Property	Signal	Fourier Transform
Linearity	$\alpha x_1(t) + \beta x_2(t)$	$\alpha X_1(f) + \beta X_2(f)$
Duality	$X(t)$	$x(-f)$
Conjugacy	$x^*(t)$	$X^*(-f)$
Time-scaling ($a \neq 0$)	$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{f}{a}\right)$
Time-shift	$x(t - t_0)$	$e^{-j2\pi f t_0} X(f)$
Modulation	$e^{j2\pi f_0 t} x(t)$	$X(f - f_0)$
Convolution	$x(t) \star y(t)$	$X(f)Y(f)$
Multiplication	$x(t)y(t)$	$X(f) \star Y(f)$
Differentiation	$\frac{d^n}{dt^n} x(t)$	$(j2\pi f)^n X(f)$
Differentiation in frequency	$t^n x(t)$	$\left(\frac{j}{2\pi}\right)^n \frac{d^n}{df^n} X(f)$
Integration	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\frac{X(f)}{j2\pi f} + \frac{1}{2} X(0)\delta(f)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)y^*(t)\, dt = \int_{-\infty}^{\infty} X(f)Y^*(f)\, df$	
Rayleigh's theorem	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\, dt = \int_{-\infty}^{\infty} \lvert X(f)\rvert^2\, df$	

$$V(f) = \mathscr{F}[v(t)] = \int_{-\infty}^{\infty} v(t) e^{-j2\pi ft}\, dt \quad v(t) = \mathscr{F}^{-1}[V(f)] = \int_{-\infty}^{\infty} V(f) e^{j2\pi ft}\, df$$

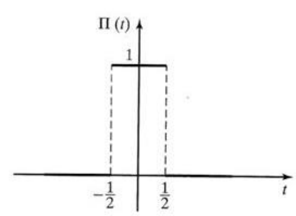
Waveform		Coefficients	
Impulse train			
$\delta(t)$	$\lvert t\rvert < T_0/2$	$1/T_0$	
Rectangular pulse train			
$\Pi(t/\tau)$	$\lvert t\rvert < T_0/2$	(τ/T_0) sinc $nf_0\tau$	
Square wave (odd symmetry)			
$+1$	$0 < t < T_0/2$	0	n even
-1	$-T_0/2 < t < 0$	$-j2/\pi n$	n odd
Triangular wave (even symmetry)			
$1 - \frac{4\lvert t\rvert}{T_0}$	$\lvert t\rvert < T_0/2$	0	n even
		$(2/\pi n)^2$	n odd
Sawtooth wave			
t/T_0	$0 < t < T_0$	$1/2$	$n = 0$
		$j/2\pi n$	$n \neq 0$

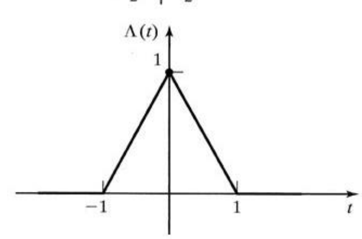
$$x(t) = \sum_{n=-\infty}^{n=+\infty} c_n e^{+j2\pi n f_0 t}\ ,\ c_n = \frac{1}{T_0}\int_{T_0} x(t) e^{-j2\pi n f_0 t} dt$$

$$X(f) = \sum_{n=-\infty}^{n=+\infty} c_n \delta(f - nf_0)$$

$$PSD: G_x(f) = \sum_{n=-\infty}^{n=+\infty} \lvert c_n\rvert^2\, \delta(f - nf_0)$$

