

OWA operator based link prediction ensemble for social network



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ABSTRACT

The objective of link prediction for social network is to estimate the likelihood that a link exists between two nodes. Although there are many local information-based algorithms which have been proposed to handle this essential problem in the social network analysis, the empirical observations show that the stability of local information-based algorithm is usually very low, i.e., the variabilities of local information-based algorithms are high. Thus, motivated by obtaining a stable link predictor with low variance, this paper proposes a kind of ordered weighted averaging (OWA) operator based link prediction ensemble algorithm (LPE_{OWA}) for social network by assigning the aggregation weights for nine local information-based link prediction algorithms with three different OWA operators. The finally experimental results on benchmark social network datasets show that LPE_{OWA} obtains a more stable prediction performance and considerably improves the prediction accuracy which is measured by the area under the receiver operating characteristic curve (AUC) in comparison with nine individual prediction algorithms.

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1. Introduction

1.1. A brief review for link prediction algorithms

With the development of information technology and big data mining (Lin & Ryaboy, 2012), the social network analysis is attracting more and more attentions and becoming a research hot-spot of sociology and statistics. The social network analysis (Carrington, Scott, & Wasserman, 2005; Knoke & Yang, 2008; Soares & Prudêncio, 2013) refers to mine and discover the underlying knowledge from a social network diagram by using the mathematical and graphical techniques. The social network is represented as a graphic structure made up of a set of nodes and links, where nodes represent the individuals within network and links denote the relationships between individuals. The main studies of social network analysis include the identification of local/global patterns, location of social units, and modeling of dynamic network, etc., where the link prediction (Hasan & Zaki, 2011; Lü & Zhou, 2011) as a branch of network pattern recognition is the most fundamental and essential problem for the social network analysis.

The link prediction for social network attempts to estimate the existence likelihood s_{xy} of a link between two nodes x and y in

social network. The essence of link prediction algorithm is to assign a score for the non-existent link in social network (Lü, Jin, & Zhou, 2009; Lü & Zhou, 2011; Zhou, Lü, & Zhang, 2009), where the score quantifies the existence likelihood of this non-existent link. Among all existing link prediction algorithms, similarity-based algorithm (Lü et al., 2009; Lü & Zhou, 2011) is a kind of simplest framework for the prediction of non-existent link. The score s_{xy} between nodes x and y is directly defined as the similarity between x and y . Similarity-based algorithms can further be classified into three categories: local information indices (Adamic & Adar, 2003; Jaccard, 1901; Leicht, Holme, & Newman, 2006; Lorrain & White, 1971; Ravasz, Somera, Mongru, Oltvai, & Barabási, 2002; Salton & McGill, 1983; Sørensen, 1948; Zhou et al., 2009), global information indices (Brin & Page, 1998; Chebotarev & Shamis, 1997; Fouss, Pirotte, Renders, & Saerens, 2007; Jeh & Widom, 2002; Klein & Randić, 1993; Leicht et al., 2006; Zhou, Ren, Medo, & Zhang, 2007) and quasi-local information indices (Liu & Lü, 2010; Lü et al., 2009; Zhou et al., 2009). In consideration of the simple implementation and less computational complexity, our tour of studies in this paper starts with the local information-based algorithms as follows.

- Common neighbors (CN) (Lorrain & White, 1971) is the simplest and most direct method to calculate nodes' similarity. CN uses the number of common neighbors of x and y to represent the link likelihood. The more common neighbors of x and y have, the larger s_{xy} is.

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- Salton index (Salton & McGill, 1983) is also called cosine similarity, which can measure the intensive relationship between x and y with local information at a global level (Wagner & Leydesdorff, 2003).
- Jaccard index (Jaccard, 1901) is an oldest index which is defined as the size of the intersection divided by the size of the union of the sets of neighbors of x and y .
- Sørensen index (Sørensen, 1948) is proposed by Sørensen to estimate the similarity for ecological community data, which can be considered a semimetric version of the Jaccard index.
- Hub promoted index (HPI) (Ravasz et al., 2002) and Hub depressed index (HDI) (Lü & Zhou, 2011; Zhou et al., 2009) are two similarity measures which are proposed to quantify the topological overlap of substrate pairs in metabolic network. These two indices assign the lower scores for the link between nodes with high degrees.
- Leicht-Holme-Newman-I index (LHN-I) (Leicht et al., 2006) is proposed to deal with the social network in which the nodes x and y have the similar structures with their common neighbors.
- Adamic-Adar index (AA) (Adamic & Adar, 2003) focuses on the link prediction by mining the information from links and text on a user's homepage to mailing lists the user subscribes.
- Resource allocation index (RA) (Zhou et al., 2009) proposed by Zhou, Lü, and Zhang considers the degrees of common neighbors of nodes x and y and the empirical results demonstrate that AA can obtain the better prediction performance in comparison with other 8 above-mentioned methods (Zhou et al., 2009).

In addition to similarity-based models, maximum likelihood models (Airoldi, Blei, Fienberg, & Xing, 2008; Clauset, Moore, & Newman, 2008) and probabilistic models (Getoor, 2000; Heckerman, Meek, & Koller, 2004; Yu, Chu, Yu, Tresp, & Xu, 2007) are two kinds of promising link prediction algorithms. Maximum likelihood methods calculate the scores for the non-existent links by maximizing the likelihood of observed structures which are presupposed according to some organization principles and include some necessary rules and parameters. The obvious disadvantage of maximum likelihood method is that it is very time-consuming. There are two representative maximum likelihood methods: hierarchical structure method (Clauset et al., 2008) and stochastic block method (Airoldi et al., 2008). Probabilistic models firstly learn the underlying structures from the observed networks and then predict the non-existent links with the learned structures. Three representative probabilistic models are probabilistic relational model (Getoor, 2000), probabilistic entity relationship model (Heckerman et al., 2004), and stochastic relational model (Yu et al., 2007). Here, we only give a brief introduction to the existing link prediction algorithms. For more detailed information, the audience can refer to Lü and Zhou's work (Lü & Zhou, 2011) which is a very good and valuable review to the link prediction in complex networks.

1.2. Research motivation and main contributions

The above-mentioned link prediction algorithms are all individual learning algorithms, that is to say, not the aggregation of different link prediction algorithms. So far, there are very few studies which have discussed the ensemble mechanism of link prediction algorithms. Here, the main question is whether it is necessary to study the link prediction ensemble? Although afore-mentioned local information-based algorithms are competent to obtain the acceptable prediction accuracy with low time-consumption (Lü et al., 2009; Zhao, Feng, Dong, Liang, & Xu, 2012), the empirical analysis in this study indicates that the stability of local information-based algorithm is poor, i.e., the prediction performances of

local information-based algorithms vary intensively. This indicates that the variability of local information-based algorithms is high. Then, can we find some strategy to reduce the likelihood of choosing a worse link prediction algorithm and hence obtain a more stable link predictor?

As stated in Zhang and Ma (2012), ensemble learning is such a strategy which is known to reduce the classifiers' variance and improve the decision system's robustness and accuracy. In recent years, the studies regarding ensemble learning have obtained many satisfactorily theoretical and practical results, e.g., (Baumgartner & Serpen, 2013; Christou, Gekas, & Kyrikou, 2012; Seni & Elder, 2010; Zhang, Ling, Yang, Wang, & Li, 2012; Zhang & Ma, 2012; Zhou, 2012). The ensembles of some well-known machine learning algorithms (e.g., decision tree (Banfield, Hall, Bowyer, & Kegelmeyer, 2007), neural network (Zhou, Wu, & Tang, 2002), support vector machine (Kim, Pang, Je, Kim, & Bang, 2003), etc.) are all well and sophisticatedly studied, while the study of ensemble of link prediction algorithms is rare and fledgling. Such works could be found from the following literatures. Comar, Tan, and Jain (2011) proposed a cost-sensitive boosting based link prediction algorithm for community-level network. Pujari and Kanawati (2012a, 2012b) presented a supervised rank aggregation based link prediction for complex networks. These ensemble methods (i.e., Comar et al., 2011; Pujari & Kanawati, 2012a, 2012b) treat the link prediction as a supervised learning problem and do not construct the ensemble based on local information-based algorithms. Thus, they do not consider how to enhance the stabilities of local information-based algorithms.

Inspired by the outlook in Lü and Zhao's work (Lü & Zhou, 2011), i.e., "we can implement many individual prediction algorithms and then try to select and organize them in a proper way. This so-called ensemble learning method can obtain better prediction performance than could be obtained from any of the individual algorithms.", we design an ordered weighted averaging (OWA) operator (Filev & Yager, 1995, 1998; Yager, 1998) based link prediction ensemble algorithm (LPE_{OWA}) in this paper. Three different OWA operators, i.e., maximum entropy method (O'Hagan, 1988), minimum variance method (Fullér & Majlender, 2003) and chi-square method (Wang, Luo, & Liu, 2007), are respectively introduced to assign the aggregation weights for nine local information-based link prediction algorithms mentioned above. Based on the benchmark social network datasets obtained from Pajek datasets (2006), we compare the prediction performances of LPE_{OWA} s with individual link prediction algorithms, where the prediction performance is measured by the area under the receiver operating characteristic curve (AUC) (Bradley, 1997; Lü & Zhou, 2011; Zhao et al., 2012). The experimental results show that LPE_{OWA} can effectively reduce the variance of local information-based link prediction algorithm and improve the stability of link predictor. The rest of this paper is organized as follows. In Section 2, the theoretical and empirical analysis to nine local information-based link prediction algorithms are given. In Section 3, the new OWA operator based link prediction ensemble model (LPE_{OWA}) is presented. In Section 4, experimental comparisons are conducted to illustrate the feasibility of proposed ensemble model. Finally, conclusions are given in Section 5.

2. Analysis to local information-based link prediction algorithms

Before giving our description and discussion about the local information-based link prediction algorithms, we firstly provide a notation-list (Table 1) to explain the meanings of mathematical symbols applied afterwards. In this section, we firstly introduce the concepts of nine commonly used local information-based link

Table 1
The notation-list.

Notation	Meaning
$G = (V, E)$	A social network graph
$A = (a_{xy})$	The adjacency matrix of G
V	The set of nodes in G
$E = E_{\text{Train}} \cup E_{\text{Test}}$	The set of links in G ($E_{\text{Train}} \cap E_{\text{Test}} = \emptyset$)
E_{Train}	The training set
E_{Test}	The testing set
U	The set containing all possible links of G
$E_{\text{Predict}} = U - E$	The set containing non-existent links of G
$x \in V$	A node x belonging to V
s_{xy}	The existence likelihood of link xy
$\Gamma(x)$	The set of neighbors of node x
$\ S\ $	The cardinality of set S
$k_x = \ \Gamma(x)\ $	The degree of node x

prediction algorithms and then analyze the characteristic of performance measure index-area under the receiver operating characteristic curve (AUC). Finally, the high prediction variances of local information-based link prediction algorithms are confirmed experimentally.

2.1. Nine basic algorithms

For a nonexistent link $xy \in E_{\text{Predict}}$, local information-based link prediction algorithms calculate the score s_{xy} for it to express the likelihood of its existence by considering the information of common nearest neighbors $\Gamma(x) \cap \Gamma(y)$ of nodes x and y . Fig. 1 presents an intuitive illustration on the local information-based link prediction. In fact, we can see that from Fig. 1 the local information-based link prediction algorithm takes into account all paths from x to y with two steps. There are nine frequently used local information-based link prediction algorithms as follows. Without loss of generality, we assume there is no isolated node in social network graph G for the sake of simplicity.

- Common neighbors (CN) (Lorrain & White, 1971) index is the most direct and simplest likelihood measure and defined as

$$s_{xy}^{\text{CN}} = \|\Gamma(x) \cap \Gamma(y)\|. \quad (1)$$

It is obvious that $s_{xy}^{\text{CN}} = \binom{A^2}{xy}$. And, s_{xy}^{CN} represents the number of paths from x to y with two steps in G . Thus, the minimum of s_{xy}^{CN} is 0, i.e., there is no any path with two steps between x and y ; the maximum of s_{xy}^{CN} is $\|V\| - 2$, i.e., all the remaining nodes are served as the intermediate nodes from x and y . In summary, we get $s_{xy}^{\text{CN}} \in [0, \|V\| - 2]$.

- Salton index (Salton & McGill, 1983) considers the degrees of nodes and is defined as

$$s_{xy}^{\text{Salton}} = \frac{\|\Gamma(x) \cap \Gamma(y)\|}{\sqrt{k_x \times k_y}}. \quad (2)$$

In Eq. (2), $k_x = \|\Gamma(x)\| \in [1, \|V\| - 1]$ and $k_y = \|\Gamma(y)\| \in [1, \|V\| - 1]$. Then, $\sqrt{k_x \times k_y} \in [1, \|V\| - 1]$. Thus, $s_{xy}^{\text{Salton}} \in [0, \|V\| - 2]$.

- Jaccard index (Jaccard, 1901) is defined as

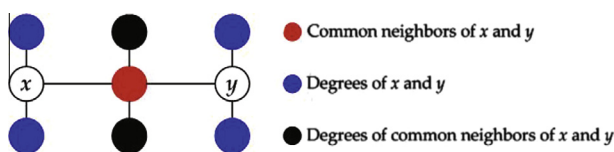


Fig. 1. Three kinds of local information.

$$s_{xy}^{\text{Jaccard}} = \frac{\|\Gamma(x) \cap \Gamma(y)\|}{\|\Gamma(x) \cup \Gamma(y)\|}. \quad (3)$$

$\|\Gamma(x) \cup \Gamma(y)\| \in [1, \|V\|]$, so $s_{xy}^{\text{Jaccard}} \in [0, \|V\| - 2]$.
- Sørensen index (Sørensen, 1948) is defined as

$$s_{xy}^{\text{Sørensen}} = \frac{2\|\Gamma(x) \cap \Gamma(y)\|}{k_x + k_y}. \quad (4)$$

$k_x + k_y \in [2, 2(\|V\| - 1)]$, so $s_{xy}^{\text{Sørensen}} \in [0, \|V\| - 2]$.
- Hub promoted index (HPI) (Ravasz et al., 2002) is said to assign a higher score for link connecting to the nodes with high degrees (Zhao et al., 2012; Zhou et al., 2009) and defined as

$$s_{xy}^{\text{HPI}} = \frac{\|\Gamma(x) \cap \Gamma(y)\|}{\min\{k_x, k_y\}} \in [0, \|V\| - 2]. \quad (5)$$

- Hub depressed index (HDI) (Lü & Zhou, 2011; Zhou et al., 2009) is opposite to HPI and assigns a lower score for link connecting to the nodes with high degrees. The definition of HDI is

$$s_{xy}^{\text{HDI}} = \frac{\|\Gamma(x) \cap \Gamma(y)\|}{\max\{k_x, k_y\}} \in [0, \|V\| - 2]. \quad (6)$$

Now, we give some different understandings to the roles of HPI and HDI by analyzing the subgraphs in Figs. 2 and 3. For Fig. 2, the scores of link existence likelihoods between nodes x and y computed with HPI are $s_{x_a y_a}^{\text{HPI}} = \frac{2}{5} < s_{x_b y_b}^{\text{HPI}} = \frac{2}{3}$. And, for Fig. 3, the scores of link existence likelihoods between nodes x and y computed with HDI are $s_{x_a y_a}^{\text{HDI}} = \frac{2}{4} > s_{x_b y_b}^{\text{HDI}} = \frac{2}{6}$. By observing these results, we can find that HPI and HDI are all inclined to assign a low score for the link adjacent to the nodes with high degrees.

- Leicht-Holme-Newman-I index (LHN-I) (Leicht et al., 2006) is similar to the Salton index and defined as

$$s_{xy}^{\text{LHN-I}} = \frac{\|\Gamma(x) \cap \Gamma(y)\|}{k_x \times k_y} \in [0, \|V\| - 2]. \quad (7)$$

The main difference between Salton index and LHN-I index is the denominator of Eqs. (2) and (7): the former is $\sqrt{k_x \times k_y}$ and the latter $k_x \times k_y$. Because $k_x \times k_y \geq 1$, $k_x \times k_y \geq \sqrt{k_x \times k_y}$. Then, we can get $s_{xy}^{\text{Salton}} > s_{xy}^{\text{LHN-I}}$ when $k_x \times k_y \neq 1$. That is to say, for a same link, Salton index always assigns a higher score compared with LHN-I index.

- Adamic-Adar index (AA) (Adamic & Adar, 2003) is defined as

$$s_{xy}^{\text{AA}} = \sum_{z \in \Gamma(x) \cap \Gamma(y)} \frac{1}{\log_2(k_z)}, \quad (8)$$

which considers the logarithms of degrees of common neighbors of x and y . Because $\|\Gamma(x) \cap \Gamma(y)\| \in [0, \|V\| - 2]$ and $k_z \in [2, \|V\| - 1]$, we can get $s_{xy}^{\text{AA}} \in [\frac{1}{\log_2(\|V\| - 1)}, \|V\| - 2]$.

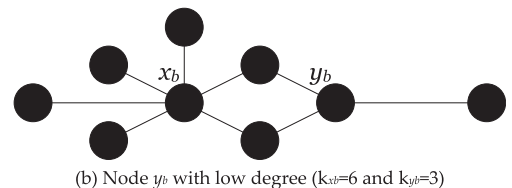
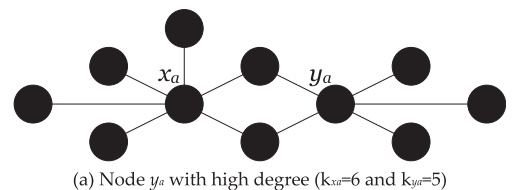


Fig. 2. Link prediction with HPI.

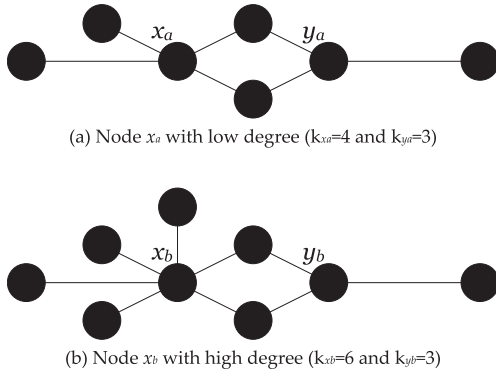


Fig. 3. Link prediction with HDI.

- Resource allocation index (RA) (Zhou et al., 2009) is similar to AA index and defined as

$$s_{xy}^{RA} = \sum_{z \in \Gamma(x) \cap \Gamma(y)} \frac{1}{k_z} \in \left[\frac{1}{\|V\| - 1}, \frac{\|V\| - 2}{2} \right]. \quad (9)$$

AA and RA indices are all inclined to assign a low score for the link between x and y which have the common neighbors with high degrees. By comparing Eqs. (8) with (9), we can find $s_{xy}^{AA} > s_{xy}^{RA}$ when $\Gamma(x) \cap \Gamma(y) \neq \emptyset$.

2.2. Performance measure index-AUC

AUC (Bradley, 1997; Lü & Zhou, 2011; Zhao et al., 2012) is the prevalently used index to measure the performance of link prediction algorithm, which is defined as

$$AUC = \frac{n_1 + 0.5n_2}{n}, \quad (10)$$

where n is the number of independent comparisons including n_1 times the missing link having a higher score, n_2 times the missing link and nonexistent link having the same score, and n_3 times the missing link having a lower score, i.e., $n = n_1 + n_2 + n_3$. The missing link denotes the link in testing set E_{Test} , and nonexistent link is the link in $E_{Predict}$. AUC assumes that a good prediction algorithm is more likely to assign a higher score for the missing link compared with the nonexistent link.

Assume there are two different link prediction algorithms: AlgoA and AlgoB. If AlgoA obtains a better performance, i.e., larger AUC, than AlgoB on the same E_{Test} and $E_{Predict}$, we want to know what conclusions can be derived from the result $AUC^{AlgoA} > AUC^{AlgoB}$.

From the definition of Eq. (10), we know

$$AUC^{AlgoA} = \frac{n_1^{AlgoA} + 0.5n_2^{AlgoA}}{n} \quad (11)$$

and

$$AUC^{AlgoB} = \frac{n_1^{AlgoB} + 0.5n_2^{AlgoB}}{n}. \quad (12)$$

Because $AUC^{AlgoA} > AUC^{AlgoB}$, we can get

$$n_1^{AlgoA} - n_1^{AlgoB} > 0.5(n_2^{AlgoB} - n_2^{AlgoA}). \quad (13)$$

Case I: If $n_1^{AlgoA} - n_1^{AlgoB} = 0, n_2^{AlgoA} > n_2^{AlgoB}$ is obtained. Considering

$$n_1^{AlgoA} + n_2^{AlgoA} + n_3^{AlgoA} = n_1^{AlgoB} + n_2^{AlgoB} + n_3^{AlgoB}, \quad (14)$$

then, we get $n_3^{AlgoA} < n_3^{AlgoB}$.

Case II: If $n_1^{AlgoA} - n_1^{AlgoB} < 0, n_2^{AlgoA} > n_2^{AlgoB}$ is also obtained. By combining Eqs. (13)–(15) can be derived as

$$n_1^{AlgoA} - n_1^{AlgoB} > n_3^{AlgoA} - n_3^{AlgoB}, \quad (15)$$

i.e., $n_3^{AlgoA} < n_3^{AlgoB}$.

Case III-1: If $n_1^{AlgoA} - n_1^{AlgoB} > 0$ and $n_2^{AlgoA} = n_2^{AlgoB}, n_3^{AlgoA} < n_3^{AlgoB}$ certainly holds.

Case III-2: If $n_1^{AlgoA} - n_1^{AlgoB} > 0$ and $n_2^{AlgoA} > n_2^{AlgoB}, n_3^{AlgoA} < n_3^{AlgoB}$ certainly holds.

Case III-3: If $n_1^{AlgoA} - n_1^{AlgoB} > 0$ and $n_2^{AlgoA} < n_2^{AlgoB}$, the relationship between n_3^{AlgoA} and n_3^{AlgoB} is undefined.

These analytical results have been summarized in Table 2. As mentioned above, a better link prediction algorithm is assumed to assign a high score for the missing link in E_{Test} more easily. Thus, we think that the Case I and Case II are inadvisable for $AUC^{AlgoA} > AUC^{AlgoB}$, because $n_1^{AlgoA} = n_1^{AlgoB}$ and $n_1^{AlgoA} < n_1^{AlgoB}$ all deviate from the previous assumption. This deduction can be demonstrated by the following experimental results and analysis.

2.3. High prediction variance

In this section, we study the prediction variances of these nine local information-based link prediction algorithms. We select four benchmark social networks (Pajek datasets, 2006) as shown in Figs. 4–7 for our experimental datasets: World Soccer Data Paris 1998-WSDP98, Food Webs-ChesLower, and Graph Drawing Contests Data-C96 and B97. The detailed descriptions of these four networks are listed in Table 3, where $\langle k \rangle$ is the average degree of network and $E(G)$ is the network efficiency (Latora & Marchiori, 2001) which measures the capability of information exchange with this network. The definition of $E(G)$ is as follows:

$$E(G) = \frac{2}{\|V\|(\|V\| - 1)} \sum_{x,y \in V} \frac{1}{d_{xy}}, \quad (16)$$

where d_{xy} is the shortest path between nodes x and y . Compared with WSDP98 and C96, we find that ChesLower and B97 are the more efficient networks. That is to say, the information can be exchanged more efficiently via the networks of ChesLower and B97.

