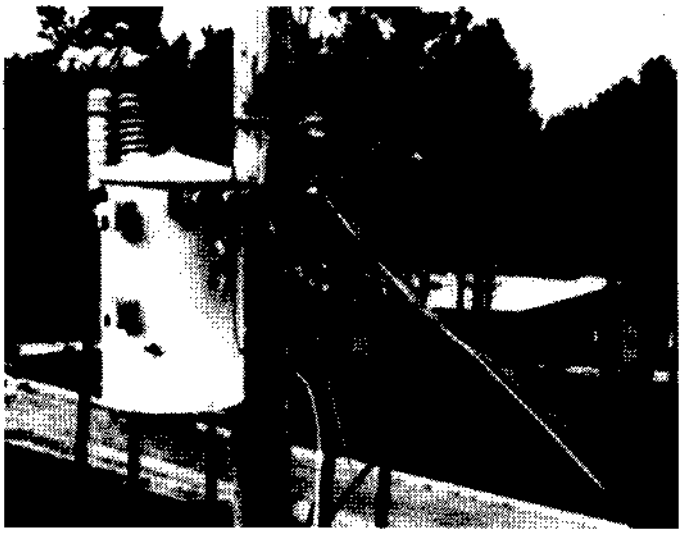


FIGURE 2.5 Distribution transformer (Courtesy of Westinghouse Electric Corporation.)

2.1 IDEAL TRANSFORMER

Consider a transformer with two windings, a primary winding of N_1 turns and a secondary winding of N_2 turns, as shown schematically in Fig. 2.6. In a schematic diagram it is a common practice to show the two windings in the two legs of the core, although in an actual transformer the windings are interleaved. Let us consider an ideal transformer that has the following properties:

1. The winding resistances are negligible.
2. All fluxes are confined to the core and link both windings; that is, no leakage fluxes are present. Core losses are assumed to be negligible.
3. Permeability of the core is infinite (i.e., $\mu \to \infty$). Therefore, the exciting current required to establish flux in the core is negligible; that is, the net mmf required to establish a flux in the core is zero.

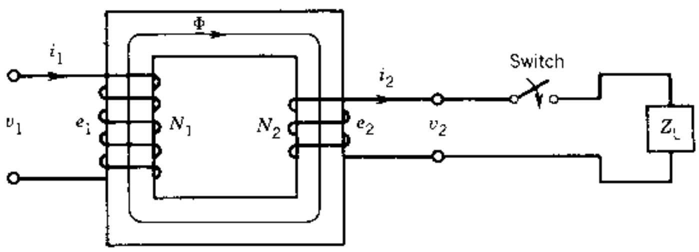


FIGURE 2.6 Ideal transformer.

When the primary winding is connected to a time-varying voltage v_1, a time-varying flux Φ is established in the core. A voltage e_1 will be induced in the winding and will equal the applied voltage if resistance of the winding is neglected:

$$v_1 = e_1 = N_1 \frac{d\Phi}{dt} \tag{2.1}$$

The core flux also links the secondary winding and induces a voltage e_2, which is the same as the terminal voltage v_2:

$$v_2 = e_2 = N_2 \frac{d\Phi}{dt} \tag{2.2}$$

From Eqs. 2.1 and 2.2,

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = a \tag{2.3}$$

where a is the turns ratio.

Equation 2.3 indicates that the voltages in the windings of an ideal transformer are directly proportional to the turns of the windings.

Let us now connect a load (by closing the switch in Fig. 2.6) to the secondary winding. A current i_2 will flow in the secondary winding, and the secondary winding will provide an mmf N_2i_2 for the core. This will immediately make a primary winding current i_1 flow so that a counter-mmf N_1i_1 can oppose N_2i_2. Otherwise N_2i_2 would make the core flux change drastically and the balance between v_1 and e_1 would be disturbed. Note in Fig. 2.6 that the current directions are shown such that their mmf's oppose each other. Because the net mmf required to establish a flux in the ideal core is zero,

$$N_1i_1 - N_2i_2 = \text{net mmf} = 0 \tag{2.4}$$

$$N_1i_1 = N_2i_2 \tag{2.5}$$

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a} \tag{2.6}$$

The currents in the windings are inversely proportional to the turns of the windings. Also note that if more current is drawn by the load, more current will flow from the supply. It is this mmf-balancing requirement (Eq. 2.5) that makes the primary know of the presence of current in the secondary.