Time Domain	Frequency Domain
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$e^{-j2\pi f t_0}$
$e^{j2\pi f_0 t}$	$\delta(f - f_0)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$
$\Pi(t)$	$\mathrm{sinc}(f)$
$\mathrm{sinc}(t)$	$\Pi(f)$
$\Lambda(t)$	$\mathrm{sinc}^2(f)$
$\mathrm{sinc}^2(t)$	$\Lambda(f)$
$e^{-\alpha t} u_{-1}(t), \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$
$t e^{-\alpha t} u_{-1}(t), \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$
$e^{-\alpha\lvert t\rvert} (\alpha > 0)$	$\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$
$e^{-\pi t^2}$	$e^{-\pi f^2}$
$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
$u_{-1}(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\frac{1}{2}\delta(t) + j\frac{1}{2\pi t}$	$u_{-1}(f)$
$\delta'(t)$	$j2\pi f$
$\delta^{(n)}(t)$	$(j2\pi f)^n$
$\frac{1}{t}$	$-j\pi\ \mathrm{sgn}(f)$
$\sum_{n=-\infty}^{\infty} \delta(t - nT_0)$	$\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$





$$< z(t) > \triangleq \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} z(t)\,dt$$

$$< \alpha + \beta z(t) > = \alpha + \beta < z(t) >$$

$$< z(t-t_0) > = < z(t) >$$

$$< v(t), w(t) > \triangleq \begin{cases} \int_{-\infty}^{+\infty} v(t)w^*(t)\,dt \ , \ \textit{for ES} \\ < v(t)w^*(t) > \ , \ \textit{for PS} \end{cases}$$

$$< x(t), x(t) > = \begin{cases} E_x \ , \ \textit{for ES} \\ P_x \ , \ \textit{for PS} \end{cases}$$

$$< x(t-t_0), y(t-t_0) > = < x(t), y(t) >$$

$$< \alpha x(t), \beta y(t) > = \alpha\beta^* < x(t), y(t) >$$

$$\left| < x(t), y(t) > \right|^2 \le \ < x(t), x(t) > < y(t), y(t) >$$

$$< x(t)+z(t), y(t)+w(t) > = < x(t), y(t) > + \\ + < x(t), w(t) > + < z(t), y(t) > + < z(t), w(t) >$$

$$R_{xy}(\tau) \triangleq < x(t), y(t-\tau) >$$

$$R_x(\tau) \triangleq R_{xx}(\tau) = < x(t), x(t-\tau) >$$

$$R_{yx}(\tau) = R_{xy}^*(-\tau)$$

$$R_x(\tau) \le R_x(0) = < x(t), x(t) > = \begin{cases} E_x \ , \ \textit{for ES} \\ P_x \ , \ \textit{for PS} \end{cases}$$

$$< x(t-t_1), y(t-t_2) > = R_{xy}(t_2 - t_1)$$

$$\left| R_{xy}(\tau) \right|^2 \le \begin{cases} E_x E_y \ , \ \textit{for ES} \\ P_x P_y \ , \ \textit{for PS} \end{cases}$$

$$G_x(f) = \begin{cases} \left| X(f) \right|^2 \ , \ \textit{ESD: for ES} \\ \sum_n \left| c_n \right|^2 \delta(f - nf_0) \ , \ \textit{PSD: for Periodic PS} \end{cases}$$

$$G_x(f) = F\{R_x(\tau)\}$$

$$\int_{-\infty}^{+\infty} G_x(f)\,df = \begin{cases} E_x \ , \ \textit{for ES} \\ P_x \ , \ \textit{for PS} \end{cases}$$

$$G_y(f) = G_x(f)\left| H(f) \right|^2$$

$$R_{yx}(\tau) = R_x(\tau) * h(\tau)$$

$$R_y(\tau) = R_{yx}(\tau) * h^*(-\tau)$$

Trigonometric identities

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$e^{j2\alpha} + e^{j2\beta} = 2\cos(\alpha - \beta)e^{j(\alpha+\beta)}$$

$$e^{j2\alpha} - e^{j2\beta} = j2\sin(\alpha - \beta)e^{j(\alpha+\beta)}$$

$$\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta}) = \sin(\theta + 90°)$$

$$\sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta}) = \cos(\theta - 90°)$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$\cos^2\theta = \tfrac{1}{2}(1 + \cos 2\theta)$$

$$\cos^3\theta = \tfrac{1}{4}(3\cos\theta + \cos 3\theta)$$

$$\sin^2\theta = \tfrac{1}{2}(1 - \cos 2\theta)$$

$$\sin^3\theta = \tfrac{1}{4}(3\sin\theta - \sin 3\theta)$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