The 10-fold cross-validation is used to test the AUC of link prediction algorithm. Firstly, the set E including all the existing links is randomly and averagely divided into 10 disjointed subsets (folds): $E = E_1 \cup E_2 \cup \dots \cup E_{10}$ and $E_1 \cap E_2 \cap \dots \cap E_{10} = \emptyset$. Then, we select the subset E_i ($1 \leq i \leq 10$) as testing set E_{Test} in sequence, the link in which is called missing link. Based on the $E_{test} = E_i$ and $\|U - E_i\|$, AUC_i in Eq. (10) is calculated for i th fold dataset. Finally, 10 AUCs on 10 folds are averaged as the evaluation result of link prediction algorithm. The detailed experimental results on these 4 networks are summarized in Tables 4–7 respectively. By observing the results in Tables 4–7, we can get the following conclusions:

- AA and RA obtain the higher AUCs, CN the medium AUC and other 6 methods the lower AUCs. From Eqs. (1)–(9), we know that AA and RA consider the degrees of common neighbors of x and y (Black nodes in the second column of Fig. 1), CN considers the number of common neighbors of x and y (Red node in the second column of Fig. 1), and other methods consider the number of common neighbors of x and y and the degrees of x and y (Blue nodes in the first and third columns of Fig. 1) synchronously (The item $\|\Gamma(x) \cup \Gamma(y)\|$ in Jaccard index equals to $k_x + k_y$ when there are no common neighbors for x and y).
- For 2 different prediction algorithms AlgoA and AlgoB, if $AUC^{AlgoA} > AUC^{AlgoB}$, we can get $n_1^{AlgoA} > n_1^{AlgoB}$. For example, $AUC^{AA} > AUC^{CN}$ holds for the 4 aforementioned networks, then, we can find

Table 2
The derived results from $AUC^{AlgoA} > AUC^{AlgoB}$.

	Missing link having higher score	Missing link and nonexistent link having same score	Missing link having lower score
Case I	$n_1^{AlgoA} = n_1^{AlgoB}$	$n_2^{AlgoA} > n_2^{AlgoB}$	$n_3^{AlgoA} < n_3^{AlgoB}$
Case II	$n_1^{AlgoA} < n_1^{AlgoB}$	$n_2^{AlgoA} > n_2^{AlgoB}$	$n_3^{AlgoA} < n_3^{AlgoB}$
Case III-1	$n_1^{AlgoA} > n_1^{AlgoB}$	$n_2^{AlgoA} = n_2^{AlgoB}$	$n_3^{AlgoA} < n_3^{AlgoB}$
Case III-2	$n_1^{AlgoA} > n_1^{AlgoB}$	$n_2^{AlgoA} > n_2^{AlgoB}$	$n_3^{AlgoA} < n_3^{AlgoB}$
Case III-3	$n_1^{AlgoA} > n_1^{AlgoB}$	$n_2^{AlgoA} < n_2^{AlgoB}$	$n_3^{AlgoA} > n_3^{AlgoB}$ OR $n_3^{AlgoA} = n_3^{AlgoB}$ OR $n_3^{AlgoA} < n_3^{AlgoB}$

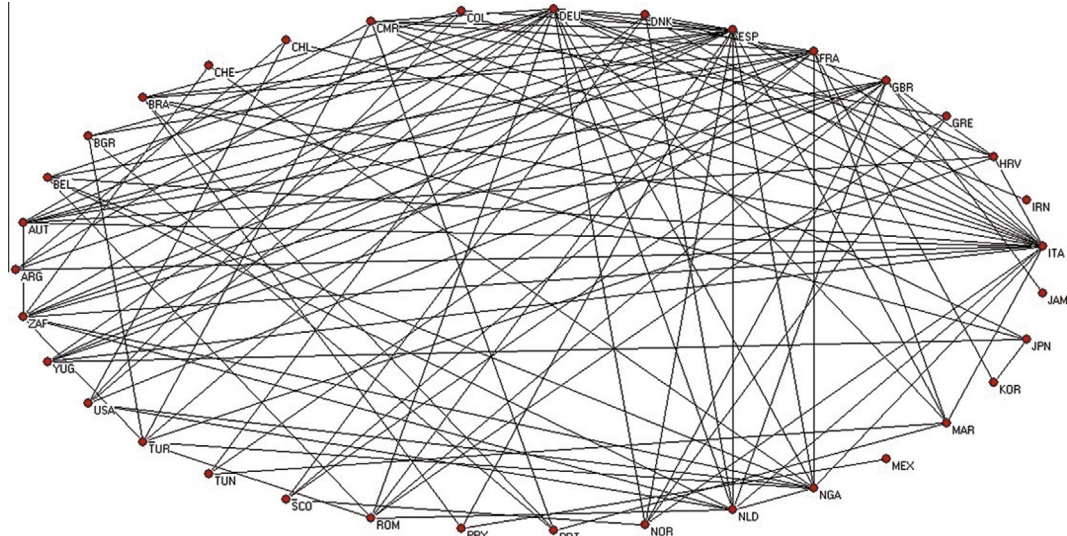


Fig. 4. Network of World Soccer Data Paris 1998-WSDP98.

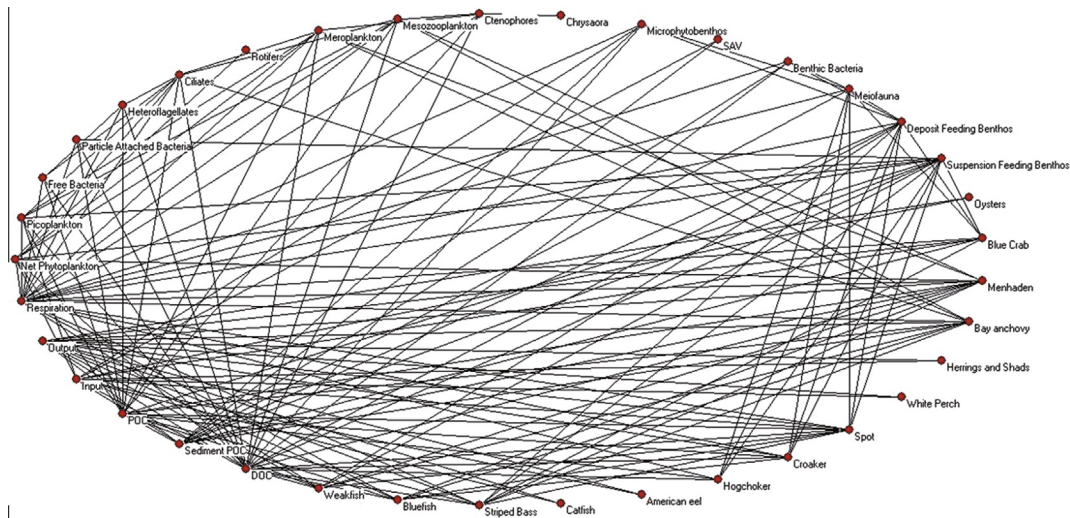


Fig. 5. Network of Food Webs-ChesLower.

$n_1^{AA}(WSDP98) = 3599 > n_1^{CN}(WSDP98) = 3345,$

$n_1^{AA}(ChesLower) = 6038 > n_1^{CN}(ChesLower) = 5424,$

$n_1^{AA}(C96) = 10,510 > n_1^{CN}(C96) = 10,200$

and

$n_1^{AA}(B97) = 19,998 > n_1^{CN}(B97) = 17,399,$

respectively. This empirical conclusion also reflects that increasing the number of missing links having higher scores is the key of improving the performance of link prediction algorithm from another perspective.

- The variability of local information-based link prediction algorithms are high. We can find that the prediction performances of different algorithms are varying dramatically. At least, this point is demonstrated on the 4 employed networks, of which

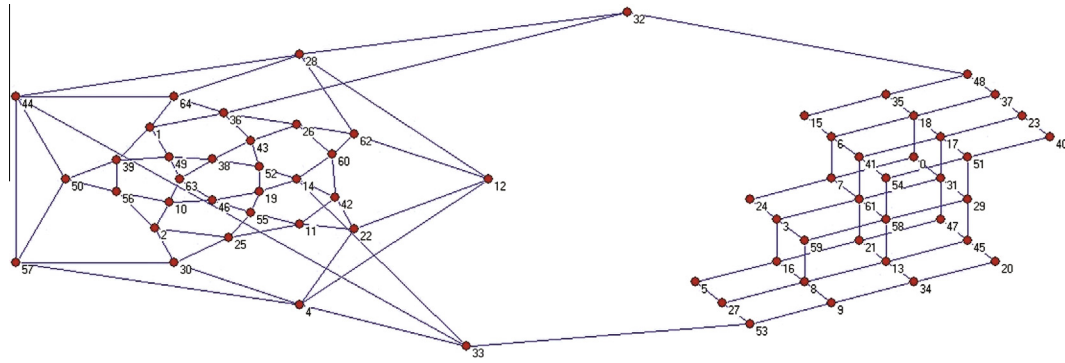


Fig. 6. Network of Graph Drawing Contests Data-C96.

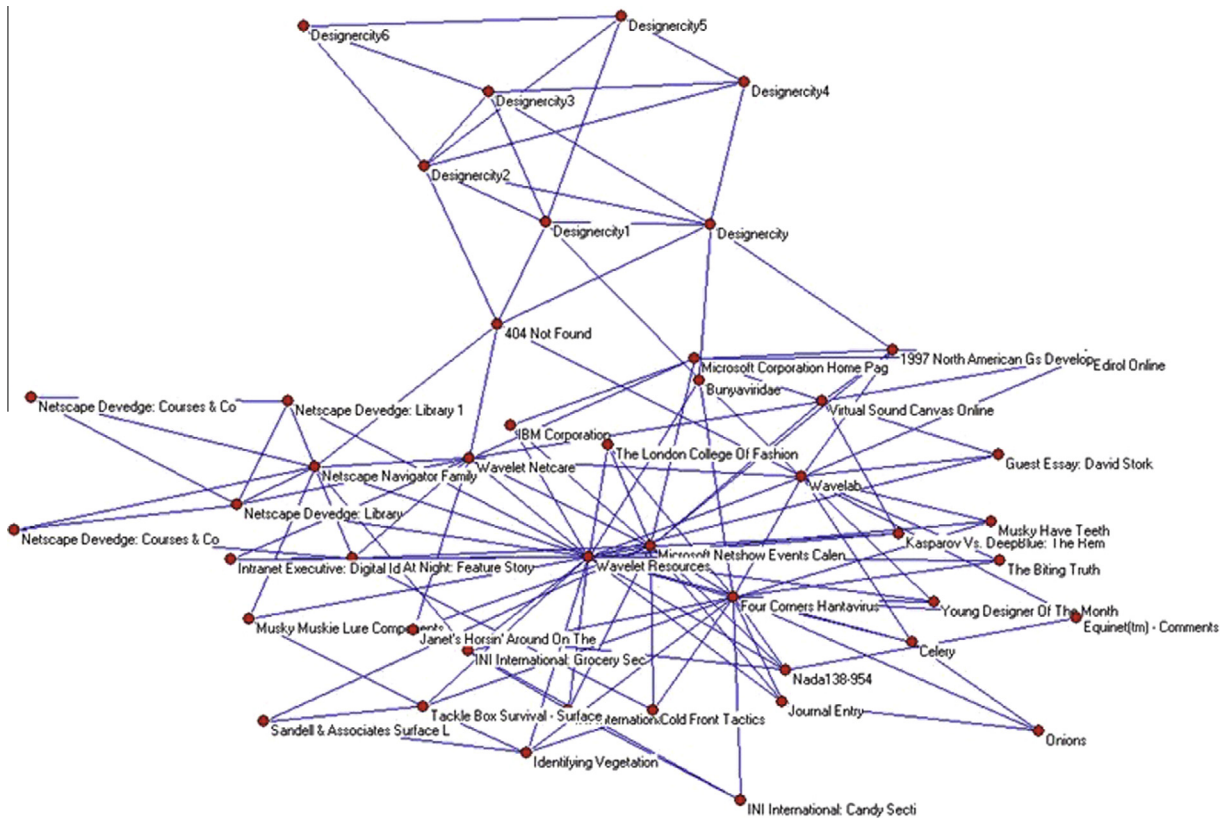


Fig. 7. Network of Graph Drawing Contests Data-B97.

Table 3
The detailed information regarding 4 networks.

Network	$\ V\ $	$\ E\ $	$\ U - E\ $	$\langle k \rangle = 2 \frac{\ E\ }{\ V\ }$	$E(G)$
WSDP98	35	118	477	6.7429	0.6149
ChesLower	37	167	499	9.0270	0.6561
C96	65	125	1955	3.8462	0.5258
B97	46	264	903	11.4783	0.6469

the network efficiency is ranged from 0.5258 to 0.6561. Especially for the networks with higher efficiency, e.g., Cheslower and B97, this kind of variability is particularly outstanding.

From the foregoing analysis, we can find that (1) not any link prediction algorithm mentioned in Section 2.1 can consider the degrees of x and y , the common neighbors of x and y , and the degrees of common neighbors of x and y simultaneously; and (2)

the link prediction algorithm is unstable and has high prediction variance. This may limit the prediction performances of local information-based link prediction algorithms to a certain degree.

3. Ordered weighted averaging (OWA) operator based link prediction ensemble

In this section, we firstly introduce the basic concept of OWA operator and then give an ensemble based link prediction algorithm (LPE_{OWA}) which relies on the OWA operator to aggregate 9 above-mentioned individual local information-based link prediction algorithms.

3.1. OWA operator

OWA operator (Yager, 1998) was firstly introduced by Yager and is a mostly used information aggregation technique. The

Table 4
Prediction performances of nine algorithms on the network of World Soccer Data Paris 1998-WSDP98.

	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
CN	[3129 1018 1577 0.6356]	[2733 1155 1836 0.5784]	[3449 862 1413 0.6778]	[4151 656 917 0.7825]	[4229 629 866 0.7938]	[3351 921 1452 0.6659]	[3184 990 1550 0.6427]	[3763 779 1182 0.7255]	[3067 827 1353 0.6633]	[2397 1078 1772 0.5596]	[3345 892 1392 0.6725±0.0060]
Salton	[2626 346 2752 0.4890]	[2217 513 2994 0.4321]	[2959 509 2256 0.5614]	[3586 199 1939 0.6439]	[4096 17 1611 0.7171]	[2718 548 2458 0.5227]	[2856 381 2487 0.5322]	[3511 368 1845 0.6455]	[2550 525 2172 0.5360]	[2088 693 2466 0.4640]	[2921 410 2298 0.5544±0.0080]
Jaccard	[2694 356 2674 0.5017]	[2299 577 2848 0.4520]	[2988 520 2216 0.5674]	[3737 255 1732 0.6751]	[4179 53 1492 0.7347]	[2761 557 2406 0.5310]	[2997 438 2289 0.5618]	[3532 400 1792 0.6520]	[2552 560 2135 0.5397]	[2057 728 2462 0.4614]	[2980 444 2205 0.5677±0.0086]
Sørensen	[2694 356 2674 0.5017]	[2299 577 2848 0.4520]	[2988 520 2216 0.5674]	[3737 255 1732 0.6751]	[4179 53 1492 0.7347]	[2761 557 2406 0.5310]	[2997 438 2289 0.5618]	[3532 400 1792 0.6520]	[2552 560 2135 0.5397]	[2057 728 2462 0.4614]	[2980 444 2205 0.5677±0.0086]
HPI	[2586 657 2481 0.5092]	[2181 739 2804 0.4456]	[2803 571 2350 0.5396]	[3182 468 2074 0.5968]	[3537 351 1836 0.6486]	[2528 815 2381 0.5128]	[2703 682 2339 0.5318]	[3238 571 1915 0.6156]	[2477 736 2034 0.5422]	[2150 937 2160 0.4990]	[2739 653 2237 0.5441±0.0037]
HDI	[2624 391 2709 0.4926]	[2345 606 2773 0.4626]	[2887 556 2281 0.5529]	[3765 322 1637 0.6859]	[4187 123 1414 0.7422]	[2768 643 2313 0.5397]	[2969 529 2226 0.5649]	[3492 469 1763 0.6510]	[2517 539 2191 0.5311]	[1975 723 2549 0.4453]	[2953 490 2186 0.5668±0.0095]
LHN-I	[1937 361 3426 0.3699]	[1768 527 3429 0.3549]	[1853 523 3348 0.3694]	[2661 242 2821 0.4860]	[3305 80 2339 0.5844]	[1847 546 3331 0.3704]	[2221 408 3095 0.4237]	[2627 396 2701 0.4935]	[1703 523 3021 0.3744]	[1606 721 2920 0.3748]	[2153 433 3043 0.4201±0.0059]
AA	[3359 402 1963 0.6219]	[3182 522 2020 0.6015]	[3555 573 1596 0.6711]	[4344 248 1132 0.7806]	[4507 81 1136 0.7945]	[3664 517 1543 0.6853]	[3543 393 1788 0.6533]	[3951 366 1407 0.7222]	[3287 515 1445 0.6755]	[2596 729 1922 0.5642]	[3599 435 1595 0.6770±0.0054]
RA	[3224 402 2098 0.5984]	[3218 522 1984 0.6078]	[3538 573 1613 0.6682]	[4254 248 1222 0.7648]	[4425 81 1218 0.7801]	[3706 518 1500 0.6927]	[3510 393 1821 0.6475]	[3870 367 1487 0.7082]	[3303 516 1428 0.6787]	[2594 729 1924 0.5638]	[3564 435 1630 0.6710±0.0049]

Note: The quadruple denotes $[n_1 n_2 n_3 AUC]$.

Table 5
Prediction performances of nine algorithms on the network of Food Webs-ChesLower.

	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
CN	[6310 660 1513 0.7827]	[4900 877 2706 0.6293]	[6045 939 1499 0.7679]	[5277 840 2366 0.6716]	[5911 708 1864 0.7385]	[5482 943 2058 0.7018]	[5442 1003 2038 0.7006]	[5311 1007 1666 0.7283]	[4980 833 2171 0.6759]	[4581 853 2550 0.6272]	[5424 866 2043 0.7024±0.0028]
Salton	[4591 30 3862 0.5430]	[3858 55 4570 0.4580]	[4549 23 3911 0.5376]	[3944 50 4489 0.4679]	[4286 56 4141 0.5085]	[3692 13 4778 0.4360]	[3522 21 4940 0.4164]	[4382 40 3562 0.5514]	[3837 27 4120 0.4823]	[3383 23 4578 0.4252]	[4004 34 4295 0.4826±0.0025]
Jaccard	[4626 227 3630 0.5587]	[3971 154 4358 0.4772]	[4441 166 3876 0.5333]	[3622 178 4683 0.4375]	[4334 110 4039 0.5174]	[3486 129 4868 0.4185]	[3292 142 5049 0.3964]	[4357 131 3496 0.5539]	[3785 105 4094 0.4806]	[3278 81 4625 0.4156]	[3919 142 4272 0.4789±0.0036]
Sørensen	[4626 227 3630 0.5587]	[3971 154 4358 0.4772]	[4441 166 3876 0.5333]	[3622 178 4683 0.4375]	[4334 110 4039 0.5174]	[3486 129 4868 0.4185]	[3292 142 5049 0.3964]	[4357 131 3496 0.5539]	[3785 105 4094 0.4806]	[3278 81 4625 0.4156]	[3919 142 4272 0.4789±0.0036]
HPI	[3745 550 4188 0.4739]	[2884 370 5229 0.3618]	[4438 620 3425 0.5597]	[4501 482 3500 0.5590]	[3570 771 4142 0.4663]	[3614 403 4466 0.4498]	[4036 536 3911 0.5074]	[4087 670 3227 0.5539]	[3451 503 4030 0.4637]	[3272 404 4308 0.4351]	[3760 531 4043 0.4831±0.0040]
HDI	[4743 214 3526 0.5717]	[4254 204 4025 0.5135]	[4383 174 3926 0.5269]	[3400 115 4968 0.4076]	[4431 145 3907 0.5309]	[3557 55 4871 0.4226]	[3187 207 5089 0.3879]	[4238 163 3583 0.5410]	[3738 122 4124 0.4758]	[3286 83 4615 0.4168]	[3922 148 4263 0.4795±0.0043]
LHN-I	[1601 89 6793 0.1940]	[2083 82 6318 0.2504]	[1980 103 6400 0.2395]	[1557 121 6805 0.1907]	[1943 89 6451 0.2343]	[1368 61 7054 0.1649]	[1324 95 7064 0.1617]	[1908 139 5937 0.2477]	[1548 68 6368 0.1981]	[1133 31 6820 0.1439]	[1645 88 6601 0.2025±0.0015]
AA	[6855 5 1623 0.8084]	[5589 40 2854 0.6612]	[6478 247 1758 0.7782]	[5896 153 2434 0.7041]	[6491 31 1961 0.7670]	[6222 52 2209 0.7365]	[6238 81 2164 0.7401]	[5986 98 1900 0.7559]	[5499 112 2373 0.6958]	[5130 100 2754 0.6488]	[6038 92 2203 0.7296±0.0026]
RA	[6992 5 1486 0.8245]	[5643 40 2800 0.6676]	[6676 247 1560 0.8015]	[6001 153 2329 0.7164]	[6482 31 1970 0.7659]	[6241 52 2190 0.7388]	[6243 81 2159 0.7407]	[6041 98 1845 0.7628]	[5528 112 2344 0.6994]	[5220 100 2664 0.6601]	[6107 92 2135 0.7378±0.0029]

n -dimensional OWA operator is a mapping $F: \mathfrak{R}^n \rightarrow \mathfrak{R}$ with an associated weight vector $\mathbf{w} = (w_1, w_2, \dots, w_n)$ such that

$$\sum_{i=1}^n w_i = 1, w_i \in [0, 1], i = 1, 2, \dots, n \tag{17}$$

and

$$F(a_1, a_2, \dots, a_n) = \sum_{i=1}^n w_i b_i, \tag{18}$$

where b_i is the i th largest value of a_1, a_2, \dots, a_n . The important issue of applying OWA operator is to determine the weight vector \mathbf{w} of OWA operator.

In order to select the weight vector \mathbf{w} , two important measures $Disp(\mathbf{w})$ and $orness(\mathbf{w})$ are defined, where $Disp(\mathbf{w})$ measures the degree to which all the aggregates are equally used and $orness(\mathbf{w})$ measures the degree to which the aggregation is like an *or* operation. Based on the different designs of $Disp(\mathbf{w})$, three commonly used methods for determining the weight vector of OWA operator can be found from the literatures.

Table 6
Prediction performances of nine algorithms on the network of Graph Drawing Contests Data-C96.