From Eqs. 2.3 and 2.6

$$v_1i_1 = v_2i_2 \tag{2.7}$$

that is, the instantaneous power input to the transformer equals the instantaneous power output from the transformer. This is expected, because all power losses are neglected in an ideal transformer. Note that although there is no physical connection between load and supply, as soon as power is consumed by the load, the same power is drawn from the supply. The transformer, therefore, provides a physical isolation between load and supply while maintaining electrical continuity.

If the supply voltage v_1 is sinusoidal, then Eqs. 2.3, 2.6, and 2.7 can be written in terms of rms values:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = a \tag{2.8}$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a} \tag{2.9}$$

$$\underset{\substack{\uparrow \\ \text{input} \\ \text{volt-amperes}}}{V_1 I_1} = \underset{\substack{\uparrow \\ \text{output} \\ \text{volt-amperes}}}{V_2 I_2} \tag{2.10}$$

2.1.1 IMPEDANCE TRANSFER

Consider the case of a sinusoidal applied voltage and a secondary impedance Z_2, as shown in Fig. 2.7*a*.

$$Z_2 = \frac{V_2}{I_2}$$

The input impedance is

$$Z_1 = \frac{V_1}{I_1} = \frac{aV_2}{I_2/a} = a^2 \frac{V_2}{I_2}$$

$$= a^2 Z_2 \tag{2.11}$$

so

$$Z_1 = a^2 Z_2 = Z_2' \tag{2.12}$$

An impedance Z_2 connected in the secondary will appear as an impedance Z_2' looking from the primary. The circuit in Fig. 2.7*a* is therefore equivalent to the circuit in Fig. 2.7*b*. Impedance can be transferred from secondary to

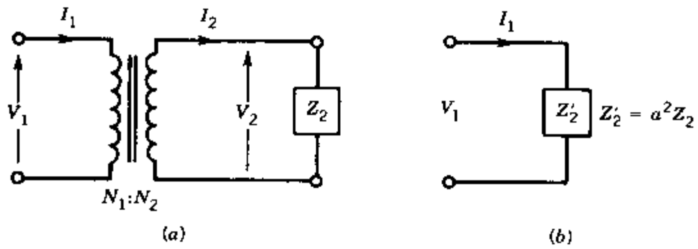


FIGURE 2.7 Impedance transfer across an ideal transformer.

primary if its value is multiplied by the square of the turns ratio. An impedance from the primary side can also be transferred to the secondary side, and in that case its value has to be divided by the square of the turns ratio:

$$Z_1' = \frac{1}{a^2} Z_1 \tag{2.13}$$

This impedance transfer is very useful because it eliminates a coupled circuit in an electrical circuit and thereby simplifies the circuit.

EXAMPLE 2.1

A speaker of 9 Ω resistive impedance is connected to a supply of 10 V with internal resistive impedance of 1 Ω, as shown in Fig. E2.1*a*.

(a) Determine the power absorbed by the speaker.

(b) To maximize the power transfer to the speaker, a transformer of 1 : 3 turns ratio is used between source and speaker as shown in Fig. E2.1*b*. Determine the power taken by the speaker.

Solution

(a) From Fig. E2.1*a*,

$$I = \frac{10}{1+9} = 1 \text{ A}$$

$$P = 1^2 \times 9 = 9 \text{ W}$$

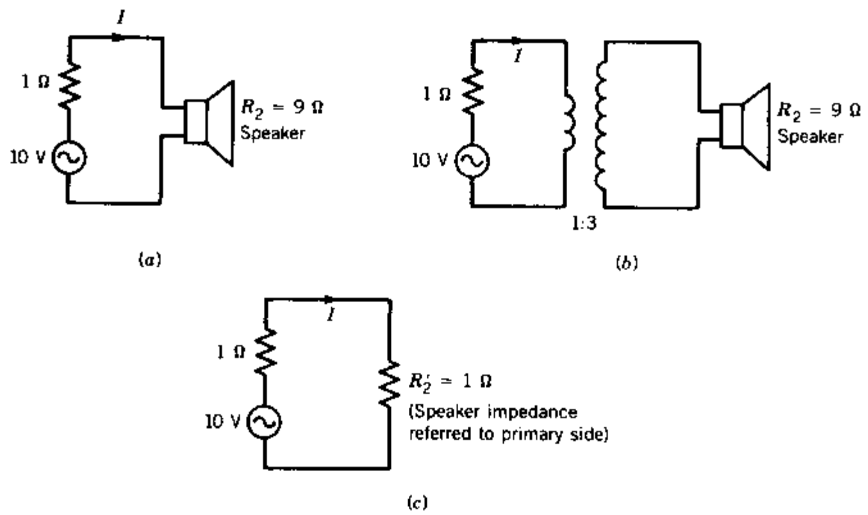


FIGURE E2.1

(b) If the resistance of the speaker is referred to the primary side, its resistance is

$$R_2' = a^2 R_2 = \left(\frac{1}{3}\right)^2 \times 9 = 1\ \Omega$$

The equivalent circuit is shown in Fig. E2.1*c*.