$$\sin\alpha\sin\beta = \tfrac{1}{2}\cos(\alpha - \beta) - \tfrac{1}{2}\cos(\alpha + \beta)$$

$$\cos\alpha\cos\beta = \tfrac{1}{2}\cos(\alpha - \beta) + \tfrac{1}{2}\cos(\alpha + \beta)$$

$$\sin\alpha\cos\beta = \tfrac{1}{2}\sin(\alpha - \beta) + \tfrac{1}{2}\sin(\alpha + \beta)$$

$$PCM: F_s \ge 2W, K = 2^k, R_b = kF_s$$

$$R \le C = B\log_2(1 + SNR), SNR = \frac{P_s}{P_n}$$

$$\log_{10}(2) \simeq 0.3, \log_{10}(3) \simeq 0.48, \log_{10}(5) \simeq 0.7$$

$$x(t)\cos(2\pi f_c t) \xrightarrow{F.T.} \frac{1}{2}X(f - f_c) + \frac{1}{2}X(f + f_c)$$

$$x(t)\sin(2\pi f_c t) \xrightarrow{F.T.} \frac{1}{2j}X(f - f_c) - \frac{1}{2j}X(f + f_c)$$

$$h_Q(t) = \frac{1}{\pi t}, \ H_Q(f) = -j\,\mathrm{sgn}(f)$$

$$H.T.\{\sin(2\pi f_0 t)\} = -\cos(2\pi f_0 t)$$

$$H.T.\{\cos(2\pi f_0 t)\} = \sin(2\pi f_0 t)$$

$$H.T.\{x(t)\sin(2\pi f_0 t)\} = -x(t)\cos(2\pi f_0 t)$$

$$H.T.\{x(t)\cos(2\pi f_0 t)\} = x(t)\sin(2\pi f_0 t)$$

$$R_{vw}(t_1, t_2) = E\left[v(t_1)w(t_2)\right]$$

$$\gamma = \frac{S_R}{N_0 W}$$

$$S_R = \frac{S_T}{L}$$

$\mathcal{T}_0 \triangleq 290$ K (63°F)

$k\mathcal{T}_0 \approx 4 \times 10^{-21}$ W-s

k = Boltzmann constant $= 1.38 \times 10^{-23}$ J/K

$N_0 = k\mathcal{T}_N = k\mathcal{T}_0(\mathcal{T}_N/\mathcal{T}_0) \approx 4 \times 10^{-21}(\mathcal{T}_N/\mathcal{T}_0)$ W/Hz

Table 10.4–1 Comparison of C

Type	$b = B_T/W$	$(S/N)_D \div \gamma$
Baseband	1	1
AM	2	$\frac{\mu^2 S_x}{1 + \mu^2 S_x}$
DSB	2	1
SSB	1	1
VSB	1+	1
VSB + C	1+	$\frac{\mu^2 S_x}{1 + \mu^2 S_x}$
< 1		
PM[3]	$2M(\phi_\Delta)$	$\phi_\Delta^2 S_x$
FM[3,4]	$2M(D)$	$3D^2 S_x$

Table 5.1–2 Selected values of $J_n(\beta)$

n	$J_n(0.1)$	$J_n(0.2)$	$J_n(0.5)$	$J_n(1.0)$	$J_n(2.0)$	$J_n(5.0)$
0	1.00	0.99	0.94	0.77	0.22	−0.18
1	0.05	0.10	0.24	0.44	0.58	−0.33
2			0.03	0.11	0.35	0.05
3				0.02	0.13	0.36
4					0.03	0.39
5						0.26
6						0.13
7						0.05
8						0.02
9						
10						
11						
12						
13						
14						

$$x_c(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$$

$$B_T = \begin{cases} 2DW = 2f_\Delta & D \gg 1 \\ 2W & D \ll 1 \end{cases}$$

$$B_T \approx 2(f_\Delta + 2W) = 2(D + 2)W \qquad D > 2$$

$$\phi(t) = \beta\sin\omega_m t$$

$$\beta \triangleq \begin{cases} \phi_\Delta A_m & \text{PM} \\ (A_m/f_m)f_\Delta & \text{FM} \end{cases}$$