	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
CN	[11900	[8500	[10200	[10200	[10200	[8500	[11900	[8500	[13600	[8500	[10200 12196
	11537	14555	13046	13046	13046	12855	9837 1723	12855	8328 1532	12855	2042
	1978	2360	2169	2169	2169	2105	0.7169]	2105	0.7572]	2105	0.6673±0.0018]
Salton	0.6952]	0.6208]	0.6580]	0.6580]	0.6580]	0.6363]		0.6363]		0.6363]	
	[12150	[8594	[10409	[10368	[10409	[8631	[12076	[8631	[13891	[8594	[10375 11515
	10819	13905	12422	12329	12422	12265	8999 2385	12265	7516 2053	12205	2548
Jaccard	2446	2916	2584	2718	2584	2564	0.7065]	2564	0.7523]	2661	0.6605±0.0019]
	0.6909]	0.6117]	0.6539]	0.6505]	0.6539]	0.6293]		0.6293]		0.6264]	
	[12150	[8594	[10409	[10368	[10409	[8631	[12076	[8631	[13891	[8594	[10375 11564
Sørensen	10891	13929	12482	12377	12482	12301	9047 2337	12301	7600 1969	12229	2498
	2374	2892	2524	2670	2524	2528	0.7076]	2528	0.7541]	2637	0.6615±0.0019]
	0.6923]	0.6122]	0.6551]	0.6514]	0.6551]	0.6301]		0.6301]		0.6270]	
HPI	[12150	[8594	[10409	[10368	[10409	[8631	[12076	[8631	[13891	[8594	[10375 11564
	10891	13929	12482	12377	12482	12301	9047 2337	12301	7600 1969	12229	2498
	2374	2892	2524	2670	2524	2528	0.7076]	2528	0.7541]	2637	0.6615±0.0019]
HDI	0.6923]	0.6122]	0.6551]	0.6514]	0.6551]	0.6301]		0.6301]		0.6270]	
	[11928	[8520	[10224	[10220	[10224	[8520	[11928	[8520	[13632	[8520	[10224 11841
	11138	14270	12704	12574	12704	12570	9438 2094	12570	7872 1956	12570	2373
LHN-I	2349	2625	2487	2621	2487	2370	0.7096]	2370	0.7488]	2370	0.6610±0.0018]
	0.6885]	0.6160]	0.6522]	0.6495]	0.6522]	0.6311]		0.6311]		0.6311]	
	[12218	[8606	[10465	[10412	[10465	[8659	[12112	[8659	[13971	[8606	[10417 11643
AA	10973	13999	12553	12486	12553	12366	9139 2209	12366	7693 1796	12299	2378
	2224	2810	2397	2517	2397	2435	0.7111]	2435	0.7595]	2555	0.6648±0.0020]
	0.6966]	0.6140]	0.6587]	0.6553]	0.6587]	0.6327]		0.6327]		0.6290]	
RA	[12150	[8594	[10409	[10368	[10409	[8631	[12076	[8631	[13891	[8594	[10375 11515
	10819	13905	12422	12329	12422	12265	8999 2385	12265	7516 2053	12205	2548
	2446	2916	2584	2718	2584	2564	0.7065]	2564	0.7523]	2661	0.6605±0.0019]
RA	0.6909]	0.6117]	0.6539]	0.6505]	0.6539]	0.6293]		0.6293]		0.6264]	
	[12215	[8693	[10510	[10510	[10566	[8805	[12271	[8749	[14032	[8749	[10510 11696
	10902	14066	12546	12546	12608	12490	9264 1925	12428	7682 1746	12428	2232
RA	2298	2656	2359	2359	2241	2165	0.7205]	2283	0.7618]	2283	0.6698±0.0019]
	0.6951]	0.6188]	0.6604]	0.6604]	0.6638]	0.6415]		0.6378]		0.6378]	
	[12215	[8693	[10510	[10510	[10566	[8805	[12271	[8749	[14032	[8749	[10510 11696
RA	10902	14066	12546	12546	12608	12490	9264 1925	12428	7682 1746	12428	2232
	2298	2656	2359	2359	2241	2165	0.7205]	2283	0.7618]	2283	0.6698±0.0019]
	0.6951]	0.6188]	0.6604]	0.6604]	0.6638]	0.6415]		0.6378]		0.6378]	

Table 7
Prediction performances of nine algorithms on the network of Graph Drawing Contests Data-B97.

	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
CN	[19232	[17312	[17147	[17597	[17754	[16973	[17304	[17866	[16617	[16190	[17399 4042 2398
	3378 1771	4318 2751	4372 2862	4200 2584	3789 1935	4151 2354	3961 2213	3482 2130	4140 2721	4631 2657	0.8146±0.0005]
	0.8581]	0.7986]	0.7930]	0.8079]	0.8369]	0.8113]	0.8214]	0.8351]	0.7959]	0.7882]	
Salton	[17989	[17122	[16694	[16575	[17803	[15560	[16895	[16434	[15095	[15839	[16601 468 6771
	610 5782	329 6930	590 7097	612 7194	212 5463	265 7653	161 6422	577 6467	955 7428	367 7272	0.7061±0.0011]
	0.7503]	0.7090]	0.6968]	0.6924]	0.7628]	0.6684]	0.7230]	0.7123]	0.6633]	0.6824]	
Jaccard	[16913	[16710	[16249	[15836	[17464	[14657	[16081	[16120	[14395	[15163	[15959 640 7241
	715 6753	514 7157	833 7299	686 7859	481 5533	300 8521	319 7078	770 6588	1206 7877	574 7741	0.6828±0.0013]
	0.7084]	0.6959]	0.6835]	0.6636]	0.7541]	0.6307]	0.6917]	0.7030]	0.6388]	0.6581]	
Sørensen	[16913	[16710	[16249	[15836	[17464	[14657	[16081	[16120	[14395	[15163	[15959 640 7241
	715 6753	514 7157	833 7299	686 7859	481 5533	300 8521	319 7078	770 6588	1206 7877	574 7741	0.6828±0.0013]
	0.7084]	0.6959]	0.6835]	0.6636]	0.7541]	0.6307]	0.6917]	0.7030]	0.6388]	0.6581]	
HPI	[19350	[17551	[17574	[18052	[17499	[17253	[17849	[16418	[16088	[16780	[17441 2174 4223
	1878 3153	2410 4420	2138 4669	2104 4225	2270 3709	2261 3964	1812 3817	1820 5240	2635 4755	2416 4282	0.7771±0.0008]
	0.8322]	0.7693]	0.7647]	0.7836]	0.7937]	0.7830]	0.7988]	0.7381]	0.7414]	0.7662]	
HDI	[16215	[16333	[15854	[14978	[16984	[13937	[15088	[16143	[14087	[14947	[15457 736 7647
	798 7368	527 7521	894 7633	1034 8369	487 6007	447 9094	713 7677	711 6624	1140 8251	607 7924	0.6638±0.0015]
	0.6814]	0.6807]	0.6686]	0.6355]	0.7338]	0.6031]	0.6578]	0.7027]	0.6243]	0.6496]	
LHN-I	[15403	[15306	[14875	[14428	[15788	[13568	[14439	[14049	[13380	[14726	[14596 528 8715
	604 8374	428 8647	631 8875	704 9249	232 7458	298 9612	389 8650	597 8832	1004 9094	389 8363	0.6233±0.0007]
	0.6441]	0.6366]	0.6230]	0.6062]	0.6774]	0.5842]	0.6233]	0.6111]	0.5913]	0.6355]	
AA	[21451	[20514	[19608	[20133	[20364	[20024	[20393	[20239	[18418	[18832	[19998 608 3234
	640 2290	330 3537	882 3891	798 3450	334 2780	288 3166	341 2744	587 2652	1284 3776	596 4050	0.8516±0.0008]
	0.8929]	0.8482]	0.8223]	0.8421]	0.8745]	0.8590]	0.8759]	0.8745]	0.8118]	0.8148]	
RA	[21386	[21125	[19915	[20254	[20461	[20459	[20750	[20607	[18277	[18996	[20223 608 3008
	639 2356	330 2926	882 3584	797 3330	334 2683	288 2731	341 2387	587 2284	1284 3917	596 3886	0.8611±0.0010]
	0.8903]	0.8732]	0.8349]	0.8471]	0.8786]	0.8775]	0.8911]	0.8902]	0.8058]	0.8218]	

- O'Hagan's maximum entropy method (MEM) (O'Hagan, 1988) is to solve the constrained nonlinear optimization model as follows:

$$\begin{aligned} \text{Maximize } \text{Disp}(\mathbf{w}) &= -\sum_{i=1}^n w_i \ln(w_i) \\ \text{s.t. } \text{orness}(\mathbf{w}) &= \alpha = \frac{1}{n-1} \sum_{i=1}^n (n-i)w_i, \\ \sum_{i=1}^n w_i &= 1, \\ w_i &\in [0, 1], \quad i = 1, 2, \dots, n. \end{aligned} \quad (19)$$

- Fuller and Majlender's minimum variance method (MVM) (Fullér & Majlender, 2003) is to solve the following mathematical programming model:

$$\begin{aligned} \text{Minimize } \text{Disp}(\mathbf{w}) &= \frac{1}{n} \sum_{i=1}^n (w_i - \frac{1}{n})^2 \\ \text{s.t. } \text{orness}(\mathbf{w}) &= \alpha = \frac{1}{n-1} \sum_{i=1}^n (n-i)w_i, \\ \sum_{i=1}^n w_i &= 1, \\ w_i &\in [0, 1], \quad i = 1, 2, \dots, n. \end{aligned} \quad (20)$$

- Wang, Luo and Liu's chi-square method (CSM) (Wang et al., 2007) is to solve the following nonlinear programming model:

$$\begin{aligned} \text{Minimize } \text{Disp}(\mathbf{w}) &= \sum_{i=1}^{n-1} \left(\frac{w_i}{w_{i+1}} + \frac{w_{i+1}}{w_i} - 2 \right) \\ \text{s.t. } \text{orness}(\mathbf{w}) &= \alpha = \frac{1}{n-1} \sum_{i=1}^n (n-i)w_i, \\ \sum_{i=1}^n w_i &= 1, \\ w_i &\in [0, 1], \quad i = 1, 2, \dots, n. \end{aligned} \quad (21)$$

where α in Eqs. (19)–(21) is the optimism level factor, which controls the desired degree of orness. There are three special OWA operator weights which should be addressed when selecting the following optimism level factors:

- For MEM and MVM, $\alpha \in [0, 1]$.
 1. When $\alpha = 0, \mathbf{w} = (0, \dots, 0, 1)$. Then, OWA operator $F(a_1, a_2, \dots, a_n) = \min_{i=1,2,\dots,n} \{a_i\}$. This expresses the aggregation of a_1, a_2, \dots, a_n is to select the minimum of a_1, a_2, \dots, a_n .
 2. When $\alpha = 1, \mathbf{w} = (1, 0, \dots, 0)$. Then, OWA operator $F(a_1, a_2, \dots, a_n) = \max_{i=1,2,\dots,n} \{a_i\}$. This expresses the aggregation of a_1, a_2, \dots, a_n is to select the maximum of a_1, a_2, \dots, a_n .
 3. When $\alpha = 0.5, \mathbf{w} = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})$. Then, OWA operator $F(a_1, a_2, \dots, a_n) = \frac{1}{n} \sum_{i=1}^n a_i$. This expresses the aggregation of a_1, a_2, \dots, a_n equals to the arithmetic mean of a_1, a_2, \dots, a_n .
- For CSM, $\alpha \in (0, 1)$. $\alpha = 0$ and $\alpha = 1$ are inapplicable for the optimization model in Eq. (21). The case of $\alpha = 0.5$ is similar with MEM and MVM.

In this paper, LINGO software is used to solve the OWA operator weights corresponding to the three above-mentioned optimization problems.

3.2. OWA operator based link prediction ensemble-LPE_{OWA}

In order to make the link prediction ensemble more effective, we firstly normalize the link prediction algorithms introduced in

Section 2.1 so that the scores representing likelihood of link existence can be located into the interval [0, 1]. Let $sn_{xy}^{CN}, sn_{xy}^{Salton}, sn_{xy}^{Jaccard}, sn_{xy}^{Sorensen}, sn_{xy}^{HPI}, sn_{xy}^{HDI}, sn_{xy}^{LHN-1}, sn_{xy}^{AA}$ and sn_{xy}^{RA} be the normalizations of $s_{xy}^{CN}, s_{xy}^{Salton}, s_{xy}^{Jaccard}, s_{xy}^{Sorensen}, s_{xy}^{HPI}, s_{xy}^{HDI}, s_{xy}^{LHN-1}, s_{xy}^{AA}$ and s_{xy}^{RA} . In Section 2, we have analyzed the boundaries of these 9 algorithms, thus, the normalized link prediction algorithms can be obtained through the original link prediction algorithms divided by the respective maxima.

Then, we try to calculate the likelihood score of link existence through our proposed link prediction ensemble method-LPE_{OWA}. We denote this score as s_{xy}^{OWA} between nodes x and y , which is calculated as:

$$s_{xy}^{OWA} = \sum_{i=1}^9 \left[w_i \left[\frac{s_{xy}^{(i)}}{\sum_{j=1}^9 s_{xy}^{(j)}} \right] \right], \quad (22)$$

where $s_{xy}^{(i)}$ is the i th largest value of $sn_{xy}^{CN}, sn_{xy}^{Salton}, sn_{xy}^{Jaccard}, sn_{xy}^{Sorensen}, sn_{xy}^{HPI}, sn_{xy}^{HDI}, sn_{xy}^{LHN-1}, sn_{xy}^{AA}$ and sn_{xy}^{RA} , the term $\frac{s_{xy}^{(i)}}{\sum_{j=1}^9 s_{xy}^{(j)}}$ is supposed to be b_i in Eq. (18); w_i is the weight of OWA operator, $i = 1, 2, \dots, 9$. In our study, we respectively use these different methods, i.e., MEM, MVM and CSM, to determine the weight w_i and further compare their performances.

The purpose of normalization is to consider the likelihood score as the probability value, which makes it possible to compare the scores corresponding to the different link prediction algorithms. For the $k_x, k_y > 2$ and $k_x \neq k_y$, we can derive

$$1 < \min \{k_x, k_y\} < \sqrt{k_x k_y} < \frac{k_x + k_y}{2} < \max \{k_x, k_y\} < \|\Gamma(x) \cup \Gamma(y)\| < k_x k_y. \quad (23)$$

Furthermore, we can get the following derivations:

$$s_{xy}^{CN} > s_{xy}^{HPI} > s_{xy}^{Salton} > s_{xy}^{Sorensen} > s_{xy}^{HDI} > s_{xy}^{Jaccard} > s_{xy}^{LHN-1} \quad (24)$$

and

$$sn_{xy}^{CN} > sn_{xy}^{HPI} > sn_{xy}^{Salton} > sn_{xy}^{Sorensen} > sn_{xy}^{HDI} > sn_{xy}^{Jaccard} > sn_{xy}^{LHN-1}. \quad (25)$$

For any node $z \in \|\Gamma(x) \cap \Gamma(y)\|$, when $k_z > 2$, we can obtain

$$1 > \frac{1}{\log_2 k_z} > \frac{1}{k_z}. \quad (26)$$

Because $s_{xy}^{CN} = \|\Gamma(x) \cap \Gamma(y)\| = \sum_{z \in \Gamma(x) \cap \Gamma(y)} 1$, we can derive

$$s_{xy}^{CN} > s_{xy}^{AA} > s_{xy}^{RA} \quad (27)$$

and

$$sn_{xy}^{CN} > sn_{xy}^{AA} > sn_{xy}^{RA}. \quad (28)$$

From Eqs. (25) and (28), we can find that the normalized link prediction algorithm based on local information entitled the number of common neighbors of nodes x and y (sn_{xy}^{CN}) can obtain a higher probability value in comparison with other link prediction algorithms based on the other two kinds of local information, i.e., the degrees of nodes x and y ($sn_{xy}^{HPI}, sn_{xy}^{Salton}, sn_{xy}^{Sorensen}, sn_{xy}^{HDI}, sn_{xy}^{Jaccard}$ and sn_{xy}^{LHN-1}) and the degrees of common neighbors of nodes x and y (sn_{xy}^{AA} and sn_{xy}^{RA}). We think it is reasonable that sn_{xy}^{CN} obtains the higher likelihood score, because it is obvious and direct that a link will more likely exist between two nodes x and y if they have more common neighbors.

Table 8
Prediction performances of LPE_{OWA} and LPE_{AM} algorithms on the network of World Soccer Data Paris 1998-WSDP98.

LPE _{OWA} orness(w) = α	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
<i>Maximum entropy method</i>											
0.55	[4567 339 818 0.8275]	[3924 507 1293 0.7298]	[4235 507 982 0.7842]	[4723 170 831 0.8400]	[4654 3 1067 0.8133]	[4130 507 1087 0.7658]	[3800 343 1581 0.6938]	[4232 339 1153 0.7690]	[3662 507 1078 0.7462]	[3066 676 1505 0.6488]	[4099 390 1140 0.7618±0.0036]
0.60	[4591 339 794 0.8317]	[3911 507 1306 0.7276]	[4239 507 978 0.7849]	[4725 170 829 0.8403]	[4661 3 1060 0.8146]	[4132 507 1085 0.7662]	[3794 343 1587 0.6928]	[4245 339 1140 0.7712]	[3666 507 1074 0.7470]	[3064 676 1507 0.6484]	[4103 390 1136 0.7624±0.0037]
0.65	[4599 339 786 0.8331]	[3894 507 1323 0.7246]	[4245 507 972 0.7859]	[4728 170 826 0.8408]	[4669 3 1052 0.8160]	[4116 507 1101 0.7634]	[3792 343 1589 0.6924]	[4253 339 1132 0.7726]	[3657 507 1083 0.7453]	[3061 676 1510 0.6478]	[4101 390 1137 0.7622±0.0038]
0.70	[4609 339 776 0.8348]	[3885 507 1332 0.7230]	[4249 507 968 0.7866]	[4749 170 805 0.8445]	[4693 3 1028 0.8201]	[4106 507 1111 0.7616]	[3788 343 1593 0.6917]	[4264 339 1121 0.7745]	[3647 507 1093 0.7434]	[3053 676 1518 0.6463]	[4104 390 1135 0.7627±0.0040]
0.75	[4615 339 770 0.8359]	[3860 507 1357 0.7186]	[4251 507 966 0.7869]	[4757 170 797 0.8459]	[4716 3 1013 0.8228]	[4078 507 1139 0.7567]	[3795 343 1586 0.6930]	[4292 339 1093 0.7794]	[3636 507 1104 0.7413]	[3035 676 1536 0.6428]	[4103 390 1136 0.7623±0.0043]
0.80	[4626 339 759 0.8378]	[3849 507 1368 0.7167]	[4252 507 965 0.7871]	[4748 170 806 0.8443]	[4716 3 1005 0.8242]	[4062 507 1155 0.7539]	[3805 343 1576 0.6947]	[4302 339 1083 0.7812]	[3617 507 1123 0.7377]	[3034 676 1537 0.6427]	[4101 390 1138 0.7620±0.0043]
0.85	[4623 339 762 0.8373]	[3796 507 1421 0.7075]	[4251 507 966 0.7869]	[4759 170 795 0.8463]	[4718 3 1003 0.8245]	[4028 507 1189 0.7480]	[3821 343 1560 0.6975]	[4299 339 1086 0.7807]	[3591 507 1149 0.7327]	[3033 676 1538 0.6425]	[4092 390 1147 0.7604±0.0045]
0.90	[4635 339 750 0.8394]	[3744 507 1473 0.6984]	[4256 507 961 0.7878]	[4751 170 803 0.8449]	[4705 3 1016 0.8222]	[3983 507 1234 0.7401]	[3840 343 1541 0.7008]	[4306 339 1079 0.7819]	[3572 507 1168 0.7291]	[3005 676 1566 0.6371]	[4080 390 1159 0.7582±0.0047]
0.92	[4637 339 748 0.8397]	[3728 507 1489 0.6956]	[4259 507 958 0.7883]	[4740 170 814 0.8429]	[4712 3 1009 0.8235]	[3951 507 1266 0.7345]	[3836 343 1545 0.7001]	[4308 339 1077 0.7822]	[3543 507 1197 0.7236]	[2994 676 1577 0.6350]	[4071 390 1168 0.7566±0.0049]
0.93	[4634 339 751 0.8392]	[3720 507 1497 0.6942]	[4259 507 958 0.7883]	[4727 170 827 0.8407]	[4696 3 1025 0.8207]	[3940 507 1277 0.7326]	[3846 343 1535 0.7019]	[4296 339 1089 0.7801]	[3542 507 1198 0.7234]	[2986 676 1585 0.6335]	[4065 390 1174 0.7555±0.0048]
0.94	[4633 339 752 0.8390]	[3695 507 1522 0.6898]	[4256 507 961 0.7878]	[4723 170 831 0.8400]	[4692 3 1029 0.8200]	[3923 507 1294 0.7296]	[3851 343 1530 0.7027]	[4294 339 1091 0.7798]	[3536 507 1204 0.7222]	[2976 676 1595 0.6316]	[4058 390 1181 0.7543±0.0049]
0.95	[4638 339 747 0.8399]	[3688 507 1529 0.6886]	[4254 507 963 0.7875]	[4717 170 837 0.8389]	[4696 3 1025 0.8207]	[3910 507 1307 0.7274]	[3855 343 1526 0.7034]	[4295 339 1090 0.7800]	[3522 507 1218 0.7196]	[2961 676 1610 0.6287]	[4054 390 1185 0.7535±0.0050]
0.96	[4636 339 749 0.8395]	[3681 507 1536 0.6874]	[4253 507 964 0.7873]	[4710 170 844 0.8377]	[4695 3 1026 0.8205]	[3895 507 1322 0.7248]	[3866 343 1515 0.7054]	[4295 339 1090 0.7800]	[3518 507 1222 0.7188]	[2945 676 1626 0.6257]	[4049 390 1189 0.7527±0.0051]
0.97	[4634 339 751 0.8392]	[3675 507 1542 0.6874]	[4249 507 968 0.7873]	[4696 170 858 0.8377]	[4697 3 1024 0.8205]	[3882 507 1335 0.7248]	[3867 343 1514 0.7054]	[4290 339 1095 0.7800]	[3517 507 1223 0.7188]	[2925 676 1646 0.6257]	[4043 390 1196 0.7516±0.0052]
0.98	[4635 339 750 0.8394]	[3678 507 1539 0.6863]	[4247 507 970 0.7866]	[4685 170 869 0.8353]	[4689 3 1032 0.8208]	[3866 507 1351 0.7225]	[3856 343 1525 0.7055]	[4289 339 1096 0.7791]	[3513 507 1227 0.7186]	[2914 676 1657 0.6219]	[4037 390 1202 0.7505±0.0052]
<i>Minimum variance method</i>											
0.55	[4544 339 841 0.8235]	[3933 507 1284 0.7314]	[4236 507 981 0.7843]	[4713 170 841 0.8382]	[4645 3 1076 0.8118]	[4121 507 1096 0.7642]	[3793 343 1588 0.6926]	[4226 339 1159 0.7679]	[3665 507 1075 0.7468]	[3070 676 1501 0.6495]	[4095 390 1144 0.7610±0.0035]
0.60	[4544 339 841 0.8235]	[3933 507 1284 0.7314]	[4236 507 981 0.7843]	[4713 170 841 0.8382]	[4645 3 1076 0.8118]	[4121 507 1096 0.7642]	[3793 343 1588 0.6926]	[4226 339 1159 0.7679]	[3665 507 1075 0.7468]	[3070 676 1501 0.6495]	[4095 390 1144 0.7610±0.0035]
0.65	[4544 339 841 0.8235]	[3933 507 1284 0.7314]	[4236 507 981 0.7843]	[4713 170 841 0.8382]	[4645 3 1076 0.8118]	[4121 507 1096 0.7642]	[3793 343 1588 0.6926]	[4226 339 1159 0.7679]	[3665 507 1075 0.7468]	[3070 676 1501 0.6495]	[4095 390 1144 0.7610±0.0035]
0.70	[4544 339 841 0.8235]	[3933 507 1284 0.7314]	[4236 507 981 0.7843]	[4713 170 841 0.8382]	[4645 3 1076 0.8118]	[4121 507 1096 0.7642]	[3793 343 1588 0.6926]	[4226 339 1159 0.7679]	[3665 507 1075 0.7468]	[3070 676 1501 0.6495]	[4095 390 1144 0.7610±0.0035]
0.75	[4541 339 844 0.8229]	[3930 507 1287 0.7309]	[4236 507 981 0.7843]	[4711 170 843 0.8379]	[4644 3 1077 0.8116]	[4119 507 1098 0.7639]	[3797 343 1584 0.6933]	[4221 339 1164 0.7670]	[3662 507 1078 0.7462]	[3071 676 1500 0.6497]	[4093 390 1146 0.7608±0.0034]
0.80	[4554 339 831 0.8252]	[3921 507 1296 0.7293]	[4234 507 983 0.7840]	[4717 170 837 0.8389]	[4642 3 1079 0.8112]	[4124 507 1093 0.7648]	[3788 343 1593 0.6917]	[4227 339 1158 0.7681]	[3662 507 1078 0.7462]	[3067 676 1504 0.6489]	[4094 390 1145 0.7608±0.0035]
0.85	[4588 339 797 0.8311]	[3893 507 1324 0.7244]	[4235 507 982 0.7842]	[4727 170 827 0.8407]	[4661 3 1060 0.8146]	[4119 507 1098 0.7639]	[3769 343 1612 0.6884]	[4246 339 1139 0.7714]	[3652 507 1088 0.7443]	[3055 676 1516 0.6467]	[4095 390 1144 0.7610±0.0039]
0.90	[4613 339 772 0.8355]	[3850 507 1367 0.7169]	[4247 507 970 0.7863]	[4764 170 790 0.8471]	[4704 3 1017 0.8221]	[4072 507 1145 0.7557]	[3789 343 1592 0.6919]	[4301 339 1084 0.7810]	[3602 507 1138 0.7348]	[3022 676 1549 0.6404]	[4096 390 1142 0.7612±0.0044]
0.92	[4626 339 759 0.8394]	[3809 507 1408 0.6868]	[4248 507 969 0.7863]	[4755 170 799 0.8333]	[4711 3 1010 0.8194]	[4042 507 1175 0.7197]	[3801 343 1580 0.7036]	[4297 339 1088 0.7789]	[3575 507 1165 0.7178]	[3015 676 1556 0.6198]	[4088 390 1151 0.7596±0.0046]

