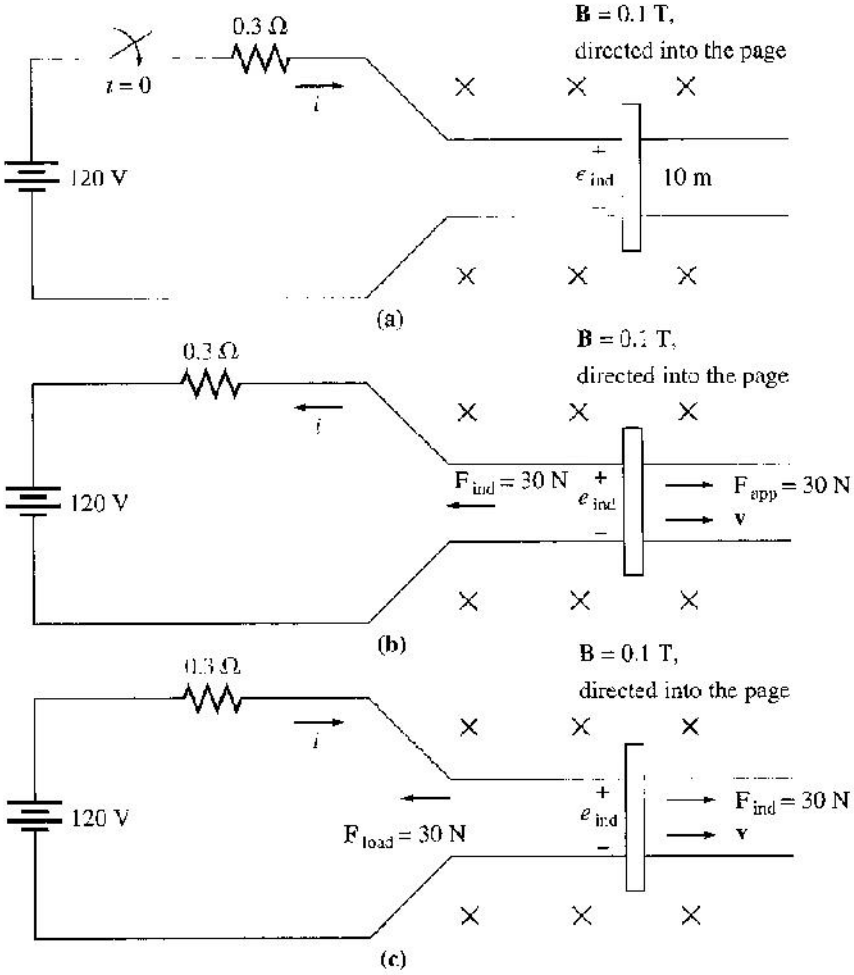
Example 1.10



The linear dc machine is as shown in (a).

(a) What is the machine's maximum starting current? What is the steady state velocity at no load?

(b) Suppose a 30N force pointing to the right were applied to the bar (figure b). What would the steady-state speed be? How much power would the bar be producing or consuming? How much power would the bar be producing or consuming? Is the machine acting as a motor or a generator?

(c) Now suppose a 30N force pointing to the left were applied to the bar (figure c). What would the new steady-state speed be? Is the machine a motor or generator now?

CHAPTER 2 – TRANSFORMERS

Summary:

1. Types and Construction of Transformers

2. The Ideal Transformer
   - Power in an Ideal Transformer
   - Impedance transformation through a transformer
   - Analysis of circuits containing ideal transformer

3. Theory of operation of real single-phase transformers.
   - The voltage ratio across a transformer
   - The magnetization current in a Real Transformer
   - The current ratio on a transformer and the Dot Convention

4. The Equivalent Circuit of a Transformer.
   - Exact equivalent circuit
   - Approximate equivalent circuit
   - Determining the values pf components in the transformer model

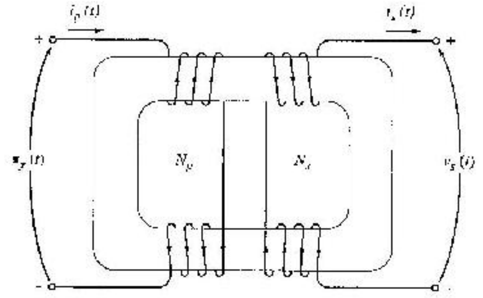
5. The Per-Unit System of Measurement

6. Transformer voltage regulation and efficiency
   - The transformer phasor diagram
   - Transformer efficiency

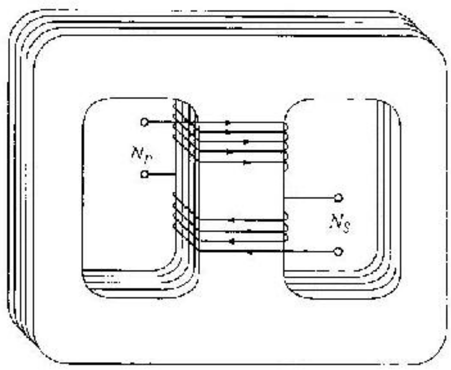
7. Three phase transformers

1. Types and Construction of Transformers

Types of cores for power transformer (both types are constructed from thin laminations electrically isolated from each other – minimize eddy currents)



i) *Core Form* : a simple rectangular laminated piece of steel with the transformer windings wrapped around two sides of the rectangle.



ii) *Shell Form* : a three legged laminated core with the windings wrapped around the centre leg.