$$I = \frac{10}{1+1} = 5\ \text{A}$$

$$P = 5^2 \times 1 = 25\ \text{W} \quad \blacksquare$$

2.1.2 POLARITY

Windings on transformers or other electrical machines are marked to indicate terminals of like polarity. Consider the two windings shown in Fig. 2.8*a*. Terminals 1 and 3 are identical, because currents entering these terminals produce fluxes in the same direction in the core that forms the common magnetic path. For the same reason, terminals 2 and 4 are identical. If these two windings are linked by a common time-varying flux, voltages will be induced in these windings such that, if at a particular instant the potential

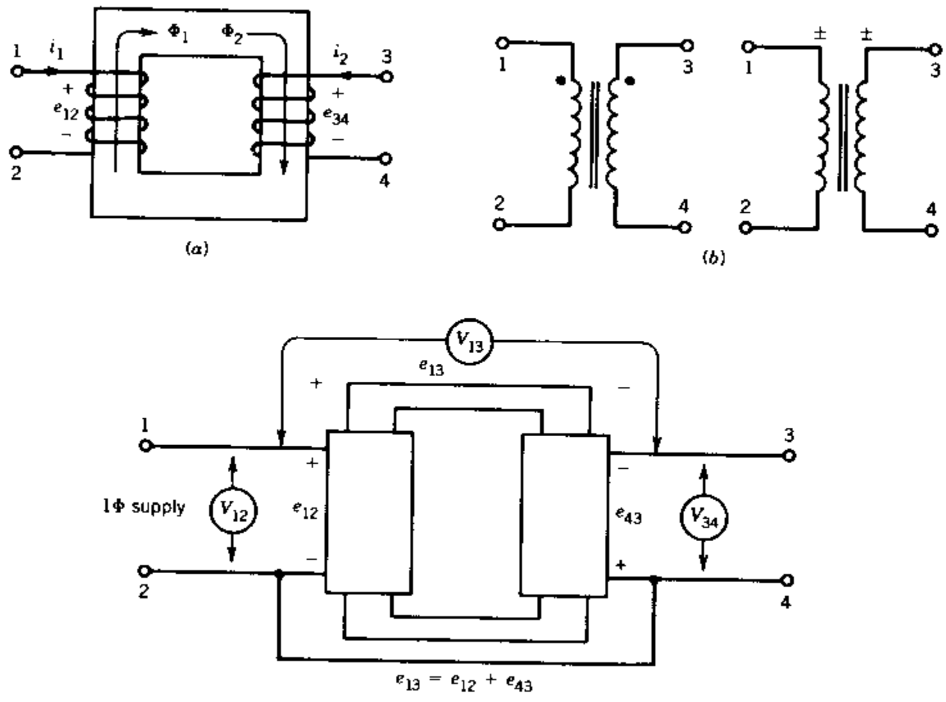


FIGURE 2.8 Polarity determination.

of terminal 1 is positive with respect to terminal 2, then at the same instant the potential of terminal 3 will be positive with respect to terminal 4. In other words, induced voltages e_{12} and e_{34} are in phase. Identical terminals such as 1 and 3 or 2 and 4 are sometimes marked by dots or ± as shown in Fig. 2.8*b*. These are called the polarity markings of the windings. They indicate how the windings are wound on the core.

If the windings can be visually seen in a machine, the polarities can be determined. However, usually only the terminals of the windings are brought outside the machine. Nevertheless, it is possible to determine the polarities of the windings experimentally. A simple method is illustrated in Fig. 2.8*c*, in which terminals 2 and 4 are connected together and winding 1–2 is connected to an ac supply.