Table 8 (continued)

LPE _{OWA} orness(w) = α	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
0.93	0.8378] [4625 339 760	0.7097] [3780 507 1437	0.7864] [4248 507 969	0.8456] [4755 170 799	0.8233] [4712 3 1009	0.7504] [4012 507 1205	0.6940] [3809 343 1572	0.7803] [4312 339 1073	0.7297] [3564 507 1176	0.6390] [2998 676 1573	[4082 390 1157 0.7585±0.0047]
0.94	0.8376] [4631 339 754	0.7047] [3750 507 1467	0.7864] [4250 507 967	0.8456] [4750 170 804	0.8235] [4705 3 1016	0.7452] [3984 507 1233	0.6954] [3815 343 1566	0.7829] [4312 339 1073	0.7276] [3548 507 1192	0.6358] [2989 676 1582	[4073 390 1165 0.7570±0.0048]
0.95	0.8387] [4635 339 750	0.6994] [3724 507 1493	0.7868] [4251 507 966	0.8447] [4750 170 824	0.8222] [4702 3 1019	0.7403] [3954 507 1263	0.6965] [3825 343 1556	0.7829] [4315 339 1070	0.7245] [3538 507 1202	0.6341] [2971 676 1600	[4065 390 1174 0.7554±0.0050]
0.96	0.8394] [4635 339 750	0.6949] [3724 507 1527	0.7869] [4255 507 962	0.8412] [4727 170 827	0.8217] [4702 3 1031	0.7351] [3930 507 1287	0.6982] [3832 343 1599	0.7835] [4325 339 1060	0.7226] [3526 507 1214	0.6306] [2965 676 1606	[4058 390 1181 0.7542±0.0051]
0.97	0.8394] [4637 339 748	0.6889] [3681 507 1536	0.7876] [4252 507 965	0.8407] [4703 170 851	0.8196] [4687 3 1034	0.7309] [3899 507 1318	0.6994] [3863 343 1518	0.7852] [4299 339 1086	0.7203] [3515 507 1225	0.6295] [2933 676 1638	[4047 390 1192 0.7522±0.0051]
0.98	0.8397] [4637 339 748	0.6874] [3675 507 1542	0.7871] [4248 507 969	0.8365] [4693 170 861	0.8191] [4694 3 1027	0.7255] [3873 507 1344	0.7048] [3868 343 1513	0.7807] [4291 339 1094	0.7182] [3513 507 1227	0.6234] [2915 676 1656	[4041 390 1198 0.7511±0.0052]
0.8397]	0.6863]	0.7864]	0.8347]	0.8203]	0.7209]	0.7057]	0.7793]	0.7178]	0.6200]		
<i>Chi-square method</i>											
0.55	[4498 339 887	[3936 507 1281	[4227 507 990	[4699 170 855	[4644 3 1077	[4114 507 1103	[3796 343 1585	[4212 339 1173	[3655 507 1085	[3061 676 1510	[4084 390 1155 0.7592±0.0033]
0.60	0.8154] [4543 339 842	0.7319] [3927 507 1290	0.7828] [4231 507 986	0.8358] [4710 170 844	0.8116] [4650 3 1071	0.7630] [4122 507 1095	0.6931] [3785 343 1596	0.7655] [4229 339 1156	0.7449] [3659 507 1081	0.6478] [3063 676 1508	[4092 390 1147 0.7605±0.0035]
0.65	0.8233] [4579 339 806	0.7303] [3908 507 1309	0.7835] [4233 507 984	0.8377] [4727 170 827	0.8126] [4654 3 1067	0.7644] [4125 507 1092	0.6912] [3774 343 1607	0.7684] [4241 339 1144	0.7457] [3657 507 1083	0.6482] [3057 676 1514	[4096 390 1143 0.7611±0.0038]
0.70	0.8296] [4600 339 785	0.7270] [3881 507 1336	0.7838] [4244 507 973	0.8407] [4741 170 813	0.8133] [4677 3 1044	0.7649] [4107 507 1110	0.6893] [3784 343 1597	0.7705] [4260 339 1125	0.7453] [3643 507 1097	0.6470] [3047 676 1524	[4098 390 1140 0.7616±0.0040]
0.75	0.8332] [4617 339 768	0.7223] [3861 507 1356	0.7857] [4250 507 967	0.8431] [4753 170 801	0.8173] [4720 3 1001	0.7618] [4076 507 1141	0.6910] [3791 343 1590	0.7738] [4296 339 1089	0.7426] [3630 507 1110	0.6451] [3030 676 1541	[4102 390 1136 0.7623±0.0043]
0.80	0.8362] [4628 339 757	0.7188] [3825 507 1392	0.7868] [4248 507 969	0.8452] [4759 170 795	0.8249] [4712 3 1009	0.7564] [4047 507 1170	0.6923] [3805 343 1576	0.7801] [4305 339 1080	0.7401] [3598 507 1142	0.6419] [3028 676 1543	[4096 390 1143 0.7610±0.0044]
0.85	0.8381] [4634 339 751	0.7125] [3761 507 1456	0.7864] [4253 507 964	0.8463] [4756 170 798	0.8235] [4710 3 1011	0.7513] [3998 507 1219	0.6947] [3820 343 1561	0.7817] [4309 339 1076	0.7340] [3573 507 1167	0.6415] [3015 676 1556	[4083 390 1156 0.7587±0.0047]
0.8392]	0.7013]	0.7873]	0.8457]	0.8231]	0.7427]	0.6973]	0.7824]	0.7293]	0.6390]		
0.90	0.8394] [4635 339 750	0.6937] [3717 507 1500	0.7883] [4259 507 958	0.8403] [4725 170 829	0.8205] [4695 3 1026	0.7317] [3935 507 1282	0.7019] [3846 343 1535	0.7801] [4296 339 1089	0.7228] [3539 507 1201	0.6331] [2984 676 1587	[4063 390 1176 0.7552±0.0048]
0.92	0.8394] [4637 339 748	0.6937] [3691 507 1526	0.7883] [4254 507 963	0.8403] [4717 170 837	0.8205] [4700 3 1021	0.7317] [3914 507 1303	0.7019] [3854 343 1527	0.7801] [4293 339 1092	0.7228] [3525 507 1215	0.6331] [2967 676 1604	[4055 390 1184 0.7538±0.0050]
0.8397]	0.6891]	0.7875]	0.8389]	0.8214]	0.7281]	0.7033]	0.7796]	0.7201]	0.6299]		
0.93	0.8397] [4637 339 748	0.6882] [3686 507 1531	0.7875] [4254 507 963	0.8391] [4718 170 836	0.8203] [4694 3 1027	0.7263] [3904 507 1313	0.7045] [3861 343 1520	0.7796] [4293 339 1092	0.7192] [3520 507 1220	0.6280] [2957 676 1614	[4052 390 1186 0.7532±0.0050]
0.94	0.8397] [4637 339 748	0.6882] [3681 507 1536	0.7875] [4253 507 964	0.8391] [4709 170 845	0.8203] [4703 3 1018	0.7263] [3893 507 1324	0.7045] [3866 343 1515	0.7796] [4294 339 1091	0.7192] [3518 507 1222	0.6280] [2945 676 1626	[4050 390 1189 0.7528±0.0051]
0.8397]	0.6874]	0.7873]	0.8375]	0.8219]	0.7244]	0.7054]	0.7798]	0.7188]	0.6257]		
0.95	0.8397] [4637 339 748	0.6874] [3679 507 1538	0.7873] [4251 507 966	0.8375] [4702 170 852	0.8219] [4700 3 1021	0.7244] [3885 507 1332	0.7054] [3867 343 1514	0.7798] [4288 339 1097	0.7188] [3519 507 1221	0.6257] [2930 676 1641	[4046 390 1193 0.7520±0.0052]
0.96	0.8397] [4637 339 748	0.6870] [3681 507 1536	0.7869] [4250 507 967	0.8363] [4692 170 862	0.8214] [4699 3 1022	0.7230] [3881 507 1336	0.7055] [3862 343 1519	0.7787] [4292 339 1093	0.7190] [3516 507 1224	0.6228] [2924 676 1647	[4043 390 1195 0.7516±0.0052]
0.8397]	0.6874]	0.7868]	0.8346]	0.8212]	0.7223]	0.7047]	0.7794]	0.7184]	0.6217]		
0.97	0.8397] [4636 339 749	0.6872] [3680 507 1537	0.7864] [4248 507 969	0.8330] [4683 170 871	0.8193] [4688 3 1033	0.7204] [3870 507 1347	0.7036] [3856 343 1525	0.7789] [4289 339 1096	0.7178] [3513 507 1227	0.6204] [2917 676 1654	[4038 390 1201 0.7507±0.0052]
0.98	0.8395] [4637 339 748	0.6872] [3673 507 1544	0.7864] [4246 507 971	0.8330] [4670 170 884	0.8193] [4680 3 1041	0.7204] [3846 507 1371	0.7036] [3860 343 1521	0.7789] [4282 339 1103	0.7178] [3509 507 1231	0.6204] [2907 676 1664	[4031 390 1208 0.7494±0.0052]
0.8397]	0.6860]	0.7861]	0.8307]	0.8179]	0.7162]	0.7043]	0.7777]	0.7171]	0.6184]		
LPE _{AM}	[2753 1008 1963 0.5690]	[2834 1003 1887 0.5827]	[2636 933 2155 0.5420]	[1955 1299 2470 0.4550]	[3569 755 1400 0.6895]	[2907 966 1851 0.5922]	[2756 1132 1836 0.5804]	[2943 1182 1599 0.6174]	[2461 1015 1771 0.5658]	[2528 1055 1664 0.5823]	[2734 1035 1860 0.5776±0.0034]

Table 9
Prediction performances of LPE_{OWA} and LPE_{AM} algorithms on the network of Food Webs-ChesLower.

LPE _{OWA} orness(w) = α	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
<i>Maximum entropy method</i>											
0.55	[7103 3 1377 0.8375]	[5531 10 2942 0.6526]	[7283 10 1190 0.8591]	[6369 14 2100 0.7516]	[6060 6 2417 0.7147]	[7282 3 1198 0.8586]	[7002 4 1477 0.8257]	[6674 4 1306 0.8362]	[6072 7 1905 0.7610]	[5911 9 2064 0.7409]	[6529 7 1798 0.7838±0.0049]
0.60	[7126 3 1354 0.8402]	[5554 10 2919 0.6553]	[7289 10 1184 0.8598]	[6343 14 2126 0.7486]	[6073 6 2404 0.7163]	[7289 3 1191 0.8594]	[6988 4 1491 0.8240]	[6683 4 1297 0.8373]	[6072 7 1905 0.7610]	[5891 9 2084 0.7384]	[6531 7 1796 0.7840±0.0049]
0.65	[7131 3 1349 0.8408]	[5578 10 2895 0.6581]	[7278 10 1195 0.8585]	[6337 14 2132 0.7478]	[6108 6 2369 0.7204]	[7311 3 1169 0.8620]	[6966 4 1513 0.8214]	[6687 4 1293 0.8378]	[6071 7 1906 0.7608]	[5899 9 2076 0.7394]	[6537 7 1790 0.7847±0.0048]
0.70	[7124 3 1356 0.8400]	[5574 10 2899 0.6577]	[7263 10 1210 0.8568]	[6316 14 2153 0.7454]	[6131 6 2346 0.7231]	[7326 3 1154 0.8638]	[6950 4 1529 0.8195]	[6680 4 1300 0.8369]	[6067 7 1910 0.7603]	[5896 9 2079 0.7390]	[6533 7 1794 0.7842±0.0047]
0.75	[7137 3 1343 0.8415]	[5614 10 2859 0.6624]	[7246 10 1227 0.8548]	[6260 14 2209 0.7388]	[6153 6 2324 0.7257]	[7346 3 1134 0.8661]	[6938 4 1541 0.8181]	[6683 4 1297 0.8373]	[6069 7 1908 0.7606]	[5878 9 2097 0.7368]	[6532 7 1794 0.7842±0.0047]
0.80	[7145 3 1335 0.8424]	[5636 10 2837 0.6650]	[7200 10 1273 0.8493]	[6202 14 2267 0.7319]	[6166 6 2311 0.7272]	[7381 3 1099 0.8703]	[6909 4 1570 0.8147]	[6682 4 1298 0.8372]	[6084 7 1893 0.7625]	[5843 9 2132 0.7324]	[6525 7 1802 0.7833±0.0047]
0.85	[7135 3 1345 0.8413]	[5686 10 2787 0.6709]	[7172 10 1301 0.8460]	[6157 14 2312 0.7266]	[6233 6 2244 0.7351]	[7367 3 1113 0.8686]	[6862 4 1617 0.8091]	[6660 4 1320 0.8344]	[6097 7 1880 0.7641]	[5849 9 2126 0.7332]	[6522 7 1805 0.7829±0.0043]
0.90	[7113 3 1367 0.8387]	[5694 10 2779 0.6718]	[7092 10 1381 0.8366]	[6059 14 2410 0.7151]	[6262 6 2215 0.7385]	[7333 3 1147 0.8646]	[6800 4 1679 0.8018]	[6621 4 1359 0.8295]	[6096 7 1881 0.7640]	[5800 9 2175 0.7270]	[6487 7 1839 0.7788±0.0042]
0.92	[7088 3 1392 0.8357]	[5702 10 2771 0.6728]	[7065 10 1408 0.8334]	[5977 14 2492 0.7054]	[6252 6 2225 0.7374]	[7321 3 1159 0.8632]	[6775 4 1704 0.7989]	[6604 4 1376 0.8274]	[6085 7 1892 0.7626]	[5758 9 2217 0.7218]	[6463 7 1864 0.7759±0.0042]
0.93	[7068 3 1412 0.8334]	[5712 10 2761 0.6739]	[7061 10 1412 0.8330]	[5950 14 2519 0.7022]	[6256 6 2221 0.7378]	[7310 3 1170 0.8619]	[6754 4 1725 0.7964]	[6584 4 1396 0.8249]	[6079 7 1898 0.7618]	[5747 9 2228 0.7204]	[6452 7 1874 0.7746±0.0041]
0.94	[7048 3 1432 0.8310]	[5700 10 2773 0.6725]	[7023 10 1450 0.8285]	[5907 14 2562 0.6972]	[6249 6 2228 0.7370]	[7298 3 1182 0.8605]	[6744 4 1735 0.7952]	[6574 4 1406 0.8236]	[6070 7 1907 0.7607]	[5733 9 2242 0.7186]	[6435 7 1892 0.7725±0.0042]
0.95	[7033 3 1447 0.8292]	[5689 10 2784 0.6712]	[6993 10 1480 0.8249]	[5877 14 2592 0.6936]	[6240 6 2237 0.7359]	[7272 3 1208 0.8574]	[6731 4 1748 0.7937]	[6551 4 1429 0.8208]	[6061 7 1916 0.7596]	[5714 9 2261 0.7162]	[6416 7 1910 0.7703±0.0041]
0.96	[7009 3 1471 0.8264]	[5680 10 2793 0.6702]	[6969 10 1504 0.8221]	[5824 14 2645 0.6874]	[6260 6 2217 0.7383]	[7256 3 1224 0.8555]	[6710 4 1769 0.7912]	[6540 4 1440 0.8194]	[6064 7 1913 0.7600]	[5692 9 2283 0.7135]	[6400 7 1926 0.7684 0.0041]
0.97	[6984 3 1496 0.8235]	[5694 10 2779 0.6718]	[6955 10 1518 0.8205]	[5761 14 2708 0.6799]	[6247 6 2230 0.7368]	[7234 3 1246 0.8529]	[6705 4 1774 0.7906]	[6531 4 1449 0.8183]	[6053 7 1924 0.7586]	[5646 9 2329 0.7077]	[6381 7 1945 0.7661±0.0042]
0.98	[6945 3 1535 0.8189]	[5679 10 2794 0.6700]	[6924 10 1549 0.8168]	[5682 14 2226 0.6706]	[6251 6 2226 0.7372]	[7199 3 1281 0.8488]	[6677 4 1802 0.7873]	[6503 4 1477 0.8148]	[6040 7 1937 0.7570]	[5618 9 2357 0.7042]	[6352 7 1975 0.7626±0.0042]
<i>Minimum variance method</i>											
0.55	[7092 3 1388 0.8362]	[5524 10 2949 0.6518]	[7274 10 1199 0.8581]	[6379 14 2090 0.7528]	[6039 6 2438 0.7122]	[7269 3 1211 0.8571]	[7018 4 1461 0.8275]	[6655 4 1325 0.8338]	[6068 7 1909 0.7605]	[5924 9 2051 0.7425]	[6524 7 1802 0.7832±0.0049]
0.60	[7092 3 1388 0.8362]	[5524 10 2949 0.6518]	[7274 10 1199 0.8581]	[6379 14 2090 0.7528]	[6039 6 2438 0.7122]	[7269 3 1211 0.8571]	[7018 4 1461 0.8275]	[6655 4 1325 0.8338]	[6068 7 1909 0.7605]	[5924 9 2051 0.7425]	[6524 7 1802 0.7832±0.0049]
0.65	[7092 3 1388 0.8362]	[5524 10 2949 0.6518]	[7274 10 1199 0.8581]	[6379 14 2090 0.7528]	[6039 6 2438 0.7122]	[7269 3 1211 0.8571]	[7018 4 1461 0.8275]	[6655 4 1325 0.8338]	[6068 7 1909 0.7605]	[5924 9 2051 0.7425]	[6524 7 1802 0.7832±0.0049]
0.70	[7092 3 1388 0.8362]	[5524 10 2949 0.6518]	[7274 10 1199 0.8581]	[6379 14 2090 0.7528]	[6039 6 2438 0.7122]	[7269 3 1211 0.8571]	[7018 4 1461 0.8275]	[6655 4 1325 0.8338]	[6068 7 1909 0.7605]	[5924 9 2051 0.7425]	[6524 7 1802 0.7832±0.0049]
0.75	[7067 3 1413 0.8333]	[5497 10 2976 0.6486]	[7270 10 1203 0.8576]	[6381 14 2088 0.7530]	[6002 6 2475 0.7079]	[7268 3 1212 0.8569]	[6991 4 1488 0.8244]	[6647 4 1333 0.8328]	[6057 7 1920 0.7591]	[5914 9 2061 0.7413]	[6509 7 1817 0.7815±0.0050]
0.80	[7064 3 1416 0.8329]	[5503 10 2970 0.6493]	[7263 10 1210 0.8568]	[6363 14 2106 0.7509]	[6020 6 2457 0.7100]	[7267 3 1213 0.8568]	[6982 4 1497 0.8233]	[6645 4 1335 0.8325]	[6053 7 1924 0.7586]	[5908 9 2067 0.7405]	[6507 7 1820 0.7812±0.0049]
0.85	[7119 3 1361 0.8394]	[5570 10 2903 0.6572]	[7267 10 1206 0.8572]	[6337 14 2132 0.7478]	[6094 6 2383 0.7187]	[7305 3 1175 0.8613]	[6952 4 1527 0.8198]	[6671 4 1309 0.8358]	[6054 7 1923 0.7587]	[5891 9 2084 0.7384]	[6526 7 1800 0.7834±0.0047]
0.90	[7149 3 1331 0.8429]	[5638 10 2835 0.6652]	[7210 10 1263 0.8505]	[6276 14 2193 0.7407]	[6162 6 2315 0.7267]	[7351 3 1129 0.8667]	[6936 4 1543 0.8179]	[6665 4 1315 0.8350]	[6046 7 1931 0.7577]	[5829 9 2146 0.7306]	[6526 7 1800 0.7834±0.0046]
0.92	[7157 3 1323	[5715 10 2758	[7171 10 1302	[6232 14 2237	[6200 6 2277	[7347 3 1133	[6910 4 1569	[6660 4 1320	[6061 7 1916	[5808 9 2167	[6526 7 1800 0.7834±0.0043]

Table 9 (continued)