The primary and secondary windings are wrapped one on top of the other with the low-voltage winding innermost, due to 2 purposes:

i) It simplifies the problem of insulating the high-voltage winding from the core.
ii) It results in much less leakage flux

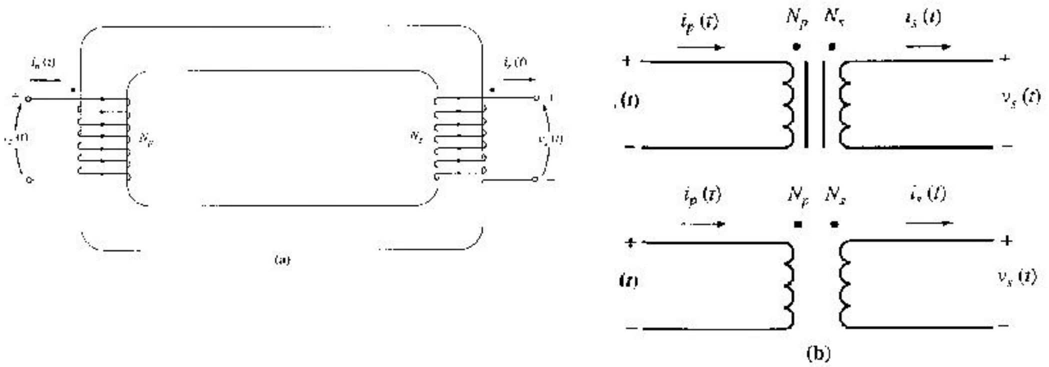
Types of transformers:

i) Step up/Unit transformers – Usually located at the output of a generator. Its function is to step up the voltage level so that transmission of power is possible.
ii) Step down/Substation transformers – Located at main distribution or secondary level transmission substations. Its function is to lower the voltage levels for distribution 1st level purposes.
iii) Distribution Transformers – located at small distribution substation. It lowers the voltage levels for 2nd level distribution purposes.
iv) Special Purpose Transformers - E.g. Potential Transformer (PT) , Current Transformer (CT)

2. The Ideal Transformer

1. Definition – a lossless device with an input winding and an output winding.

2. Figures below show an ideal transformer and schematic symbols of a transformer.



3. The transformer has N_p turns of wire on its primary side and N_s turns of wire on its secondary sides. The relationship between the primary and secondary voltage is as follows:

$$\frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a$$

where *a* is the turns ratio of the transformer.

4. The relationship between primary and secondary current is:

$$N_p i_p(t) = N_s i_s(t)$$

$$\frac{i_p(t)}{i_s(t)} = \frac{1}{a}$$

5. Note that since both type of relations gives a constant ratio, hence the transformer only changes ONLY the magnitude value of current and voltage. Phase angles are not affected.

6. The dot convention in schematic diagram for transformers has the following relationship:

   i) If the primary **voltage** is +ve at the dotted end of the winding wrt the undotted end, then the secondary voltage will be positive at the dotted end also. Voltage polarities are the same wrt the dots on each side of the core.

   ii) If the primary **current** of the transformer flows **into** the dotted end of the primary winding, the secondary current will flow **out** of the dotted end of the secondary winding.

Power in an Ideal Transformer

1. The power supplied to the transformer by the primary circuit:

$$P_{in} = V_p I_p \cos \theta_p$$

Where θ_p = the angle between the primary voltage and the primary current. The power supplied by the transformer secondary circuit to its loads is given by:

$$P_{out} = V_s I_s \cos \theta_s$$

Where θ_s = the angle between the secondary voltage and the secondary current.

2. The primary and secondary windings of an ideal transformer have the SAME power factor – because voltage and current angles are unaffected $\theta_p - \theta_s = \theta$

3. How does power going into the primary circuit compare to the power coming out?

$P_{out} = V_s I_s \cos \theta$

Also, $V_s = V_p/a$ and $I_s = a\, I_p$

So, $P_{out} = \dfrac{V_p}{a}\left(aI_p\right)\cos\theta$

$P_{out} = V_p I_p \cos \theta = P_{in}$

The same idea can be applied for reactive power Q and apparent power S.

