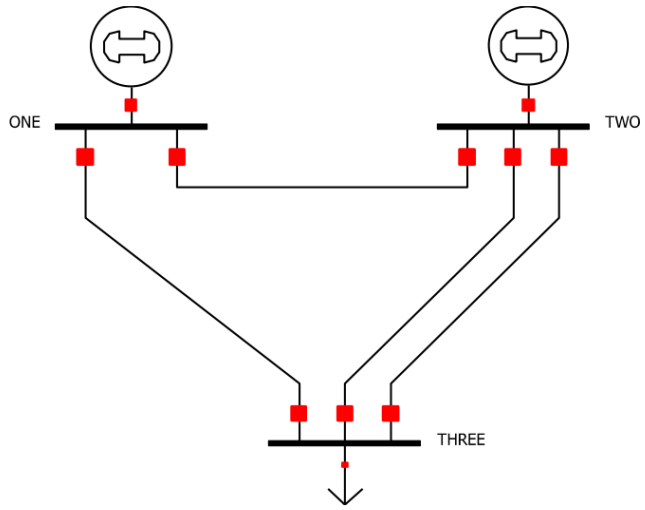
Gauss-Seidel example (Base power is S_B=100 MVA):



Bus	Voltage (p.u.)	Power
1	1.02	(slack)
2	1.02	P_G=50 MW
3	-	P_C=100 MW Q_C=60 MVAr

Lin	Impedance (p.u.)
1-2	0.02+0.04j
1-3	0.02+0.06j
2-3	0.02+0.04j (each)

Solution procedure:

1. Data and unknown:

Bus	Type	Data	Unknown
1	*Slack*	V_1=1.02 δ_1=0.0	P_1 Q_1
2	*PV*	V_2=1.02 P_2=0.5	δ_2 Q_2
3	*PQ*	P_3=-1.0 Q_2=-0.6	δ_3 V_3

2. Y^{BUS} calculation:

$$Y_{BUS} = \begin{bmatrix} 15-35\,j & -10+20\,j & -5+15\,j \\ -10+20\,j & 30-60\,j & -20+40\,j \\ -5+15\,j & -20+40\,j & 25-55\,j \end{bmatrix}$$

3. Voltage magnitude initialization (iteration 0):

$V_3 = V_3^0 = 1$ PQ bus

$\left.\begin{aligned}\delta_2 &= \delta_2^0 = 0\\ \delta_3 &= \delta_3^0 = 0\end{aligned}\right\}$ All buses but the reference one

Vector form:

$$V^0 = \begin{bmatrix}1.02\\1.02\\1\end{bmatrix} \qquad \delta^0 = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

Per iteration:

- PV buses (Q, δ) i=2,...,m (bus 2)

- PQ buses (V, δ) i=m+1,...,n (bus 3)

- Stopping criterios:
  a) convergence → S_{slack} and power flows;
  b) no converge → new iteration

4. PV buses: iteration (r+1) :

→bus 2:

$$Q_2^{(r+1)} = -\Im\left\{(\overline{V}_2^{(r)})^* \overline{Y}_{21}\overline{V}_1^{(r+1)} + (\overline{V}_2^{(r)})^* \overline{Y}_{22}\overline{V}_2^{(r)} + (\overline{V}_2^{(r)})^* \overline{Y}_{23}\overline{V}_3^{(r)}\right\}$$

$$\delta_2 = \text{angle}[\overline{V}_2] \qquad \overline{A}_2^{(r+1)} = \frac{P_2 - jQ_2^{(r+1)}}{\overline{Y}_{22}}$$

$$\delta_2^{(r+1)} = \text{angle}\left\{ \frac{\overline{A}_2^{(r+1)}}{(V_2^{(r+1)})^*} - \overline{B}_{21}\overline{V}_1^{(r+1)} - \overline{B}_{23}\overline{V}_3^{(r)} \right\}$$

where

$$\overline{B}_{ik} = \frac{\overline{Y}_{ik}}{\overline{Y}_{ii}} \text{ is a constant}$$

5. Buses PQ iteration (r+1):

→bus 3:

$$\overline{V}_3^{(r+1)} = \frac{\overline{A}_3}{(\overline{V}_3^{(r)})^*} - \overline{B}_{31}\overline{V}_1^{(r+1)} - \overline{B}_{32}\overline{V}_2^{(r+1)}$$

where

$$\left.\begin{aligned} \overline{A}_3 &= \frac{P_3 - jQ_3}{\overline{Y}_{33}} \\ \overline{B}_{ik} &= \frac{\overline{Y}_{ik}}{\overline{Y}_{ii}} \end{aligned}\right\} \text{ Are constants for PQ buses}$$

6. Stopping criterion:

$$\varepsilon = 10^{-3}$$

$$\left.\begin{aligned} &|V_i^{(r+1)} - V_i^{(r)}| \\ &\quad \& \\ &|Q_j^{(r+1)} - Q_j^{(r)}| \end{aligned}\right\} < \varepsilon \qquad i = 2,3;\ j = 2$$

If convergence:

6.1) Slack power:

$$\overline{S}_{slack}^* = P_1 - jQ_1 = \overline{V}_1^*(\overline{Y}_{11}\overline{V}_1 + \overline{Y}_{12}\overline{V}_2 + \overline{Y}_{13}\overline{V}_3)$$

6.2) Power flows:

$$\overline{S}_{12}, \overline{S}_{21}, \overline{S}_{13}, \overline{S}_{23}, \overline{S}_{31}, \overline{S}_{32}$$

$$\overline{S}_{ik} = \overline{V}_i\ (\overline{V}_i^* - \overline{V}^*_k)\overline{Y}^*_{Lik}$$

6.3) Line losses:

$$\overline{S}_{losss,ik} = \overline{S}_{ik} + \overline{S}_{ki} \qquad k,i = 1,2,3$$

$$\overline{S}_{loss} = \overline{S}_{loss,12} + \overline{S}_{loss,13} + \overline{S}_{loss,23}$$

7. If no convergence, the procedure continues in Step 4.

Solution:

11 iterations needed to attain the solution:

Iteration (pu)	1	2	3	...	10	11
P_1, Q_1(slack)	-	-	-	...	-	***0.5083*** ***0.0716***
P_2	0.5	0.5	0.5	...	0.5	0.5
Q_2	***0.81***	***0.4084***	***0.4696***	***...***	***0.5493***	***0.5501***
P_3	-1.0	-1.0	-1.0	...	-1.0	-1.0
Q_3	-0.6	-0.6	-0.6	...	-0.6	-0.6

Iteration	**1**	**2**	**3**	**...**	**10**	**11**
V_1	1.02	1.02	1.02	**...**	1.02	1.02
$\delta_1(^o)$	0	0	0	**...**	0	0
V_2	1.02	1.02	1.02	**...**	1.02	1.02
$\delta_2(^o)$	***0.0675***	***-0.1596***	***-0.2885***	***...***	***-0.4667***	***-0.4685***
V_3	***1.0041***	***1.0042***	***1.0043***	***...***	***1.0043***	***1.0043***
$\delta_3(^o)$	***-0.5746***	***-0.7336***	***-0.8278***	***...***	***-0.9580***	***-0.9593***

Final Solution:

Bus	P (MW)	Q (MVAr)	$\overline{V}$ (pu)
1	50.83	7.16	$1.02 \angle 0^o$
2	50.00	55.01	$1.02 \angle -0.4685^o$
3	-100	-60.00	$1.0043 \angle -0.9593^o$