LPE _{OWA} orness(w) = α	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
0.93	0.8439] [7158 3 1322	0.6743] [5716 10 2757	0.8459] [7118 10 1355	0.7355] [6159 14 2310	0.7312] [6248 6 2229	0.8663] [7340 3 1140	0.8148] [6874 4 1605	0.8344] [6648 4 1332	0.7596] [6087 7 1890	0.7280] [5802 9 2173	[6515 7 1811 0.7821±0.0042]
0.94	0.8440] [7133 3 1347	0.6744] [5720 10 2753	0.8397] [7107 10 1366	0.7269] [6103 14 2366	0.7369] [6262 6 2215	0.8654] [7324 3 1156	0.8106] [6811 4 1668	0.8329] [6634 4 1346	0.7628] [6081 7 1896	0.7273] [5776 9 2199	[6495 7 1831 0.7797±0.0041]
0.95	0.8410] [7112 3 1368	0.6749] [5722 10 2751	0.8384] [7089 10 1384	0.7203] [6043 14 2426	0.7385] [6282 6 2195	0.8636] [7307 3 1173	0.8031] [6771 4 1708	0.8312] [6611 4 1369	0.7621] [6088 7 1889	0.7240] [5757 9 2218	[6478 7 1848 0.7777±0.0041]
0.96	0.8386] [7084 3 1396	0.6751] [5715 10 2758	0.8363] [7035 10 1438	0.7132] [5978 14 2491	0.7409] [6271 6 2206	0.8615] [7294 3 1186	0.7984] [6714 4 1765	0.8283] [6592 4 1388	0.7630] [6089 7 1888	0.7216] [5743 9 2232	[6452 7 1875 0.7745±0.0040]
0.97	0.8353] [7031 3 1449	0.6743] [5703 10 2770	0.8299] [6985 10 1488	0.7055] [5847 14 2622	0.7396] [6262 6 2215	0.8600] [7256 3 1224	0.7917] [6696 4 1783	0.8259] [6556 4 1424	0.7631] [6060 7 1917	0.7199] [5691 9 2284	[6409 7 1918 0.7694±0.0041]
0.98	0.8290] [6967 3 1513	0.6729] [5698 10 2775	0.8240] [6933 10 1540	0.6901] [5727 14 2742	0.7385] [6245 6 2232	0.8555] [7213 3 1267	0.7896] [6680 4 1799	0.8214] [6518 4 1462	0.7595] [6061 7 1916	0.7134] [5635 9 2340	[6368 7 1959 0.7645±0.0041]
	0.8215]	0.6723]	0.8179]	0.6759]	0.7365]	0.8505]	0.7877]	0.8166]	0.7596]	0.7064]	
<i>Chi-square method</i>											
0.55	[7034 3 1446	[5462 10 3011	[7281 10 1192	[6371 14 2098	[5955 6 2522	[7219 3 1261	[6976 4 1503	[6635 4 1345	[6017 7 1960	[5885 9 2090	[6484 7 1843 0.7784±0.0051]
0.60	0.8294] [7087 3 1393	0.6445] [5507 10 2966	0.8589] [7270 10 1203	0.7519] [6378 14 2091	0.7023] [6021 6 2456	0.8512] [7258 3 1222	0.8226] [6982 4 1497	0.8313] [6643 4 1337	0.7541] [6038 7 1939	0.7377] [5899 9 2076	[6508 7 1818 0.7813±0.0049]
0.65	0.8356] [7118 3 1362	0.6498] [5554 10 2919	0.8576] [7265 10 1208	0.7527] [6361 14 2108	0.7101] [6077 6 2400	0.8558] [7287 3 1193	0.8233] [6964 4 1515	0.8323] [6663 4 1317	0.7567] [6053 7 1924	0.7394] [5883 9 2092	[6523 7 1804 0.7830±0.0048]
0.70	0.8393] [7138 3 1342	0.6553] [5594 10 2879	0.8570] [7252 10 1221	0.7507] [6324 14 2145	0.7167] [6112 6 2365	0.8592] [7324 3 1156	0.8212] [6954 4 1525	0.8348] [6680 4 1300	0.7586] [6057 7 1920	0.7374] [5885 9 2090	[6532 7 1794 0.7842±0.0047]
0.75	0.8416] [7144 3 1336	0.6600] [5623 10 2850	0.8555] [7235 10 1238	0.7463] [6283 14 2186	0.7209] [6157 6 2320	0.8636] [7353 3 1127	0.8200] [6932 4 1547	0.8369] [6679 4 1301	0.7591] [6073 7 1904	0.7377] [5865 9 2110	[6534 7 1792 0.7844±0.0046]
0.80	0.8423] [7154 3 1326	0.6634] [5655 10 2818	0.8535] [7207 10 1266	0.7415] [6177 14 2292	0.7262] [6205 6 2272	0.8670] [7366 3 1114	0.8174] [6885 4 1594	0.8368] [6662 4 1318	0.7611] [6075 7 1902	0.7352] [5850 9 2125	[6524 7 1803 0.7831±0.0045]
0.85	0.8435] [7120 3 1360	0.6672] [5705 10 2768	0.8502] [7103 10 1370	0.7290] [6071 14 2398	0.7318] [6260 6 2217	0.8685] [7339 3 1141	0.8119] [6820 4 1659	0.8347] [6631 4 1349	0.7613] [6098 7 1879	0.7333] [5806 9 2169	[6495 7 1831 0.7798±0.0042]
0.90	0.8395] [7066 3 1414	0.6731] [5713 10 2760	0.8379] [7051 10 1422	0.7165] [5935 14 2534	0.7383] [6246 6 2231	0.8653] [7315 3 1165	0.8042] [6752 4 1727	0.8308] [6589 4 1391	0.7642] [6079 7 1898	0.7278] [5744 9 2231	[6449 7 1877 0.7742±0.0042]
0.92	0.8331] [7041 3 1439	0.6741] [5689 10 2784	0.8318] [7006 10 1467	0.7005] [5883 14 2586	0.7366] [6242 6 2235	0.8625] [7280 3 1200	0.7962] [6737 4 1742	0.8255] [6559 4 1421	0.7618] [6068 7 1909	0.7200] [5726 9 2249	[6423 7 1903 0.7711±0.0041]
0.93	0.8302] [7019 3 1461	0.6712] [5685 10 2788	0.8265] [6989 10 1484	0.6943] [5848 14 2621	0.7362] [6243 6 2234	0.8584] [7276 3 1204	0.7944] [6737 4 1742	0.8218] [6550 4 1430	0.7605] [6067 7 1910	0.7177] [5700 9 2275	[6411 7 1915 0.7697±0.0042]
0.94	0.8276] [7007 3 1473	0.6708] [5681 10 2792	0.8245] [6970 10 1503	0.6902] [5824 14 2645	0.7363] [6258 6 2219	0.8579] [7254 3 1226	0.7944] [6710 4 1769	0.8206] [6541 4 1439	0.7603] [6064 7 1913	0.7145] [5673 9 2302	[6398 7 1928 0.7681±0.0042]
0.95	0.8262] [6986 3 1494	0.6703] [5700 10 2773	0.8222] [6960 10 1513	0.6874] [5769 14 2700	0.7381] [6250 6 2227	0.8553] [7244 3 1236	0.7912] [6706 4 1773	0.8195] [6540 4 1440	0.7600] [6062 7 1915	0.7111] [5650 9 2325	[6387 7 1940 0.7667±0.0042]
0.96	0.8237] [6964 3 1516	0.6725] [5692 10 2781	0.8211] [6942 10 1531	0.6809] [5733 14 2736	0.7371] [6255 6 2222	0.8541] [7225 3 1255	0.7908] [6697 4 1782	0.8194] [6520 4 1460	0.7597] [6058 7 1919	0.7082] [5635 9 2340	[6372 7 1954 0.7650±0.0042]
0.97	0.8211] [6947 3 1533	0.6716] [5673 10 2800	0.8189] [6924 10 1549	0.6766] [5678 14 2791	0.7377] [6250 6 2227	0.8519] [7201 3 1279	0.7897] [6680 4 1799	0.8169] [6501 4 1479	0.7592] [6037 7 1940	0.7064] [5620 9 2355	[6351 7 1975 0.7625±0.0042]
0.98	0.8191] [6921 3 1559	0.6693] [5663 10 2810	0.8168] [6914 10 1559	0.6702] [5623 14 2846	0.7371] [6244 6 2233	0.8491] [7175 3 1305	0.7877] [6645 4 1834	0.8145] [6482 4 1498	0.7566] [6022 7 1955	0.7045] [5604 9 2371	[6329 7 1997 0.7599±0.0043]
	0.8160]	0.6682]	0.8156]	0.6637]	0.7364]	0.8460]	0.7836]	0.8121]	0.7547]	0.7025]	
LPE _{AM}	[3505 1495 3483 0.5013]	[4024 2019 2440 0.5934]	[3069 1735 3679 0.4640]	[2842 1888 3753 0.4463]	[3437 1974 3072 0.5215]	[2738 1796 3949 0.4286]	[2700 1883 3900 0.4293]	[4286 1514 2184 0.6316]	[3562 1855 2567 0.5623]	[3233 1644 3107 0.5079]	[3340 1780 3213 0.5086 0.0049]

Table 10
Prediction performances of LPE_{OWA} and LPE_{AM} algorithms on the network of Graph Drawing Contests Data-C96.

LPE _{OWA} orness(w) = α	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
<i>Maximum entropy method</i>											
0.55	[12643	[9282	[10878	[11025	[10914	[9221	[12895	[9210	[14480	[9293	[10984 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2109
	2218	2376	2308	2226	2228	2076	1749	2108	1713	2089	0.6820±0.0019]
0.60	0.7051]	0.6359]	0.6686]	0.6731]	0.6709]	0.6523]	0.7376]	0.6514]	0.7721]	0.6535]	
	[12659	[9286	[10894	[11037	[10934	[9233	[12907	[9222	[14504	[9297	[10997 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2096
0.65	2202	2372	2292	2214	2208	2064	1737	2096	1689	2085	0.6826±0.0019]
	0.7057]	0.6360]	0.6692]	0.6736]	0.6717]	0.6528]	0.7381]	0.6519]	0.7731]	0.6537]	
	[12663	[9298	[10898	[11042	[10938	[9241	[12919	[9230	[14508	[9309	[11005 11344
0.70	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2089
	2198	2360	2288	2209	2204	2056	1725	2088	1685	2073	0.6829±0.0019]
	0.7059]	0.6365]	0.6694]	0.6738]	0.6718]	0.6531]	0.7386]	0.6522]	0.7733]	0.6542]	
0.75	[12679	[9310	[10906	[11050	[10938	[9241	[12927	[9234	[14516	[9317	[11012 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2081
	2182	2348	2280	2201	2204	2056	1717	2084	1677	2065	0.6832±0.0019]
0.80	0.7065]	0.6370]	0.6697]	0.6741]	0.6718]	0.6531]	0.7389]	0.6524]	0.7736]	0.6546]	
	[12704	[9330	[10924	[11063	[10954	[9238	[12922	[9266	[14544	[9302	[11025 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2069
0.85	2157	2328	2262	2188	2188	2059	1722	2052	1649	2080	0.6837±0.0019]
	0.7075]	0.6378]	0.6704]	0.6746]	0.6725]	0.6530]	0.7387]	0.6538]	0.7748]	0.6539]	
	[12750	[9375	[10896	[11058	[10827	[9159	[12915	[9189	[14466	[9345	[10998 11344
0.90	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2095
	2111	2283	2290	2193	2315	2138	1729	2129	1727	2037	0.6826±0.0019]
	0.7093]	0.6395]	0.6693]	0.6744]	0.6675]	0.6496]	0.7384]	0.6505]	0.7715]	0.6558]	
0.92	[12718	[9367	[10864	[11034	[10787	[9135	[12891	[9165	[14418	[9337	[10972 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2122
	2143	2291	2322	2217	2355	2162	1753	2153	1775	2045	0.6815±0.0018]
0.94	0.7080]	0.6392]	0.6681]	0.6735]	0.6659]	0.6486]	0.7374]	0.6494]	0.7695]	0.6554]	
	[12702	[9348	[10848	[11020	[10769	[9113	[12861	[9150	[14398	[9311	[10952 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2141
0.95	2159	2310	2338	2231	2373	2184	1783	2168	1795	2071	0.6807±0.0018]
	0.7074]	0.6385]	0.6674]	0.6729]	0.6652]	0.6477]	0.7361]	0.6488]	0.7686]	0.6543]	
	[12694	[9346	[10840	[11012	[10759	[9107	[12855	[9144	[14386	[9309	[10945 11344
0.96	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2148
	2167	2312	2346	2239	2383	2190	1789	2174	1807	2073	0.6804±0.0018]
	0.7071]	0.6384]	0.6671]	0.6726]	0.6648]	0.6474]	0.7358]	0.6486]	0.7681]	0.6542]	
0.97	[12702	[9352	[10844	[11016	[10759	[9107	[12859	[9146	[14390	[9313	[10949 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2144
	2159	2306	2342	2235	2383	2190	1785	2172	1803	2069	0.6806±0.0018]
0.98	0.7074]	0.6386]	0.6673]	0.6728]	0.6648]	0.6474]	0.7360]	0.6486]	0.7683]	0.6544]	
	[12699	[9348	[10841	[11013	[10756	[9101	[12850	[9145	[14387	[9304	[10944 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2149
0.99	2162	2310	2345	2238	2386	2196	1794	2173	1806	2078	0.6804±0.0018]
	0.7073]	0.6385]	0.6671]	0.6726]	0.6647]	0.6472]	0.7356]	0.6486]	0.7681]	0.6540]	
	[12699	[9348	[10841	[11013	[10756	[9101	[12850	[9145	[14387	[9304	[10944 11344
0.99	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2149
	2162	2310	2345	2238	2386	2196	1794	2173	1806	2078	0.6804±0.0018]
	0.7073]	0.6385]	0.6671]	0.6726]	0.6647]	0.6472]	0.7356]	0.6486]	0.7681]	0.6540]	
0.99	[12707	[9353	[10845	[11015	[10756	[9101	[12854	[9146	[14391	[9308	[10948 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2146
	2154	2305	2341	2236	2386	2196	1790	2172	1802	2074	0.6805±0.0018]
0.99	0.7076]	0.6387]	0.6673]	0.6727]	0.6647]	0.6472]	0.7358]	0.6486]	0.7683]	0.6542]	
	[12715	[9361	[10849	[11012	[10756	[9101	[12858	[9150	[14395	[9312	[10951 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2142
0.99	2146	2297	2337	2239	2386	2196	1786	2168	1798	2070	0.6806±0.0018]
	0.7079]	0.6390]	0.6675]	0.6726]	0.6647]	0.6472]	0.7360]	0.6488]	0.7685]	0.6543]	
	[12749	[9342	[10854	[11017	[10732	[9053	[12815	[9126	[14400	[9269	[10936 11344
0.99	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2158
	2112	2316	2332	2234	2410	2244	1829	2192	1793	2113	0.6800±0.0019]
	0.7093]	0.6382]	0.6677]	0.6728]	0.6637]	0.6451]	0.7341]	0.6478]	0.7687]	0.6525]	
<i>Minimum variance method</i>											
0.55	[12643	[9282	[10878	[11025	[10914	[9221	[12895	[9210	[14480	[9293	[10984 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2109
	2218	2376	2308	2226	2228	2076	1749	2108	1713	2089	0.6820±0.0019]
0.60	0.7051]	0.6359]	0.6686]	0.6731]	0.6709]	0.6523]	0.7376]	0.6514]	0.7721]	0.6535]	
	[12643	[9282	[10878	[11025	[10914	[9221	[12895	[9210	[14480	[9293	[10984 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2109
0.65	2218	2376	2308	2226	2228	2076	1749	2108	1713	2089	0.6820±0.0019]
	0.7051]	0.6359]	0.6686]	0.6731]	0.6709]	0.6523]	0.7376]	0.6514]	0.7721]	0.6535]	
	[12643	[9282	[10878	[11025	[10914	[9221	[12895	[9210	[14480	[9293	[10984 11344
0.65	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2109
	2218	2376	2308	2226	2228	2076	1749	2108	1713	2089	0.6820±0.0019]

Table 10 (continued)

LPE _{OWA} orness(w) = α	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
0.70	0.7051]	0.6359]	0.6686]	0.6731]	0.6709]	0.6523]	0.7376]	0.6514]	0.7721]	0.6535]	[10984 11344 2109 0.6820±0.0019]
	[12643	[9282	[10878	[11025	[10914	[9221	[12895	[9210	[14480	[9293	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2218	2376	2308	2226	2228	2076	1749	2108	1713	2089	
0.75	0.7051]	0.6359]	0.6686]	0.6731]	0.6709]	0.6523]	0.7376]	0.6514]	0.7721]	0.6535]	[10983 11344 2110 0.6820±0.0019]
	[12641	[9281	[10877	[11022	[10914	[9221	[12894	[9210	[14479	[9292	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2220	2377	2309	2229	2228	2076	1750	2108	1714	2090	
0.80	0.7050]	0.6358]	0.6686]	0.6730]	0.6709]	0.6523]	0.7375]	0.6514]	0.7721]	0.6535]	[10982 11344 2111 0.6820±0.0019]
	[12639	[9280	[10876	[11021	[10914	[9221	[12893	[9210	[14478	[9291	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2222	2378	2310	2230	2228	2076	1751	2108	1715	2091	
0.85	0.7049]	0.6358]	0.6685]	0.6729]	0.6709]	0.6523]	0.7375]	0.6514]	0.7720]	0.6535]	[10997 11344 2096 0.6826±0.0019]
	[12659	[9286	[10894	[11037	[10934	[9233	[12907	[9222	[14504	[9297	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2202	2372	2292	2214	2208	2064	1737	2096	1689	2085	
0.90	0.7057]	0.6360]	0.6692]	0.6736]	0.6717]	0.6528]	0.7381]	0.6519]	0.7731]	0.6537]	[11036 11344 2057 0.6842±0.0019]
	[12719	[9325	[10936	[11075	[10963	[9256	[12952	[9249	[14556	[9332	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2142	2333	2250	2176	2179	2041	1692	2069	1637	2050	
0.92	0.7081]	0.6376]	0.6709]	0.6751]	0.6728]	0.6538]	0.7400]	0.6530]	0.7753]	0.6552]	[10998 11344 2095 0.6826±0.0019]
	[12750	[9375	[10896	[11059	[10827	[9159	[12915	[9189	[14466	[9345	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2111	2283	2290	2192	2315	2138	1729	2129	1727	2037	
0.93	0.7093]	0.6395]	0.6693]	0.6744]	0.6675]	0.6496]	0.7384]	0.6505]	0.7715]	0.6558]	[10995 11344 2099 0.6824±0.0019]
	[12746	[9374	[10892	[11055	[10822	[9156	[12912	[9186	[14460	[9344	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2115	2284	2294	2196	2320	2141	1732	2132	1733	2038	
0.94	0.7091]	0.6395]	0.6692]	0.6743]	0.6673]	0.6495]	0.7383]	0.6503]	0.7712]	0.6557]	[10952 11344 2141 0.6807±0.0018]
	[12702	[9347	[10848	[11020	[10769	[9113	[12861	[9149	[14398	[9311	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2159	2311	2338	2231	2373	2184	1783	2169	1795	2071	
0.95	0.7074]	0.6384]	0.6674]	0.6729]	0.6652]	0.6477]	0.7361]	0.6488]	0.7686]	0.6543]	[10952 11344 2141 0.6807±0.0018]
	[12702	[9348	[10848	[11018	[10769	[9113	[12861	[9150	[14398	[9311	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2159	2310	2338	2233	2373	2184	1783	2168	1795	2071	
0.96	0.7074]	0.6385]	0.6674]	0.6728]	0.6652]	0.6477]	0.7361]	0.6488]	0.7686]	0.6543]	[10944 11344 2149 0.6804±0.0018]
	[12699	[9348	[10841	[11013	[10756	[9101	[12850	[9145	[14387	[9304	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2162	2310	2345	2238	2386	2196	1794	2173	1806	2078	
0.97	0.7073]	0.6385]	0.6671]	0.6726]	0.6647]	0.6472]	0.7356]	0.6486]	0.7681]	0.6540]	[10948 11344 2146 0.6805±0.0018]
	[12707	[9353	[10845	[11015	[10756	[9101	[12854	[9146	[14391	[9308	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2154	2305	2341	2236	2386	2196	1790	2172	1802	2074	
0.98	0.7076]	0.6387]	0.6673]	0.6727]	0.6647]	0.6472]	0.7358]	0.6486]	0.7683]	0.6542]	[10951 11344 2142 0.6806±0.0018]
	[12715	[9361	[10849	[11012	[10756	[9101	[12858	[9150	[14395	[9312	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2146	2297	2337	2239	2386	2196	1786	2168	1798	2070	
0.7079]	0.6390]	0.6675]	0.6726]	0.6647]	0.6472]	0.7360]	0.6488]	0.7685]	0.6543]		
<i>Chi-square method</i>											
0.55	[12629	[9269	[10871	[11016	[10914	[9221	[12888	[9204	[14473	[9286	[10977 11344 2116 0.6818±0.0019]
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2232	2389	2315	2235	2228	2076	1756	2114	1720	2096	
0.60	0.7045]	0.6354]	0.6683]	0.6728]	0.6709]	0.6523]	0.7373]	0.6511]	0.7718]	0.6532]	[10983 11344 2111 0.6820±0.0019]
	[12639	[9280	[10876	[11023	[10914	[9221	[12893	[9210	[14478	[9291	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
0.65	0.7049]	0.6358]	0.6685]	0.6730]	0.6709]	0.6523]	0.7375]	0.6514]	0.7720]	0.6535]	[10997 11344 2096 0.6826±0.0019]
	[12659	[9286	[10894	[11037	[10934	[9233	[12907	[9222	[14504	[9297	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2202	2372	2292	2214	2208	2064	1737	2096	1689	2085	
0.70	0.7057]	0.6360]	0.6692]	0.6736]	0.6717]	0.6528]	0.7381]	0.6519]	0.7731]	0.6537]	[11000 11344 2094 0.6827±0.0019]
	[12663	[9292	[10896	[11039	[10934	[9233	[12909	[9226	[14506	[9299	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2198	2366	2290	2212	2208	2064	1735	2092	1687	2083	
0.75	0.7059]	0.6363]	0.6693]	0.6737]	0.6717]	0.6528]	0.7382]	0.6520]	0.7732]	0.6538]	[11027 11344 2066 0.6838±0.0019]
	[12710	[9333	[10927	[11066	[10954	[9238	[12925	[9266	[14547	[9305	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2151	2325	2259	2185	2188	2059	1719	2052	1646	2077	
0.80	0.7077]	0.6379]	0.6705]	0.6747]	0.6725]	0.6530]	0.7388]	0.6538]	0.7750]	0.6540]	[10998 11344 2095 0.6826±0.0019]
	[12750	[9375	[10896	[11058	[10827	[9159	[12915	[9189	[14466	[9345	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2111	2283	2290	2193	2315	2138	1729	2129	1727	2037	

(continued on next page)

Table 10 (continued)