The voltages across 1–2, 3–4, and 1–3 are measured by a voltmeter. Let these voltage readings be called V_{12}, V_{34}, and V_{13}, respectively. If a voltmeter reading V_{13} is the sum of voltmeter readings V_{12} and V_{34} (i.e., $V_{13} \simeq V_{12} + V_{34}$), it means that at any instant when the potential of terminal 1 is positive with respect to terminal 2, the potential of terminal 4 is positive with respect to terminal 3. The induced voltages e_{12} and e_{43} are in phase, as shown in Fig. 2.8*c*, making $e_{13} = e_{12} + e_{43}$. Consequently, terminals 1 and 4 are identical (or same polarity) terminals. If the voltmeter reading V_{13} is the difference between voltmeter readings V_{12} and V_{34} (i.e., $V_{13} \simeq V_{12} - V_{34}$), then 1 and 3 are terminals of the same polarity.

Polarities of windings must be known if transformers are connected in parallel to share a common load. Figure 2.9*a* shows the parallel connection of two single-phase (1ϕ) transformers. This is the correct connection because secondary voltages e_{21} and e_{22} oppose each other internally. The connection shown in Fig. 2.9*b* is wrong, because e_{21} and e_{22} aid each other internally and a large circulating current i_{cir} will flow in the windings and may damage the transformers. For three-phase connection of transformers (see Section 2.6), the winding polarities must also be known.

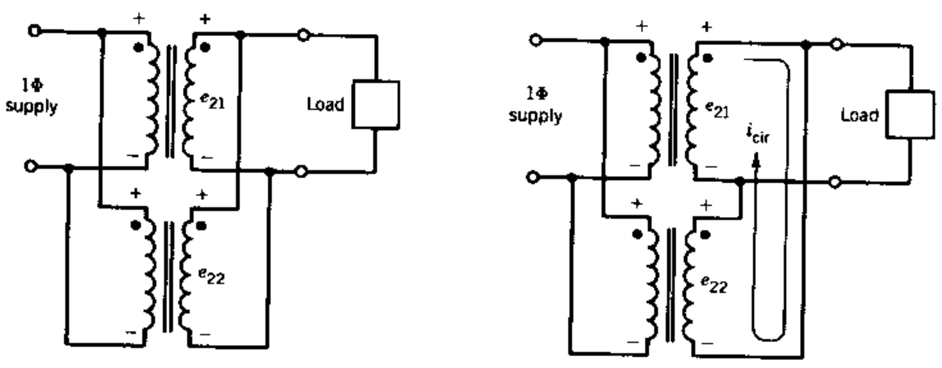


FIGURE 2.9 Parallel operation of single-phase transformers. (*a*) Correct connection. (*b*) Wrong connection.

2.2 PRACTICAL TRANSFORMER

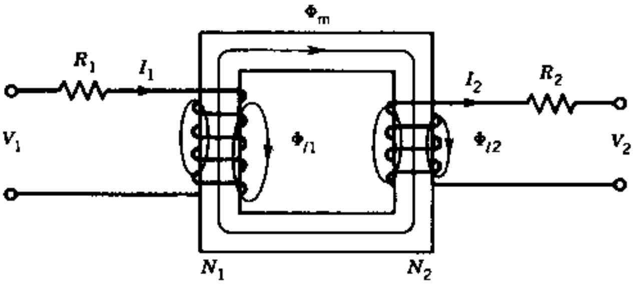
In Section 2.1 the properties of an ideal transformer were discussed. Certain assumptions were made which are not valid in a practical transformer. For example, in a practical transformer the windings have resistances, not all windings link the same flux, permeability of the core material is not infinite, and core losses occur when the core material is subjected to time-varying flux. In the analysis of a practical transformer, all these imperfections must be considered.

Two methods of analysis can be used to account for the departures from the ideal transformer:

1. An equivalent circuit model based on physical reasoning.
2. A mathematical model based on the classical theory of magnetically coupled circuits.

Both methods will provide the same performance characteristics for the practical transformer. However, the equivalent circuit approach provides a better appreciation and understanding of the physical phenomena involved, and this technique will be presented here.

