(X,Y)$ converges uniformly on K, where $L(X,Y)$ denoted the space of all bounded operators from X into Y [17, Corollary 1.1.2].

It is easily seen that every relatively compact subset of a Banach space is limited. But the converse is not true, in general. If every limited subset of Banach space X is relatively compact, then X has the Gelfand-Phillips (GP) property. For example, the classical Banach space c_0 and ℓ_1 have the GP property and every reflexive space and dual space containing no copy of ℓ_1 have the same property.

A sequence (x_n) in Banach space X is called weakly p-summable with $1 \leq p < \infty$, if for each $x^* \in X^*$, the sequence $(\langle x_n, x^* \rangle) \in \ell_p$ and a sequence (x_n) in X is said to be weakly p-convergent to $x \in X$ if the sequence $(x_n - x) \in \ell_p^{weak}(X)$, where $\ell_p^{weak}(X)$ denoted the space of all weakly p-summable sequence in X. Also a bounded set K in a Banach space is said to be relatively weakly p-compact, $1 \leq p \leq \infty$ if every sequence in K has a weakly p-convergent subsequence. If the limit point of each weakly p-convergent subsequence is in K, then we call K weakly p-compact set. Also, a Banach space X is weakly p-compact if the closed unit ball B_X of X is a weakly p-compact set. An operator $T \in L(X,Y)$ is said to be p-converging if it transfers weakly p-summable sequence into norm null sequences. The class of all p-converging operators from X into Y is denoted by $C_p(X,Y)$.

An operator $T \in L(X,Y)$ is limited p-converging if it transfers limited and weakly p-summable sequences into norm null sequences. we denote the space of all limited p-converging operators from X into Y by $C_{lp}(X,Y)$ [7].

A Banach space X has the Schur property if every weakly null sequence in X converges in norm. The simplest Banach space with the Schur property is ℓ_1. Also a banach space X has the p-Schur property $(1 \leq p \leq \infty)$ if every weakly p-summable subset of X is compact. In other words, if $1 \leq p < \infty$, X has the p-Schur property if and only if every sequence $(x_n) \in \ell_p^{weak}(X)$ is a norm null sequence, for example, ℓ_p has the 1-Schur property. Moreover, X has the ∞-Schur property if and only if every sequence in $c_0^{weak}(X)$ in norm null where $c_0^{weak}(X)$ containing all weakly null sequences in X. So ∞-Schur property coincides with the Schur property. Also one note that every Schur space has the p-Schur property [6].

The reader is referred to [2, 11, 14–16] for more information about these concepts.

In this note, we study the limited p-Schur property of some operator spaces, specially, the space of compact operators. We prove that if X and Y are two Banach spaces such that X contains no copy of ℓ_1 and Y has the limited p-Schur property, then $K(X,Y)$ has the limited p-Schur property. Finally, we conclude that if $(X_\alpha)_{\alpha \in I}$ are Banach spaces and $X = (\oplus_{\alpha \in I} X_\alpha)_1$ their ℓ_1-sum, then the space X has the p-Schur property if and only if each factor X_α has the same property.