LPE_{OWA} orness(\mathbf{w}) = α	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
0.85	0.7093]	0.6395]	0.6693]	0.6744]	0.6675]	0.6496]	0.7384]	0.6505]	0.7715]	0.6558]	[10952 11344 2141 0.6807±0.0018]
	[12702	[9349	[10848	[11020	[10769	[9113	[12861	[9151	[14398	[9311	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2159	2309	2338	2231	2373	2184	1783	2167	1795	2071	
0.90	0.7074]	0.6385]	0.6674]	0.6729]	0.6652]	0.6477]	0.7361]	0.6488]	0.7686]	0.6543]	[10949 11344 2144 0.6806±0.0018]
	[12702	[9354	[10844	[11016	[10759	[9107	[12859	[9148	[14390	[9313	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2159	2304	2342	2235	2383	2190	1785	2170	1803	2069	
0.92	0.7074]	0.6387]	0.6673]	0.6728]	0.6648]	0.6474]	0.7360]	0.6487]	0.7683]	0.6544]	[10944 11344 2149 0.6804±0.0018]
	[12699	[9348	[10841	[11013	[10756	[9101	[12850	[9145	[14387	[9304	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2162	2310	2345	2238	2386	2196	1794	2173	1806	2078	
0.93	0.7073]	0.6385]	0.6671]	0.6726]	0.6647]	0.6472]	0.7356]	0.6486]	0.7681]	0.6540]	[10944 11344 2149 0.6804±0.0018]
	[12699	[9349	[10841	[11011	[10756	[9101	[12850	[9146	[14387	[9304	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2162	2309	2345	2240	2386	2196	1794	2172	1806	2078	
0.94	0.7073]	0.6385]	0.6671]	0.6726]	0.6647]	0.6472]	0.7356]	0.6486]	0.7681]	0.6540]	[10948 11344 2146 0.6805±0.0018]
	[12707	[9353	[10845	[11015	[10756	[9101	[12854	[9146	[14391	[9308	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2154	2305	2341	2236	2386	2196	1790	2172	1802	2074	
0.95	0.7076]	0.6387]	0.6673]	0.6727]	0.6647]	0.6472]	0.7358]	0.6486]	0.7683]	0.6542]	[10951 11344 2142 0.6806±0.0018]
	[12715	[9361	[10849	[11012	[10756	[9101	[12858	[9150	[14395	[9312	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2146	2297	2337	2239	2386	2196	1786	2168	1798	2070	
0.96	0.7079]	0.6390]	0.6675]	0.6726]	0.6647]	0.6472]	0.7360]	0.6488]	0.7685]	0.6543]	[10951 11344 2142 0.6806±0.0018]
	[12715	[9361	[10849	[11012	[10756	[9101	[12858	[9150	[14395	[9312	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2146	2297	2337	2239	2386	2196	1786	2168	1798	2070	
0.97	0.7079]	0.6390]	0.6675]	0.6726]	0.6647]	0.6472]	0.7360]	0.6488]	0.7685]	0.6543]	[10936 11344 2158 0.6800±0.0019]
	[12749	[9342	[10854	[11017	[10732	[9053	[12815	[9126	[14400	[9269	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2112	2316	2332	2234	2410	2244	1829	2192	1793	2113	
0.98	0.7093]	0.6382]	0.6677]	0.6728]	0.6637]	0.6451]	0.7341]	0.6478]	0.7687]	0.6525]	[10949 11344 2144 0.6805±0.0019]
	[12765	[9346	[10870	[11029	[10752	[9065	[12827	[9138	[14424	[9273	
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	
	2096	2312	2316	2222	2390	2232	1817	2180	1769	2109	
LPE_{AM}	0.7099]	0.6384]	0.6683]	0.6733]	0.6645]	0.6456]	0.7347]	0.6483]	0.7697]	0.6527]	[10660 11738 2039 0.6762 0.0061]
	[17646	[8823	[10715	[10715	[8914	[12516	[5221	[8914	[8823	[14317	
	6120 1649	14110	12645	12645	14205	9385	15530	12505	12410	7825	
	0.8147]	2482	2055	2055	2296	1559	2709	2041	2227	1318	
	0.6247]	0.6704]	0.6704]	0.6704]	0.6302]	0.7335]	0.5535]	0.6465]	0.6406]	0.7770]	

4. Experimental comparison and analysis

4.1. Ensemble with all 9 individual algorithms

In this section, we test the prediction performance (i.e., AUC) of LPE_{OWA} based on the social networks mentioned in Section 2.3. The 10-fold cross-validation is also used to conduct the experimental comparisons. The detailed results are summarized in Tables 8–11. According to the comparative results in Tables 8–11, we explain the experimental setup and further give our analysis. The optimal weights of MEM, MVM and CSM corresponding to 15 different optimal level factors (i.e., $\alpha=0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.92, 0.93, 0.94, 0.95, 0.96, 0.97$ and 0.98) are solved respectively by using LINGO software. The LPE_{OWAS} with different associated weights are evaluated on the same testing set. There is no special reason for the selection of α . We only want to check the impact of this parameter on the performance of LPE_{OWA} . In addition, we also conduct the comparison among LPE_{OWA} , arithmetic mean based link prediction ensemble (LPE_{AM}), and weighted arithmetic mean based link prediction ensemble (LPE_{WAM}). The illustrations based on the experimental results in Tables 8–11 are also presented in Figs. 8–11. By comparing the detailed results in Tables 8–11 with ones in Tables 4–7, we can get the following observations:

- The performances of LPE_{OWAS} on WSDP98, ChesLower and C96 networks are better than any individual local-information based link prediction algorithm. This indicates that our proposed ensemble strategy is not only feasible but also effective for these 3 networks. The ensemble with these 9 algorithms can not obtain higher AUCs on B97 network.
- For WSDP98, ChesLower and C96 networks (Figs. 8–10), AUCs of LPE_{OWAS} based on the associated weights solved with MEM, MVM and CSM all keep the tendencies of first increase and then decrease with the increase of α . The performances of MEM based LPE_{OWAS} are superior to MVM and CSM based ones. For B97 network, the AUCs of MEM, MVM and CSM based LPE_{OWAS} all decrease gradually with the increase of α . The performances of MVM based LPE_{OWAS} are better than MVM and CSM based ones.
- LPE_{OWA} is obviously better than LPE_{AM} . For example, the prediction accuracies of LPE_{AM} on WSDP98, ChesLower, C96, and B97 are 0.5776, 0.5086, 0.6762, and 0.7746 respectively. However, the lowest accuracies of LPE_{OWA} on these four networks, i.e., 0.7494, 0.7599, 0.6800, and 0.8056, are also higher than LPE_{AM} .
- In order to compare LPE_{OWA} with LPE_{WAM} , we generated 1000 different weight vectors to construct LPE_{WAM} . The comparative results on WSDP98 (Fig. 12) and ChesLower (Fig. 13) show that LPE_{OWA} is statistically better than LPE_{WAM} : the numbers of

Table 11
Prediction performances of LPE_{OWA} and LPE_{AM} algorithms on the network of Graph Drawing Contests Data-B97.

LPE _{OWA} orness(w) = α	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
<i>Maximum entropy method</i>											
0.55	[20048 431 3902 0.8311]	[20456 15 3910 0.8393]	[20070 447 3864 0.8323]	[20670 446 3265 0.8569]	[19487 16 3975 0.8304]	[20965 14 2499 0.8933]	[20176 13 3289 0.8596]	[19998 423 3057 0.8608]	[18953 870 3655 0.8258]	[19553 51 3874 0.8339]	[20038 273 3529 0.8463±0.0004]
0.60	[19984 431 3966 0.8285]	[20433 15 3933 0.8384]	[20028 447 3906 0.8306]	[20612 446 3323 0.8546]	[19448 16 4014 0.8287]	[20936 14 2528 0.8920]	[20133 13 3332 0.8578]	[19982 423 3073 0.8601]	[18939 870 3669 0.8252]	[19541 51 3886 0.8334]	[20004 273 3563 0.8449±0.0004]
0.65	[19909 431 4041 0.8254]	[20416 15 3950 0.8377]	[19968 447 3966 0.8282]	[20532 446 3403 0.8513]	[19412 16 4050 0.8272]	[20913 14 2551 0.8910]	[20077 13 3388 0.8554]	[19979 423 3076 0.8600]	[18936 870 3672 0.8251]	[19552 51 3875 0.8339]	[19969 273 3597 0.8435±0.0004]
0.70	[19808 431 4142 0.8213]	[20396 15 3970 0.8369]	[19841 447 4093 0.8230]	[20458 446 3477 0.8482]	[19356 16 4106 0.8248]	[20872 14 2592 0.8893]	[20018 13 3447 0.8529]	[19926 423 3129 0.8577]	[18924 870 3684 0.8246]	[19569 51 3858 0.8346]	[19917 273 3650 0.8413±0.0005]
0.75	[19748 431 4202 0.8188]	[20322 15 4044 0.8338]	[19788 447 4146 0.8208]	[20433 446 3502 0.8472]	[19329 16 4133 0.8236]	[20853 14 2611 0.8885]	[19959 13 3506 0.8504]	[19899 423 3156 0.8566]	[18906 870 3702 0.8238]	[19563 51 3864 0.8343]	[19880 273 3687 0.8398±0.0005]
0.80	[19646 431 4304 0.8146]	[20292 15 4074 0.8326]	[19730 447 4204 0.8184]	[20261 446 3674 0.8402]	[19284 16 4178 0.8217]	[20798 14 2666 0.8861]	[19852 13 3613 0.8458]	[19809 423 3246 0.8527]	[18855 870 3753 0.8216]	[19503 51 3924 0.8318]	[19803 273 3764 0.8366±0.0005]
0.85	[19420 431 4530 0.8054]	[20215 15 4151 0.8294]	[19592 447 4342 0.8127]	[20157 446 3778 0.8359]	[19181 16 4281 0.8173]	[20709 14 2755 0.8824]	[19714 13 3751 0.8400]	[19725 423 3330 0.8492]	[18800 870 3808 0.8193]	[19467 51 3969 0.8302]	[19698 273 3869 0.8322±0.0005]
0.90	[19251 431 4699 0.7984]	[20039 15 4327 0.8222]	[19447 447 4487 0.8068]	[19953 446 3982 0.8275]	[19027 16 4435 0.8108]	[20602 14 2862 0.8778]	[19523 13 3942 0.8318]	[19595 423 3460 0.8436]	[18751 870 3857 0.8172]	[19424 51 4003 0.8284]	[19561 273 4005 0.8265±0.0005]
0.92	[19106 431 4844 0.7925]	[19952 15 4414 0.8186]	[19360 447 4574 0.8032]	[19884 446 4051 0.8247]	[18929 16 4533 0.8066]	[20536 14 2928 0.8750]	[19414 13 4051 0.8272]	[19524 423 3531 0.8406]	[18710 870 3898 0.8154]	[19391 51 4036 0.8270]	[19481 273 4086 0.8231±0.0005]
0.93	[19041 431 4909 0.7898]	[19908 15 4458 0.8168]	[19341 447 4593 0.8024]	[19830 446 4105 0.8225]	[18892 16 4570 0.8050]	[20505 14 2959 0.8737]	[19332 13 4133 0.8237]	[19477 423 3578 0.8386]	[18692 870 3916 0.8147]	[19372 51 4055 0.8262]	[19439 273 4128 0.8213±0.0005]
0.94	[19003 431 4947 0.7883]	[19855 15 4511 0.8147]	[19305 447 4629 0.8010]	[19790 446 4145 0.8208]	[18855 16 4607 0.8034]	[20436 14 3028 0.8707]	[19292 13 4173 0.8220]	[19377 423 3678 0.8343]	[18643 870 3965 0.8126]	[19344 51 4083 0.8250]	[19390 273 4177 0.8193±0.0005]
0.95	[18909 431 5041 0.7844]	[19812 15 4554 0.8129]	[19228 447 4706 0.7978]	[19738 446 4197 0.8187]	[18814 16 4648 0.8017]	[20373 14 3091 0.8680]	[19239 13 4226 0.8197]	[19324 423 3731 0.8321]	[18615 870 3993 0.8114]	[19316 51 4111 0.8238]	[19337 273 4230 0.8171±0.0005]
0.96	[18765 431 5185 0.7785]	[19699 15 4667 0.8083]	[19152 447 4782 0.7947]	[19660 446 4275 0.8155]	[18742 16 4720 0.7986]	[20324 14 3140 0.8660]	[19174 13 4291 0.8170]	[19252 423 3803 0.8290]	[18581 870 4027 0.8099]	[19277 51 4150 0.8222]	[19263 273 4304 0.8140±0.0005]
0.97	[18696 431 5254 0.7757]	[19623 15 4743 0.8052]	[19057 447 4877 0.7908]	[19534 446 4401 0.8103]	[18647 16 4815 0.7946]	[20253 14 3211 0.8629]	[19065 13 4400 0.8123]	[19189 423 3866 0.8263]	[18534 870 4074 0.8079]	[19256 51 4171 0.8213]	[19185 273 4381 0.8107±0.0006]
0.98	[18557 431 5393 0.7700]	[19523 15 4843 0.8011]	[18990 447 4944 0.7881]	[19437 446 4498 0.8064]	[18567 16 4895 0.7912]	[20215 14 3249 0.8613]	[18955 13 4510 0.8076]	[19125 423 3930 0.8236]	[18516 870 4092 0.8072]	[19231 51 4196 0.8202]	[19112 273 4455 0.8077±0.0006]
<i>Minimum variance method</i>											
0.55	[20095 431 3855 0.8330]	[20489 15 3877 0.8407]	[20082 447 3852 0.8328]	[20719 446 3216 0.8589]	[19493 16 3969 0.8306]	[21000 14 2464 0.8948]	[20235 13 3230 0.8621]	[20013 423 3042 0.8614]	[18964 870 3644 0.8263]	[19562 51 3865 0.8343]	[20065 273 3501 0.8475±0.0005]
0.60	[20095 431 3855 0.8330]	[20489 15 3877 0.8407]	[20082 447 3852 0.8328]	[20719 446 3216 0.8589]	[19493 16 3969 0.8306]	[21000 14 2464 0.8948]	[20235 13 3230 0.8621]	[20013 423 3042 0.8614]	[18964 870 3644 0.8263]	[19562 51 3865 0.8343]	[20065 273 3501 0.8475±0.0005]
0.65	[20095 431 3855 0.8330]	[20489 15 3877 0.8407]	[20082 447 3852 0.8328]	[20719 446 3216 0.8589]	[19493 16 3969 0.8306]	[21000 14 2464 0.8948]	[20235 13 3230 0.8621]	[20013 423 3042 0.8614]	[18964 870 3644 0.8263]	[19562 51 3865 0.8343]	[20065 273 3501 0.8475±0.0005]
0.70	[20095 431 3855 0.8330]	[20489 15 3877 0.8407]	[20082 447 3852 0.8328]	[20719 446 3216 0.8589]	[19493 16 3969 0.8306]	[21000 14 2464 0.8948]	[20235 13 3230 0.8621]	[20013 423 3042 0.8614]	[18964 870 3644 0.8263]	[19562 51 3865 0.8343]	[20065 273 3501 0.8475±0.0005]
0.75	[20074 431 3876 0.8322]	[20437 15 3929 0.8385]	[20056 447 3878 0.8318]	[20710 446 3225 0.8586]	[19492 16 3970 0.8306]	[21004 14 2460 0.8949]	[20215 13 3250 0.8613]	[19993 423 3062 0.8606]	[18962 870 3646 0.8262]	[19565 51 3862 0.8344]	[20051 273 3516 0.8469±0.0005]
0.80	[20061 431 3889 0.8317]	[20464 15 3902 0.8396]	[20096 447 3838 0.8334]	[20708 446 3227 0.8585]	[19494 16 3968 0.8306]	[20972 14 2492 0.8936]	[20186 13 3279 0.8601]	[19993 423 3062 0.8606]	[18963 870 3645 0.8262]	[19545 51 3882 0.8336]	[20048 273 3518 0.8468±0.0004]
0.85	[20006 431 3944 0.8294]	[20449 15 3917 0.8390]	[20036 447 3898 0.8310]	[20614 446 3321 0.8546]	[19488 16 3974 0.8304]	[20923 14 2541 0.8915]	[20146 13 3319 0.8584]	[20020 423 3035 0.8617]	[18955 870 3653 0.8259]	[19589 51 3838 0.8354]	[20023 273 3544 0.8457±0.0004]
0.90	[19832 431 4118 0.8223]	[20399 15 3967 0.8370]	[19885 447 4049 0.8248]	[20417 446 3518 0.8466]	[19396 16 4066 0.8265]	[20840 14 2624 0.8879]	[19955 13 3510 0.8502]	[20033 423 3022 0.8623]	[18958 870 3650 0.8260]	[19577 51 3850 0.8349]	[19929 273 3637 0.8418±0.0004]
0.92	[19637 431 4313	[20320 15 4046	[19752 447 4182	[20304 446 3631	[19326 16 4136	[20763 14 2701	[19811 13 3654	[19947 423 3108	[18915 870 3693	[19553 51 3874	[19833 273 3734

(continued on next page)

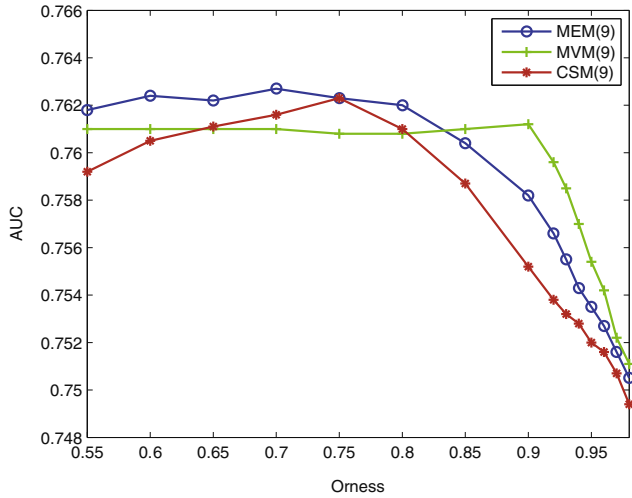


Fig. 8. Comparative results of LPE_{OWA_S} on WSDP98.

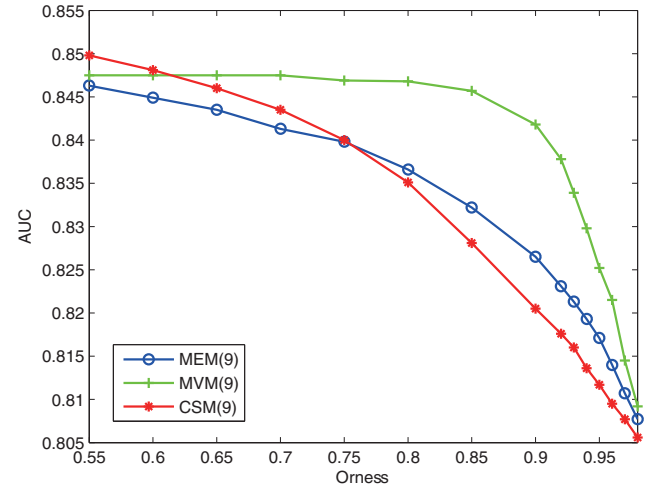


Fig. 11. Comparative results of LPE_{OWA_S} on B97.

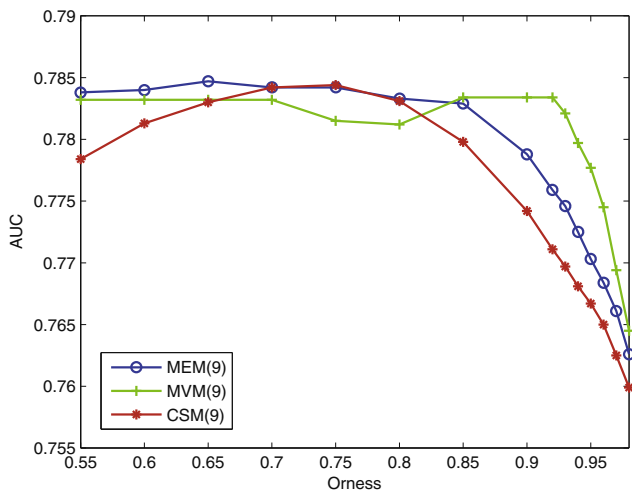


Fig. 9. Comparative results of LPE_{OWA_S} on ChesLower.

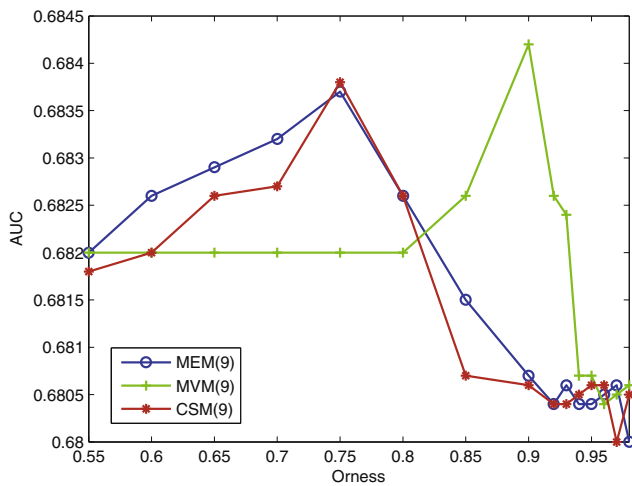


Fig. 10. Comparative results of LPE_{OWA_S} on C96.

LPE_{OWA} obtaining higher AUCs than LPE_{WAM} are 992 and 991 respectively on WSDP98 and ChesLower networks. The experimental results on WSDP98 as shown in Fig. 12 present that

there are 992 and 910 pentagrams (AUCs of LPE_{WAM}) which are located below red line (Maximal AUCs of LPE_{OWA}) and black line (Minimal AUCs of LPE_{OWA}) respectively. And, the results on ChesLower as shown in Fig. 13 show that there are 991 and 898 pentagrams which are located below red line and black line respectively. This reflects that the lowest prediction accuracies of LPE_{OWA} are also obviously better than LPE_{WAM} . The experiment tells us that the exhaustive search must find the optimal weight vectors to design the link prediction ensemble, but it is infeasible due to high running time. OWA can indeed provide us a feasible and efficient weight vector to aggregate different individual prediction algorithms with acceptable accuracy and low computational complexity.

Three advantages of LPE_{OWA} can be concluded by summarizing these experimental results: (1) LPE_{OWA} obtains the higher prediction accuracies compared with any individual algorithm through increasing the numbers (i.e., n_1 s) of missing links having higher scores. For example, n_1 s on any fold in Tables 4–6 are larger than the corresponding ones in Tables 8–10. (2) LPE_{OWA} reduces the possibility that user selects a weak link prediction algorithm and thus improve the high variability. (3) LPE_{OWA} is more stable in comparison with individual algorithm because of the lower prediction variances. In addition, the computational complexity of LPE_{OWA} is $O(\|V\|)$ which is same as the individual algorithm. The selection of parameter α plays a positive impact on the performance of LPE_{OWA} , i.e., the larger α gives rise to higher prediction accuracy through more emphasizing the individual algorithm with higher probability.

4.2. Ensemble with some of 9 individual algorithms

Our experiments also test the performances of three different kinds of ensemble algorithms: the ensemble of sn_{xy}^{CN} , sn_{xy}^{HPI} , sn_{xy}^{Salton} , $sn_{xy}^{Sorensen}$, sn_{xy}^{HDI} , $sn_{xy}^{Jaccard}$ and sn_{xy}^{LHN-1} , the ensemble of sn_{xy}^{AA} , sn_{xy}^{HPI} , sn_{xy}^{Salton} , $sn_{xy}^{Sorensen}$, sn_{xy}^{HDI} , $sn_{xy}^{Jaccard}$ and sn_{xy}^{LHN-1} , and the ensemble of sn_{xy}^{RA} , sn_{xy}^{HPI} , sn_{xy}^{Salton} , $sn_{xy}^{Sorensen}$, sn_{xy}^{HDI} , $sn_{xy}^{Jaccard}$ and sn_{xy}^{LHN-1} . We denote these 3 ensemble algorithms as $LPE_{OWA}(7a)$, $LPE_{OWA}(7b)$ and $LPE_{OWA}(7c)$ respectively. In fact, we can find that $LPE_{OWA}(7a)$ is the ensemble of a number of common neighbors of x and y and degrees of x and y , while $LPE_{OWA}(7b)$ and $LPE_{OWA}(7c)$ are the ensembles of degrees of x and y and degrees of common neighbors of x and y .