A practical winding has a resistance, and this resistance can be shown as a lumped quantity in series with the winding (Fig. 2.10*a*). When currents flow through windings in the transformer, they establish a resultant mutual



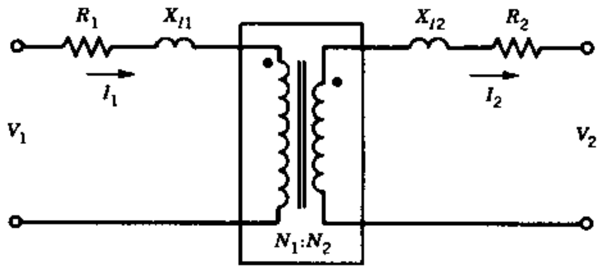
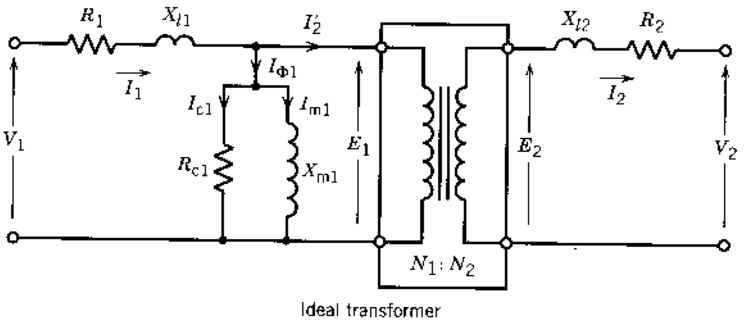


FIGURE 2.10 Development of the transformer equivalent circuits.

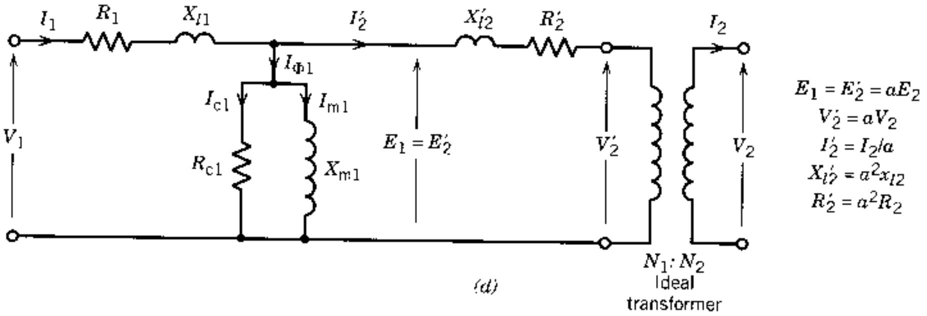
(or common) flux Φ_m that is confined essentially to the magnetic core. However, a small amount of flux known as leakage flux, Φ_l (shown in Fig. 2.10*a*), links only one winding and does not link the other winding. The leakage path is primarily in air, and therefore the leakage flux varies linearly with current. The effects of leakage flux can be accounted for by an inductance, called leakage inductance:

$$L_{l1} = \frac{N_1\Phi_{l1}}{i_1} = \text{leakage inductance of winding 1}$$

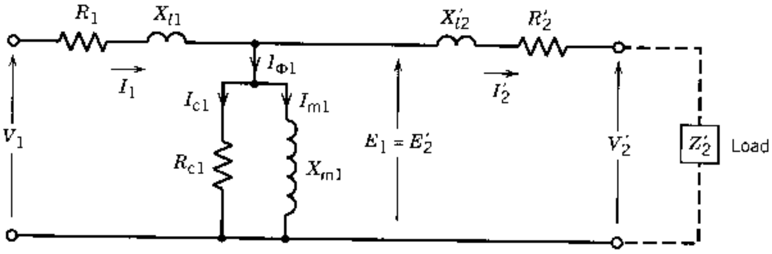
$$L_{l2} = \frac{N_2\Phi_{l2}}{i_2} = \text{leakage inductance of winding 2}$$



(*c*)



(*d*)



(*e*)

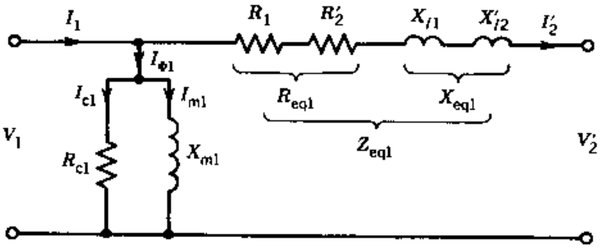
FIGURE 2.10 (*Continued*)

If the effects of winding resistance and leakage flux are respectively accounted for by resistance R and leakage reactance $X_l(= 2\pi f L_l)$, as shown in Fig. 2.10*b*, the transformer windings are tightly coupled by a mutual flux.