Here, we only consider using MEM in Eq. (19) to determine the associate weights for OWA operator due to its better performance

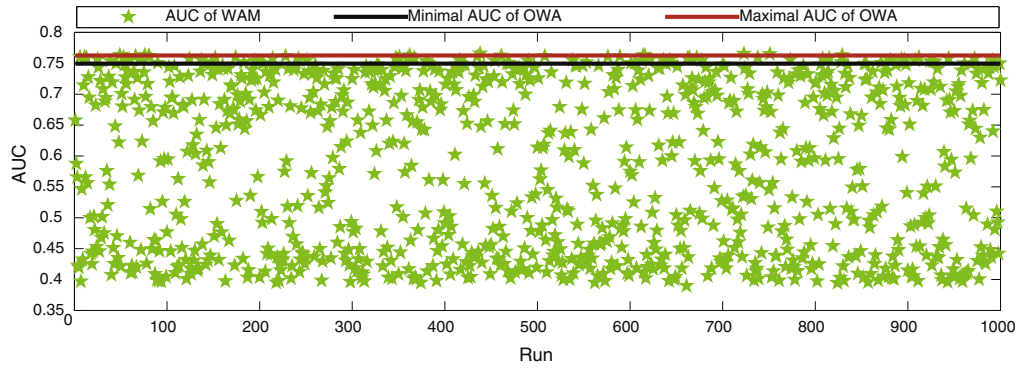


Fig. 12. 1000 AUCs of LPE_{WAM} on WSDP98.

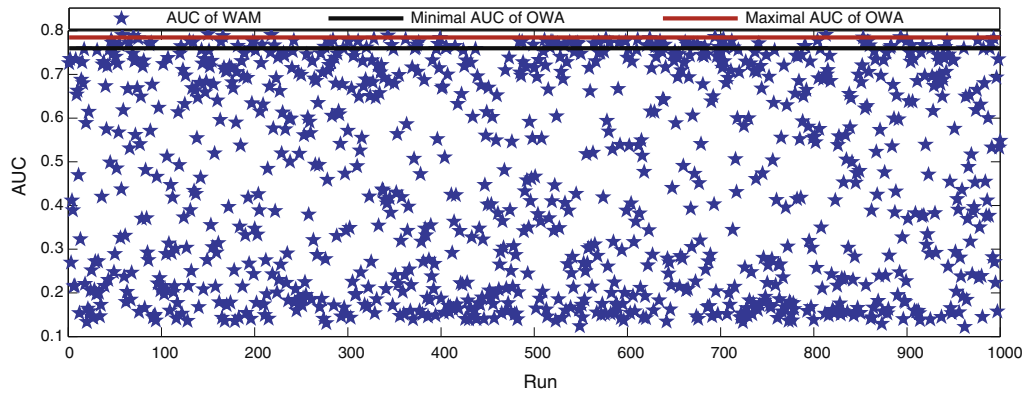


Fig. 13. 1000 AUCs of LPE_{WAM} on ChesLower.

Table 12
Prediction performances of LPE_{OWA}(7a) algorithm on 4 social networks.

Orness(w) = α	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
<i>WSDP98 network</i>											
0.55	[4512 343 869	[3983 509 1232	[4218 508 998	[4689 192 843	[4643 7 1074	[4126 511 1087	[3879 362 1483	[4226 344 1154	[3684 508 1055	[3096 691 1460	[4106 398 1126 0.7637±0.0029]
	0.8182]	0.7403]	0.7813]	0.8360]	0.8118]	0.7655]	0.7093]	0.7683]	0.7505]	0.6559]	
0.60	[4517 343 864	[3984 509 1231	[4219 508 997	[4693 192 839	[4648 7 1069	[4137 511 1076	[3876 362 1486	[4234 344 1146	[3688 508 1051	[3096 691 1460	[4109 398 1122 0.7643±0.0029]
	0.8191]	0.7405]	0.7814]	0.8367]	0.8126]	0.7674]	0.7088]	0.7697]	0.7513]	0.6559]	
0.65	[4520 343 861	[3979 509 1236	[4220 508 996	[4699 192 833	[4651 7 1066	[4137 511 1076	[3882 362 1480	[4241 344 1139	[3684 508 1055	[3095 691 1461	[4111 398 1120 0.7646±0.0030]
	0.8196]	0.7396]	0.7816]	0.8377]	0.8132]	0.7674]	0.7098]	0.7710]	0.7505]	0.6557]	
0.70	[4532 343 849	[3983 509 1232	[4220 508 996	[4710 192 822	[4668 7 1049	[4140 511 1073	[3885 362 1477	[4245 344 1135	[3683 508 1056	[3095 691 1461	[4116 398 1115 0.7655±0.0031]
	0.8217]	0.7403]	0.7816]	0.8396]	0.8161]	0.7679]	0.7103]	0.7717]	0.7503]	0.6557]	
0.75	[4543 343 838	[3986 509 1229	[4222 508 994	[4734 192 798	[4667 7 1050	[4147 511 1066	[3889 362 1473	[4248 344 1132	[3686 508 1053	[3092 691 1464	[4121 398 1110 0.7665±0.0031]
	0.8236]	0.7408]	0.7820]	0.8438]	0.8160]	0.7691]	0.7110]	0.7722]	0.7509]	0.6551]	
0.80	[4543 343 838	[3978 509 1237	[4229 508 987	[4739 192 793	[4683 7 1034	[4152 511 1061	[3891 362 1471	[4247 344 1133	[3684 508 1055	[3097 691 1459	[4124 398 1107 0.7670±0.0032]
	0.8236]	0.7394]	0.7832]	0.8447]	0.8187]	0.7700]	0.7114]	0.7720]	0.7505]	0.6561]	
0.85	[4545 343 836	[3974 509 1241	[4232 508 984	[4753 192 779	[4682 7 1035	[4154 511 1059	[3890 362 1472	[4248 344 1132	[3681 508 1058	[3093 691 1463	[4125 398 1106 0.7671±0.0033]
	0.8240]	0.7387]	0.7837]	0.8471]	0.8186]	0.7704]	0.7112]	0.7722]	0.7500]	0.6553]	
0.90	[4559 343 822	[3977 509 1238	[4229 508 987	[4754 192 778	[4714 7 1003	[4162 511 1051	[3890 362 1472	[4277 344 1103	[3680 508 1059	[3085 691 1471	[4133 398 1098 0.7684±0.0034]
	0.8264]	0.7393]	0.7832]	0.8473]	0.8242]	0.7718]	0.7112]	0.7773]	0.7498]	0.6538]	
0.92	[4558 343 823	[3984 509 1231	[4227 508 989	[4759 192 773	[4722 7 995	[4160 511 1053	[3891 362 1471	[4279 344 1101	[3674 508 1065	[3078 691 1478	[4133 398 1098 0.7685±0.0035]
	0.8263]	0.7405]	0.7828]	0.8482]	0.8256]	0.7714]	0.7114]	0.7776]	0.7486]	0.6525]	
0.93	[4559 343 822	[3981 509 1234	[4227 508 989	[4760 192 772	[4730 7 987	[4158 511 1055	[3886 362 1476	[4271 344 1109	[3672 508 1067	[3078 691 1478	[4132 398 1099 0.7683±0.0035]
	0.8264]	0.7400]	0.7828]	0.8484]	0.8270]	0.7711]	0.7105]	0.7762]	0.7482]	0.6525]	
0.94	[4562 343	[3984 509	[4225 508	[4769 192	[4735 7	[4157 511	[3888 362	[4266 344	[3661 508	[3079 691	[4133 398 1099

Table 12 (continued)

Orness(\mathbf{w}) = α	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
0.93	[20554	[20935	[20432	[20846	[19866	[21080	[20645	[20357	[19081	[19718	[20351 397
	516 3311	277 3169	573 3376	538 2997	153 3459	136 2262	134 2699	467 2654	930 3467	250 3510	3090
0.94	[0.8536]	[0.8643]	[0.8498]	[0.8660]	[0.8494]	[0.9008]	[0.8822]	[0.8770]	[0.8325]	[0.8452]	[0.8621±0.0004]
	[20545	[20908	[20415	[20845	[19899	[21110	[20649	[20394	[19081	[19744	[20359 397
0.95	516 3320	277 3196	573 3393	538 2998	153 3426	136 2232	134 2695	467 2617	930 3467	250 3484	3083
	[0.8532]	[0.8632]	[0.8491]	[0.8660]	[0.8508]	[0.9020]	[0.8824]	[0.8786]	[0.8325]	[0.8463]	[0.8624±0.0004]
0.96	[20521	[20908	[20406	[20833	[19878	[21092	[20625	[20401	[19086	[19736	[20349 397
	516 3344	277 3196	573 3402	538 3010	153 3447	136 2250	134 2719	467 2610	930 3462	250 3492	3093
0.97	[0.8523]	[0.8632]	[0.8487]	[0.8655]	[0.8499]	[0.9013]	[0.8813]	[0.8789]	[0.8327]	[0.8459]	[0.8620±0.0004]
	[20503	[20893	[20395	[20810	[19895	[21099	[20604	[20412	[19068	[19733	[20341 397
0.98	516 3362	277 3211	573 3413	538 3033	153 3430	136 2243	134 2740	467 2599	930 3480	250 3495	3101
	[0.8515]	[0.8626]	[0.8483]	[0.8646]	[0.8506]	[0.9016]	[0.8804]	[0.8794]	[0.8320]	[0.8458]	[0.8617±0.0004]

in comparison with MVM and CSM in the aforementioned experiments. The 10-fold cross-validation is also performed for the 4 above-mentioned social networks. The experimental results are summarized in Tables 12–14. The vivid comparisons corresponding to the 4 networks are depicted in Figs. 14–17, where MEM(9) denotes the MEM based LPE_{OWA}, MEM(7a), MEM(7b) and MEM(7c) denote MEM based LPE_{OWA}(7a), LPE_{OWA}(7b) and LPE_{OWA}(7c) respectively. By observing Tables 12–14 and Figs. 14–17, we can find that

- For WSDP98 and ChesLower networks (Figs. 14 and 15), LPE_{OWA}(7a) further improves the performances of link prediction ensemble algorithm, i.e. MEM based LPE_{OWA}(7a) obtains the higher AUCs than MEM based LPE_{OWA}.
- For C96 network (Fig. 16), AUCs of MEM based LPE_{OWA}(7a), LPE_{OWA}(7b) and LPE_{OWA}(7c) gradually increase with the increase of α . MEM based LPE_{OWA} obtains the better performances than other three ensemble algorithms.

Table 13 Prediction performances of LPE_{OWA}(7b) algorithm on 4 social networks.

Orness(\mathbf{w}) = α	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
<i>WSDP98 network</i>											
0.55	[4046 339	[3709	[3843 507	[4285 170	[4352 3	[3872	[3640	[3916	[3438	[2913	[3801 390 1437
	1339	507 1508	1374	1269	1369	507 1345	343 1741	339 1469	507 1302	676 1658	0.7092±0.0019
0.60	[0.7365]	[0.6923]	[0.7157]	[0.7635]	[0.7606]	[0.7207]	[0.6659]	[0.7137]	[0.7035]	[0.6196]	[3800 390 1439
	[4034 339	[3702	[3851 507	[4292 170	[4348 3	[3872	[3644	[3909	[3442	[2904	[3800 390 1439
0.65	1351	507 1515	1366	1262	1373	507 1345	343 1737	339 1476	507 1298	676 1667	0.7089±0.0019
	[0.7344]	[0.6910]	[0.7171]	[0.7647]	[0.7599]	[0.7207]	[0.6666]	[0.7125]	[0.7043]	[0.6179]	[3795 390 1444
0.70	[4021 339	[3690	[3861 507	[4292 170	[4351 3	[3875	[3636	[3893	[3443	[2888	[3795 390 1444
	1364	507 1527	1356	1262	1370	507 1342	343 1745	339 1492	507 1297	676 1683	0.7080±0.0020
0.75	[0.7321]	[0.6889]	[0.7188]	[0.7647]	[0.7604]	[0.7213]	[0.6652]	[0.7097]	[0.7045]	[0.6148]	[3794 390 1445
	[4013 339	[3679	[3865 507	[4297 170	[4367 3	[3882	[3625	[3898	[3448	[2867	[3794 390 1445
0.80	1372	507 1538	1352	1257	1354	507 1335	343 1756	339 1487	507 1292	676 1704	0.7079±0.0021
	[0.7307]	[0.6870]	[0.7195]	[0.7655]	[0.7632]	[0.7225]	[0.6633]	[0.7106]	[0.7055]	[0.6108]	[3794 390 1445
0.85	[4004 339	[3682	[3869 507	[4289 170	[4379 3	[3888	[3613	[3903	[3451	[2861	[3794 390 1445
	1381	507 1535	1348	1265	1342	507 1329	343 1768	339 1482	507 1289	676 1710	0.7078±0.0022
0.90	[0.7291]	[0.6875]	[0.7202]	[0.7642]	[0.7653]	[0.7235]	[0.6612]	[0.7115]	[0.7060]	[0.6097]	[3794 390 1445
	[3990 339	[3679	[3873 507	[4298 170	[4393 3	[3899	[3597	[3917	[3454	[2842	[3794 390 1445
0.95	1395	507 1538	1344	1256	1328	507 1318	343 1784	339 1468	507 1286	676 1729	0.7078±0.0023
	[0.7267]	[0.6870]	[0.7209]	[0.7657]	[0.7677]	[0.7255]	[0.6584]	[0.7139]	[0.7066]	[0.6061]	[3797 390 1442
0.90	[3975 339	[3678	[3882 507	[4309 170	[4420 3	[3902	[3582	[3924	[3469	[2826	[3797 390 1442
	1410	507 1539	1335	1245	1301	507 1315	343 1799	339 1461	507 1271	676 1745	0.7083±0.0025
0.92	[0.7241]	[0.6868]	[0.7225]	[0.7676]	[0.7724]	[0.7260]	[0.6557]	[0.7151]	[0.7095]	[0.6030]	[3802 390 1437
	[3966 339	[3690	[3884 507	[4337 170	[4445 3	[3911	[3593	[3926	[3448	[2818	[3802 390 1437
0.93	1419	507 1527	1333	1217	1276	507 1306	343 1788	339 1459	507 1292	676 1753	0.7091±0.0027
	[0.7225]	[0.6889]	[0.7228]	[0.7725]	[0.7768]	[0.7276]	[0.6577]	[0.7155]	[0.7055]	[0.6015]	[3803 390 1435
0.94	[3967 339	[3690	[3894 507	[4345 170	[4452 3	[3913	[3585	[3936	[3436	[2816	[3803 390 1435
	1418	507 1527	1323	1209	1269	507 1304	343 1796	339 1449	507 1304	676 1755	0.7094±0.0027
0.95	[0.7227]	[0.6889]	[0.7246]	[0.7739]	[0.7780]	[0.7279]	[0.6563]	[0.7172]	[0.7032]	[0.6011]	[3804 390 1435
	[3969 339	[3692	[3896 507	[4350 170	[4453 3	[3911	[3589	[3936	[3435	[2811	[3804 390 1435
0.96	1416	507 1525	1321	1204	1268	507 1306	343 1792	339 1449	507 1305	676 1760	0.7095±0.0028
	[0.7230]	[0.6893]	[0.7249]	[0.7748]	[0.7782]	[0.7276]	[0.6570]	[0.7172]	[0.7030]	[0.6002]	[3805 390 1434
0.97	[3973 339	[3694	[3895 507	[4350 170	[4459 3	[3908	[3582	[3940	[3433	[2812	[3805 390 1434
	1412	507 1523	1322	1204	1262	507 1309	343 1799	339 1445	507 1307	676 1759	0.7096±0.0028
0.98	[0.7237]	[0.6896]	[0.7248]	[0.7748]	[0.7793]	[0.7270]	[0.6557]	[0.7179]	[0.7026]	[0.6003]	[3809 390 1430
	[3976 339	[3693	[3890 507	[4357 170	[4467 3	[3921	[3582	[3952	[3436	[2811	[3809 390 1430
0.99	1409	507 1524	1327	1197	1254	507 1296	343 1799	339 1433	507 1304	676 1760	0.7103±0.0029
	[0.7242]	[0.6895]	[0.7239]	[0.7760]	[0.7807]	[0.7293]	[0.6557]	[0.7200]	[0.7032]	[0.6002]	[3809 390 1430

(continued on next page)

Table 13 (continued)

Orness(w) = α	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
0.96	[3983 339 1402 0.7255]	[3693 507 1524 0.6895]	[3895 507 1322 0.7248]	[4361 170 1193 0.7767]	[4467 3 1254 0.7807]	[3922 507 1295 0.7295]	[3574 343 1807 0.6544]	[3956 339 1429 0.7207]	[3441 507 1299 0.7041]	[2805 676 1766 0.5990]	[3810 390 1429 0.7105±0.0029]
0.97	[3979 339 1406 0.7248]	[3695 507 1522 0.6898]	[3900 507 1317 0.7256]	[4363 170 1191 0.7771]	[4479 3 1242 0.7828]	[3929 507 1288 0.7307]	[3572 343 1809 0.6540]	[3966 339 1419 0.7225]	[3440 507 1300 0.7039]	[2802 676 1769 0.5984]	[3813 390 1426 0.7110±0.0030]
0.98	[3975 339 1410 0.7241]	[3693 507 1524 0.6895]	[3907 507 1310 0.7269]	[4371 170 1183 0.7785]	[4494 3 1227 0.7854]	[3928 507 1289 0.7305]	[3566 343 1815 0.6530]	[3975 339 1410 0.7241]	[3439 507 1301 0.7037]	[2795 676 1776 0.5971]	[3814 390 1425 0.7113±0.0031]
<i>ChesLower network</i>											
0.55	[6730 3 1750 0.7935]	[5227 10 3246 0.6168]	[6881 10 1592 0.8117]	[6339 14 2130 0.7481]	[5660 6 2817 0.6676]	[6682 3 1798 0.7879]	[6840 4 1639 0.8066]	[6274 4 1706 0.7861]	[5612 7 2365 0.7033]	[5532 9 2443 0.6934]	[6178 7 2149 0.7415±0.0045]
0.60	[6735 3 1745 0.7941]	[5238 10 3235 0.6181]	[6908 10 1565 0.8149]	[6350 14 2119 0.7494]	[5688 6 2789 0.6709]	[6698 3 1782 0.7898]	[6848 4 1631 0.8075]	[6282 4 1698 0.7871]	[5615 7 2362 0.7037]	[5524 9 2451 0.6924]	[6189 7 2138 0.7428±0.0046]
0.65	[6751 3 1729 0.7960]	[5281 10 3192 0.6231]	[6949 10 1524 0.8198]	[6363 14 2106 0.7509]	[5689 6 2788 0.6710]	[6733 3 1747 0.7939]	[6826 4 1653 0.8049]	[6284 4 1696 0.7873]	[5586 7 2391 0.7001]	[5539 9 2436 0.6943]	[6200 7 2126 0.7441±0.0045]
0.70	[6760 3 1720 0.7971]	[5284 10 3189 0.6235]	[6981 10 1492 0.8235]	[6370 14 2099 0.7517]	[5700 6 2777 0.6723]	[6749 3 1731 0.7958]	[6822 4 1657 0.8044]	[6305 4 1675 0.7900]	[5590 7 2387 0.7006]	[5535 9 2440 0.6938]	[6210 7 2117 0.7453±0.0046]
0.75	[6775 3 1705 0.7988]	[5305 10 3168 0.6260]	[7021 10 1452 0.8282]	[6365 14 2104 0.7511]	[5736 6 2741 0.6765]	[6775 3 1705 0.7988]	[6797 4 1682 0.8015]	[6336 4 1644 0.7938]	[5588 7 2389 0.7003]	[5542 9 2433 0.6947]	[6224 7 2102 0.7470±0.0046]
0.80	[6791 3 1689 0.8007]	[5344 10 3129 0.6306]	[7034 10 1439 0.8298]	[6379 14 2090 0.7528]	[5774 6 2703 0.6810]	[6786 3 1694 0.8001]	[6767 4 1712 0.7979]	[6362 4 1618 0.7971]	[5603 7 2374 0.7022]	[5564 9 2411 0.6975]	[6240 7 2086 0.7490±0.0044]
0.85	[6814 3 1666 0.8034]	[5404 10 3069 0.6376]	[7081 10 1392 0.8353]	[6421 14 2048 0.7578]	[5813 6 2664 0.6856]	[6832 3 1648 0.8056]	[6742 4 1737 0.7950]	[6378 4 1602 0.7991]	[5627 7 2350 0.7052]	[5571 9 2404 0.6983]	[6268 7 2058 0.7523±0.0043]
0.90	[6864 3 1616 0.8093]	[5440 10 3033 0.6419]	[7122 10 1351 0.8402]	[6436 14 2033 0.7595]	[5832 6 2645 0.6878]	[6860 3 1620 0.8089]	[6713 4 1766 0.7916]	[6382 4 1598 0.7996]	[5656 7 2321 0.7089]	[5555 9 2420 0.6963]	[6286 7 2040 0.7544±0.0044]
0.92	[6886 3 1594 0.8119]	[5467 10 3006 0.6451]	[7146 10 1327 0.8430]	[6435 14 2034 0.7594]	[5846 6 2631 0.6895]	[6854 3 1626 0.8081]	[6687 4 1792 0.7885]	[6381 4 1599 0.7995]	[5663 7 2314 0.7097]	[5555 9 2420 0.6963]	[6292 7 2034 0.7551±0.0043]
0.93	[6883 3 1597 0.8116]	[5478 10 2995 0.6464]	[7169 10 1304 0.8457]	[6442 14 2027 0.7602]	[5841 6 2636 0.6889]	[6856 3 1624 0.8084]	[6684 4 1795 0.7882]	[6377 4 1603 0.7990]	[5677 7 2300 0.7115]	[5561 9 2414 0.6971]	[6297 7 2030 0.7557±0.0043]
0.94	[6910 3 1570 0.8147]	[5499 10 2974 0.6488]	[7173 10 1300 0.8462]	[6440 14 2029 0.7600]	[5841 6 2636 0.6889]	[6861 3 1619 0.8090]	[6673 4 1806 0.7869]	[6376 4 1604 0.7988]	[5694 7 2283 0.7136]	[5572 9 2403 0.6985]	[6304 7 2022 0.7565±0.0043]
0.95	[6931 3 1549 0.8172]	[5514 10 2959 0.6506]	[7172 10 1301 0.8460]	[6465 14 2004 0.7629]	[5854 6 2623 0.6904]	[6869 3 1611 0.8099]	[6679 4 1800 0.7876]	[6390 4 1590 0.8006]	[5704 7 2273 0.7149]	[5569 9 2406 0.6981]	[6315 7 2012 0.7578±0.0042]
0.96	[6941 3 1539 0.8184]	[5533 10 2940 0.6528]	[7172 10 1301 0.8460]	[6467 14 2002 0.7632]	[5871 6 2606 0.6924]	[6873 3 1607 0.8104]	[6664 4 1815 0.7858]	[6404 4 1576 0.8024]	[5680 7 2297 0.7119]	[5561 9 2414 0.6971]	[6317 7 2010 0.7580±0.0042]
0.97	[6956 3 1524 0.8202]	[5544 10 2929 0.6541]	[7176 10 1297 0.8465]	[6460 14 2009 0.7623]	[5874 6 2603 0.6928]	[6875 3 1605 0.8106]	[6614 4 1865 0.7799]	[6400 4 1580 0.8019]	[5681 7 2296 0.7120]	[5563 9 2412 0.6973]	[6314 7 2012 0.7578±0.0042]
0.98	[6964 3 1516 0.8211]	[5560 10 2913 0.6560]	[7192 10 1281 0.8484]	[6466 14 2003 0.7631]	[5874 6 2603 0.6928]	[6862 3 1618 0.8091]	[6594 4 1885 0.7776]	[6398 4 1582 0.8016]	[5684 7 2293 0.7124]	[5566 9 2409 0.6977]	[6316 7 2010 0.7580±0.0042]
<i>C96 network</i>											
0.55	[12513 10554 2348 0.7000]	[9192 13757 2466 0.6323]	[10775 12229 2411 0.6645]	[10927 12164 2324 0.6693]	[10820 12273 2322 0.6672]	[9159 12163 2138 0.6496]	[12802 8816 1842 0.7336]	[9115 12142 2203 0.6473]	[14341 7267 1852 0.7662]	[9236 12078 2146 0.6511]	[10888 11344 2205 0.6781±0.0018]
0.60	[12526 10554 2335 0.7005]	[9194 13757 2464 0.6324]	[10780 12229 2406 0.6647]	[10932 12164 2319 0.6694]	[10817 12273 2325 0.6671]	[9153 12163 2144 0.6494]	[12801 8816 1843 0.7335]	[9112 12142 2206 0.6472]	[14346 7267 1847 0.7664]	[9235 12078 2147 0.6511]	[10890 11344 2204 0.6782±0.0018]
0.65	[12554 10554 2307 0.7016]	[9201 13757 2457 0.6327]	[10808 12229 2378 0.6658]	[10953 12164 2298 0.6703]	[10852 12273 2290 0.6684]	[9174 12163 2123 0.6503]	[12822 8816 1822 0.7344]	[9133 12142 2185 0.6481]	[14388 7267 1805 0.7682]	[9242 12078 2140 0.6514]	[10913 11344 2181 0.6791±0.0018]
0.70	[12554 10554 2307 0.7016]	[9201 13757 2457 0.6327]	[10808 12229 2378 0.6658]	[10953 12164 2298 0.6703]	[10852 12273 2290 0.6684]	[9174 12163 2123 0.6503]	[12822 8816 1822 0.7344]	[9133 12142 2185 0.6481]	[14388 7267 1805 0.7682]	[9242 12078 2140 0.6514]	[10913 11344 2181 0.6791±0.0018]
0.75	[12554 10554 2307 0.7016]	[9201 13757 2457 0.6327]	[10808 12229 2378 0.6658]	[10953 12164 2298 0.6703]	[10852 12273 2290 0.6684]	[9174 12163 2123 0.6503]	[12822 8816 1822 0.7344]	[9133 12142 2185 0.6481]	[14388 7267 1805 0.7682]	[9242 12078 2140 0.6514]	[10913 11344 2181 0.6791±0.0018]

Table 14 (continued)

Orness(w) = α	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
0.65	[4925 3 3555 0.5807]	[2649 10 5824 0.3129]	[5289 10 3184 0.6241]	[5321 14 3148 0.6281]	[4254 6 4223 0.5018]	[4672 3 3808 0.5509]	[5793 4 2686 0.6831]	[4646 4 3334 0.5822]	[3703 7 4274 0.4642]	[4226 9 3749 0.5299]	[4548 7 3779 0.5458±0.0108]
	0.70	[4902 3 3578 0.5780]	[2636 10 5837 0.3113]	[5253 10 3220 0.6198]	[5287 14 3182 0.6241]	[4239 6 4238 0.5001]	[4651 3 3829 0.5484]	[5747 4 2732 0.6777]	[4622 4 3358 0.5792]	[3663 7 4314 0.4592]	[4193 9 3782 0.5257]
0.75		[4855 3 3625 0.5725]	[2646 10 5827 0.3125]	[5225 10 3248 0.6165]	[5272 14 3197 0.6223]	[4180 6 4297 0.4931]	[4636 3 3844 0.5467]	[5717 4 2762 0.6742]	[4610 4 3370 0.5777]	[3641 7 4336 0.4565]	[4162 9 3813 0.5219]
	0.80	[4829 3 3651 0.5694]	[2634 10 5839 0.3111]	[5177 10 3296 0.6109]	[5255 14 3214 0.6203]	[4130 6 4347 0.4872]	[4609 3 3871 0.5435]	[5673 4 2806 0.6690]	[4596 4 3384 0.5759]	[3651 7 4326 0.4577]	[4121 9 3854 0.5167]
0.85		[4827 3 3653 0.5692]	[2654 10 5819 0.3135]	[5145 10 3328 0.6071]	[5238 14 3231 0.6183]	[4080 6 4397 0.4813]	[4604 3 3876 0.5429]	[5657 4 2822 0.6671]	[4592 4 3388 0.5754]	[3642 7 4335 0.4566]	[4086 9 3889 0.5123]
	0.90	[4802 3 3678 0.5663]	[2710 10 5759 0.3201]	[5103 10 3393 0.6021]	[5238 14 3236 0.6183]	[4039 6 4445 0.4765]	[4592 3 3887 0.5415]	[5614 4 2867 0.6620]	[4584 4 3409 0.5744]	[3612 7 4376 0.4528]	[4053 9 3945 0.5082]
0.92		[4786 3 3694 0.5644]	[2714 10 5759 0.3205]	[5080 10 3393 0.5994]	[5233 14 3236 0.6177]	[4032 6 4445 0.4757]	[4593 3 3887 0.5416]	[5612 4 2867 0.6618]	[4571 4 3409 0.5728]	[3601 7 4376 0.4515]	[4030 9 3945 0.5053]
	0.93	[4777 3 3703 0.5633]	[2725 10 5748 0.3218]	[5059 10 3414 0.5970]	[5224 14 3245 0.6166]	[4023 6 4454 0.4746]	[4576 3 3904 0.5396]	[5601 4 2878 0.6605]	[4569 4 3411 0.5725]	[3597 7 4380 0.4510]	[4013 9 3962 0.5032]
0.94		[4758 3 3722 0.5611]	[2735 10 5738 0.3230]	[5047 10 3426 0.5955]	[5213 14 3256 0.6153]	[4015 6 4462 0.4737]	[4588 3 3892 0.5410]	[5583 4 2896 0.6584]	[4571 4 3409 0.5728]	[3590 7 4387 0.4501]	[3998 9 3977 0.5013]
	0.95	[4767 3 3713 0.5621]	[2731 10 5742 0.3225]	[5031 10 3442 0.5937]	[5208 14 3261 0.6148]	[4002 6 4475 0.4721]	[4590 3 3890 0.5413]	[5580 4 2899 0.6580]	[4544 4 3436 0.5694]	[3568 7 4409 0.4473]	[3983 9 3992 0.4994]
0.96		[4765 3 3715 0.5619]	[2736 10 5737 0.3231]	[5013 10 3460 0.5915]	[5193 14 3276 0.6130]	[3986 6 4491 0.4702]	[4571 3 3909 0.5390]	[5574 4 2905 0.6573]	[4525 4 3455 0.5670]	[3551 7 4426 0.4452]	[3977 9 3998 0.4987]
	0.97	[4770 3 3710 0.5625]	[2734 10 5739 0.3229]	[4985 10 3488 0.5882]	[5175 14 3294 0.6109]	[3987 6 4490 0.4704]	[4560 3 3920 0.5377]	[5564 4 2915 0.6561]	[4512 4 3468 0.5654]	[3528 7 4449 0.4423]	[3952 9 4023 0.4956]
0.98		[4782 3 3698 0.5639]	[2782 10 5691 0.3285]	[4972 10 3501 0.5867]	[5157 14 3312 0.6087]	[3979 6 4498 0.4694]	[4551 3 3929 0.5367]	[5563 4 2916 0.6560]	[4509 4 3471 0.5650]	[3522 7 4455 0.4416]	[3868 9 4107 0.4850]
	<i>C96 network</i>										
0.55	[12561 10554 2300 0.7019]	[9131 13757 2527 0.6299]	[10827 12229 2359 0.6666]	[10966 12164 2285 0.6708]	[10890 12273 2252 0.6699]	[9201 12163 2096 0.6514]	[12840 8816 1804 0.7352]	[9086 12142 2232 0.6461]	[14421 7267 1772 0.7696]	[9246 12078 2136 0.6515]	[10917 11344 2176 0.6793±0.0019]
	0.60	[12562 10554 2299 0.7019]	[9133 13757 2525 0.6300]	[10828 12229 2358 0.6666]	[10967 12164 2284 0.6708]	[10891 12273 2251 0.6700]	[9203 12163 2094 0.6515]	[12843 8816 1801 0.7353]	[9087 12142 2231 0.6461]	[14422 7267 1771 0.7696]	[9249 12078 2133 0.6517]
0.65		[12560 10554 2301 0.7018]	[9129 13757 2529 0.6298]	[10826 12229 2360 0.6666]	[10965 12164 2286 0.6707]	[10889 12273 2253 0.6699]	[9199 12163 2098 0.6513]	[12837 8816 1807 0.7351]	[9085 12142 2233 0.6460]	[14420 7267 1773 0.7695]	[9243 12078 2139 0.6514]
	0.70	[12556 10554 2305 0.7017]	[9121 13757 2537 0.6295]	[10822 12229 2364 0.6664]	[10961 12164 2290 0.6706]	[10885 12273 2257 0.6697]	[9191 12163 2106 0.6510]	[12825 8816 1819 0.7346]	[9081 12142 2237 0.6459]	[14416 7267 1777 0.7694]	[9231 12078 2151 0.6509]
0.75		[12556 10554 2305 0.7017]	[9121 13757 2537 0.6295]	[10822 12229 2364 0.6664]	[10961 12164 2290 0.6706]	[10885 12273 2257 0.6697]	[9191 12163 2106 0.6510]	[12825 8816 1819 0.7345]	[9081 12142 2237 0.6459]	[14416 7267 1777 0.7693]	[9231 12078 2151 0.6508]
	0.80	[12552 10554 2309 0.7015]	[9119 13757 2539 0.6295]	[10820 12229 2366 0.6663]	[10958 12164 2293 0.6705]	[10885 12273 2257 0.6697]	[9191 12163 2106 0.6510]	[12823 8816 1821 0.7345]	[9081 12142 2237 0.6459]	[14414 7267 1779 0.7693]	[9229 12078 2153 0.6508]
0.85		[12556 10554 2305 0.7017]	[9117 13757 2541 0.6294]	[10822 12229 2364 0.6664]	[10960 12164 2291 0.6705]	[10885 12273 2257 0.6697]	[9191 12163 2106 0.6510]	[12825 8816 1819 0.7346]	[9077 12142 2241 0.6457]	[14416 7267 1777 0.7694]	[9231 12078 2151 0.6509]
	0.90	[12572 10554 2289 0.7023]	[9125 13757 2533 0.6297]	[10830 12229 2356 0.6667]	[10970 12164 2281 0.6709]	[10885 12273 2257 0.6697]	[9191 12163 2106 0.6510]	[12833 8816 1811 0.7349]	[9077 12142 2241 0.6457]	[14424 7267 1769 0.7697]	[9239 12078 2143 0.6512]
0.92		[12592 10554 2289 0.7023]	[9130 13757 2533 0.6297]	[10850 12229 2356 0.6667]	[10985 12164 2281 0.6709]	[10910 12273 2257 0.6697]	[9206 12163 2106 0.6510]	[12848 8816 1811 0.7349]	[9092 12142 2241 0.6457]	[14454 7267 1769 0.7697]	[9244 12078 2143 0.6512]

(continued on next page)

Table 14 (continued)

Orness(w) = α	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	Average
0.93	2269	2528	2336	2266	2232	2091	1796	2226	1739	2138	0.6799±0.0019
	0.7031]	0.6299]	0.6675]	0.6715]	0.6707]	0.6516]	0.7355]	0.6463]	0.7710]	0.6514]	
	[12592	[9130	[10850	[10985	[10910	[9206	[12848	[9092	[14454	[9244	[10931 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2162
0.94	2269	2528	2336	2266	2232	2091	1796	2226	1739	2138	0.6799±0.0019]
	0.7031]	0.6299]	0.6675]	0.6715]	0.6707]	0.6516]	0.7355]	0.6463]	0.7710]	0.6514]	
	[12600	[9132	[10858	[10993	[10920	[9212	[12854	[9098	[14466	[9246	[10938 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2155
0.95	2261	2526	2328	2258	2222	2085	1790	2220	1727	2136	0.6801±0.0019]
	0.7034]	0.6300]	0.6678]	0.6718]	0.6711]	0.6519]	0.7358]	0.6466]	0.7715]	0.6515]	
	[12604	[9134	[10860	[10995	[10920	[9212	[12856	[9098	[14468	[9248	[10940 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2154
0.96	2257	2524	2326	2256	2222	2085	1788	2220	1725	2134	0.6802±0.0020]
	0.7036]	0.6300]	0.6679]	0.6719]	0.6711]	0.6519]	0.7359]	0.6466]	0.7716]	0.6516]	
	[12620	[9139	[10876	[11007	[10940	[9224	[12868	[9111	[14492	[9252	[10953 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2140
0.97	2241	2519	2310	2244	2202	2073	1776	2207	1701	2130	0.6808±0.0020]
	0.7042]	0.6302]	0.6685]	0.6724]	0.6719]	0.6524]	0.7364]	0.6471]	0.7726]	0.6518]	
	[12640	[9146	[10892	[11020	[10955	[9233	[12881	[9120	[14514	[9259	[10966 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2127
0.98	2221	2512	2294	2231	2187	2064	1763	2198	1679	2123	0.6813±0.0020]
	0.7050]	0.6305]	0.6692]	0.6729]	0.6725]	0.6528]	0.7370]	0.6475]	0.7736]	0.6521]	
	[12658	[9154	[10903	[11030	[10960	[9236	[12891	[9123	[14527	[9267]	[10975 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2118
B97 network	2203	2504	2283	2221	2182	2061	1753	2195	1666	2115	0.6817±0.0020]
	0.7057]	0.6308]	0.6696]	0.6733]	0.6727]	0.6529]	0.7374]	0.6477]	0.7741]	0.6524]	
	[12657	[9154	[10903	[11030	[10960	[9236	[12891	[9123	[14527	[9267]	[10975 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2118
0.55	2203	2504	2283	2221	2182	2061	1753	2195	1666	2115	0.6817±0.0020]
	0.7057]	0.6308]	0.6696]	0.6733]	0.6727]	0.6529]	0.7374]	0.6477]	0.7741]	0.6524]	
	[12657	[9154	[10903	[11030	[10960	[9236	[12891	[9123	[14527	[9267]	[10975 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2118
0.60	2203	2504	2283	2221	2182	2061	1753	2195	1666	2115	0.6817±0.0020]
	0.7057]	0.6308]	0.6696]	0.6733]	0.6727]	0.6529]	0.7374]	0.6477]	0.7741]	0.6524]	
	[12657	[9154	[10903	[11030	[10960	[9236	[12891	[9123	[14527	[9267]	[10975 11344
	10554	13757	12229	12164	12273	12163	8816	12142	7267	12078	2118

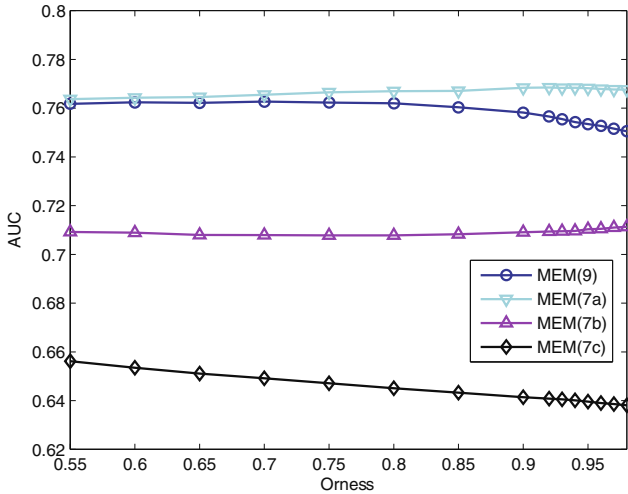


Fig. 14. Comparative results of LPE_{OWA}^S on WSDP98.

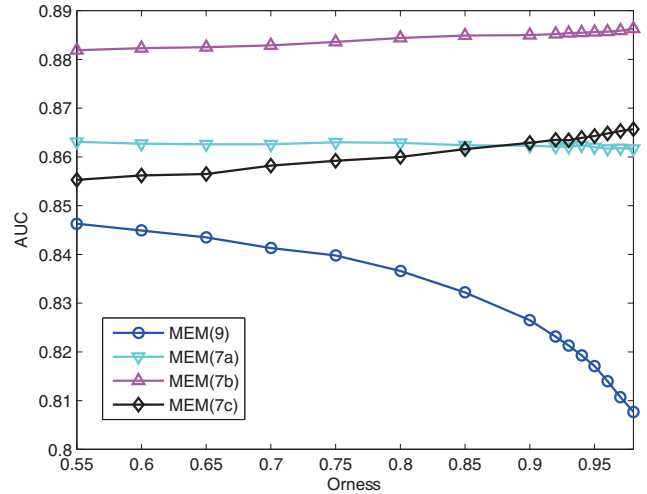


Fig. 17. Comparative results of LPE_{OWA}^S on B97.

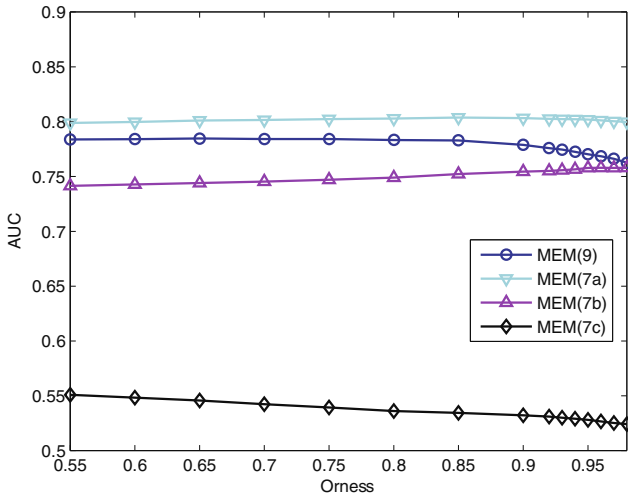


Fig. 15. Comparative results of LPE_{OWA}^S on ChesLower.

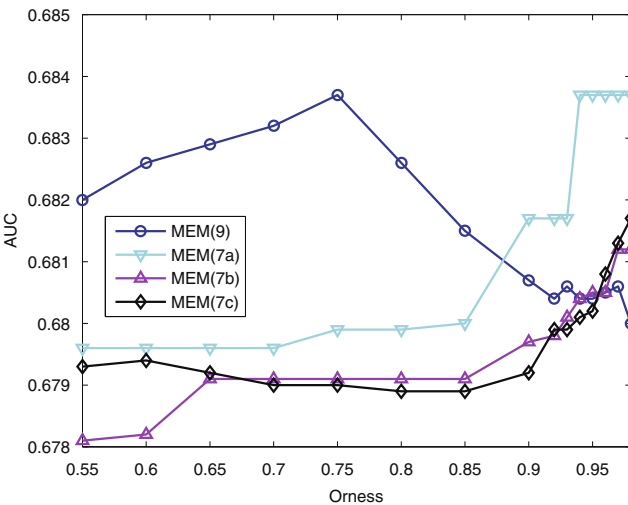


Fig. 16. Comparative results of LPE_{OWA}^S on C96.

- For B97 network (Fig. 17), the AUCs of MEM based $LPE_{OWA}(7a)$, $LPE_{OWA}(7b)$ and $LPE_{OWA}(7c)$ also gradually increase with the increase of α . This is opposite to the variation tendency of AUCs of MEM based LPE_{OWA} . MEM based LPE_{OWA} obtains the worse performances than other three ensemble algorithms.

4.3. Analysis to the experimental results

Here, we try to give some elementary explanations to the above-mentioned results. We think the prediction performance of LPE_{OWA} is dependent on the specific characteristics of the social network.

Firstly, LPE_{OWA} obtains the lower AUCs on B97 network. From Table 3 we can find the average degree of B97 is 11.4783. The statistical result shows that there are only 4 nodes in B97 of which the degrees are larger than 11.4783. However, the efficiency of B97 is not low. This indicates that the nodes in B97 network have no obvious characteristic of local information, i.e. the algorithm considering the number of common neighbors of nodes x and y , the degrees of nodes x and y and the degrees of common neighbors of nodes x and y simultaneously is not suitable for the link prediction of network with high average degree and high efficiency. This is also demonstrated by the better performances of $LPE_{OWA}(7a)$ (the number of common neighbors of nodes x and y and the degrees of nodes x and y), $LPE_{OWA}(7b)$ (the number of common neighbors of nodes x and y and the degrees of common neighbors of nodes x and y) and $LPE_{OWA}(7c)$ (the number of common neighbors of nodes x and y and the degrees of common neighbors of nodes x and y) obtain the better prediction performances on this network.

Secondly, LPE_{OWA} obtains the better performances on WSDP98 and ChesLower networks and its performances are further improved by $LPE_{OWA}(7a)$ on these two networks. This indicates that the number of common neighbors of nodes x and y and the degrees of nodes x and y are two more typical characteristics of WSDP98 and ChesLower networks in comparison with the degrees of common neighbors of nodes x and y . The statistical results of WSDP98 and ChesLower reflect that there are about half nodes (14 and 14 respectively) of which the degrees are larger than 6.7429 and 9.0270.

Thirdly, LPE_{OWA} on C96 network obtains the better performances than all individual algorithms and $LPE_{OWA}(7a)$, $LPE_{OWA}(7b)$ and $LPE_{OWA}(7c)$. C96 has 47 nodes whose degrees are larger than 3.8463. It means that the nodes in C96 have the similar local information, i.e., three aforementioned local information all

play the important roles on the performances of link prediction algorithms. LPE_{OWA} merges these three kinds of local information together and thus increases the prediction accuracies.

5. Conclusion and future work

In this paper, we design the ensemble strategy for the local information-based link prediction algorithms based on OWA operator. The experimental results demonstrate the feasibility and effectiveness of our method. A number of enhancements and future research can be summarized as follows: (1) test the performance of proposed ensemble algorithm on the large scale social networks, (2) improve the ensemble learning process with the supervised learning methods (e.g., bagging and boosting), and (3) consider to improve the prediction performance of event-based link prediction algorithm (Soares & Prudêncio, 2013) with OWA operator based aggregation.

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