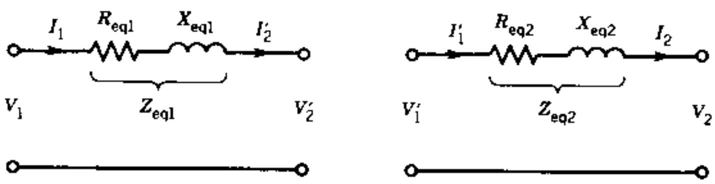
In a practical magnetic core having finite permeability, a magnetizing current I_m is required to establish a flux in the core. This effect can be represented by a magnetizing inductance L_m. Also, the core loss in the magnetic material can be represented by a resistance R_c. If these imperfections are also accounted for, then what we are left with is an ideal transformer, as shown in Fig. 2.10*c*. A practical transformer is therefore equivalent to an ideal transformer plus external impedances that represent imperfections of an actual transformer.

2.2.1 REFERRED EQUIVALENT CIRCUITS

The ideal transformer in Fig. 2.10*c* can be moved to the right or left by referring all quantities to the primary or secondary side, respectively. This is almost invariably done. The equivalent circuit with the ideal transformer moved to the right is shown in Fig. 2.10*d*. For convenience, the ideal transformer is usually not shown and the equivalent circuit is drawn, as shown in Fig. 2.10*e*, with all quantities (voltages, currents, and impedances) referred to one side. The referred quantities are indicated with primes. By analyzing



(*a*) $V_2' = aV_2$, $I_2' = I_2/a$



(*b*) Referred to side 1, $Z_{eq1} = R_{eq1} + jX_{eq1}$ (*c*) Referred to side 2, $Z_{eq2} = R_{eq2} + jX_{eq2}$

$$R_{eq2} = \frac{R_{eq1}}{a^2} = R_2 + R_1'$$

$$X_{eq2} = \frac{X_{eq1}}{a^2} = X_{l2} + X_{l1}'$$

$$V_1' = \frac{V_1}{a}, \quad I_1' = I_2 = aI_1$$

FIGURE 2.11 Approximate equivalent circuits.

this equivalent circuit the referred quantities can be evaluated, and the actual quantities can be determined from them if the turns ratio is known.

Approximate Equivalent Circuits

The voltage drops I_1R_1 and I_1X_{l1} (Fig. 2.10*e*) are normally small and $|E_1| \simeq |V_1|$. If this is true then the shunt branch (composed of R_{c1} and X_{m1}) can be moved to the supply terminal, as shown in Fig. 2.11*a*. This approximate equivalent circuit simplifies computation of currents, because both the exciting branch impedance and the load branch impedance are directly connected across the supply voltage. Besides, the winding resistances and leakage reactances can be lumped together. This equivalent circuit (Fig. 2.11*a*) is frequently used to determine the performance characteristics of a practical transformer.

In a transformer, the exciting current I_ϕ is a small percentage of the rated current of the transformer (less than 5%). A further approximation of the equivalent circuit can be made by removing the excitation branch, as shown in Fig. 2.11*b*. The equivalent circuit referred to side 2 is also shown in Fig. 2.11*c*.

2.2.2 DETERMINATION OF EQUIVALENT CIRCUIT PARAMETERS

The equivalent circuit model (Fig. 2.10*e*) for the actual transformer can be used to predict the behavior of the transformer. The parameters R_1, X_{l1}, R_{c1}, X_{m1}, R_2, X_{l2}, and a $(= N_1/N_2)$ must be known so that the equivalent circuit model can be used.

If the complete design data of a transformer are available, these parameters can be calculated from the dimensions and properties of the materials used. For example, the winding resistances (R_1, R_2) can be calculated from the resistivity of copper wires, the total length, and the cross-sectional area of the winding. The magnetizing inductances L_m can be calculated from the number of turns of the winding and the reluctance of the magnetic path. The calculation of the leakage inductance (L_l) will involve accounting for partial flux linkages and is therefore complicated. However, formulas are available from which a reliable determination of these quantities can be made.

These parameters can be directly and more easily determined by performing tests that involve little power consumption. Two tests, a no-load test (or open-circuit test) and a short-circuit test, will provide information for determining the parameters of the equivalent circuit of a transformer, as will be illustrated by an example.

Transformer Rating

The kilovolt–ampere (kVA) rating and voltage ratings of a transformer are marked on its nameplate. For example, a typical transformer may carry the following information on the nameplate: 10 kVA, 1100/110 volts. What are