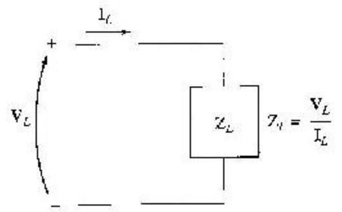
Output power = Input power

Impedance Transformation through a Transformer

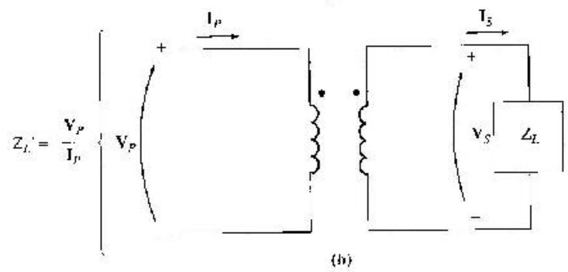
1. The impedance of a device or an element is defined as the ratio of the phasor voltage across it to the phasor current flowing through it:

$$Z_L = \frac{V_L}{I_L}$$

2. Definition of impedance and impedance scaling through a transformer:



(a)



(b)

3. Hence, the impedance of the load is: $$Z_L = \frac{V_S}{I_S}$$

4. The apparent impedance of the primary circuit of the transformer is: $$Z_L' = \frac{V_P}{I_P}$$

5. Since primary voltage can be expressed as $V_P = aV_S$, and primary current as $I_P = I_S/a$, thus the apparent impedance of the primary is

$$Z_L' = \frac{V_P}{I_P} = \frac{aV_S}{I_S / a} = a^2 \frac{V_S}{I_S}$$

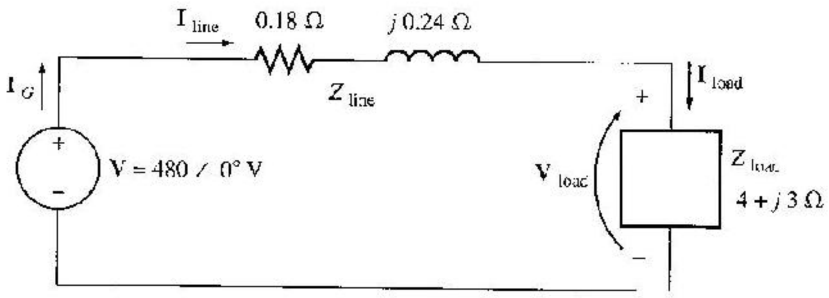
$$Z_L' = a^2 Z_L$$

Analysis of Circuits containing Ideal Transformers

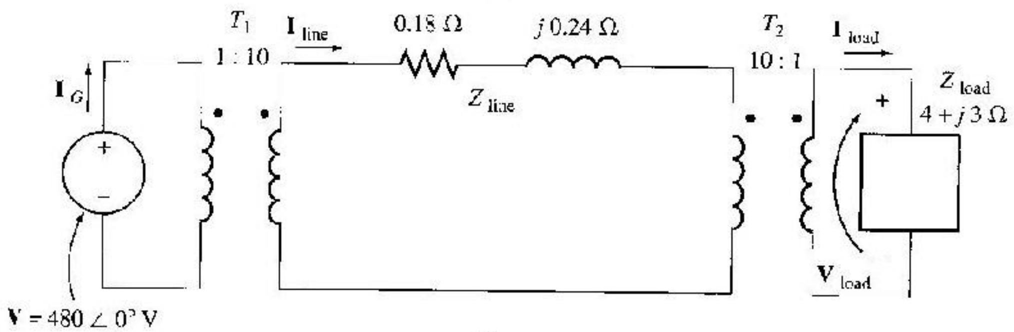
The easiest way for circuit analysis that has a transformer incorporated is by simplifying the transformer into an equivalent circuit.

Example 2.1

A generator rated at 480V, 60 Hz is connected a transmission line with an impedance of 0.18+j0.24Ω. At the end of the transmission line there is a load of 4+j3Ω.



(a)



(b)

(a) If the power system is exactly as described above in Figure (a), what will the voltage at the load be? What will the transmission line losses be?

(b) Suppose a 1:10 step-up transformer is placed at the generator end of the transmission line and a 10:1 step-down transformer is placed at the load end of the line (Figure (b)). What will the load voltage be now? What will the transmission line losses be now?

3. Theory of Operation of Real Single-Phase Transformers

Ideal transformers may never exist due to the fact that there are losses associated to the operation of transformers. Hence there is a need to actually look into losses and calculation of real single phase transformers.

Assume that there is a transformer with its primary windings connected to a varying single phase voltage supply, and the output is open circuit.

Right after we activate the power supply, flux will be generated in the primary coils, based upon Faraday's law,

$$e_{ind} = \frac{d\lambda}{dt}$$

where λ is the flux linkage in the coil across which the voltage is being induced. The flux linkage λ is the sum of the flux passing through each turn in the coil added over all the turns of the coil.

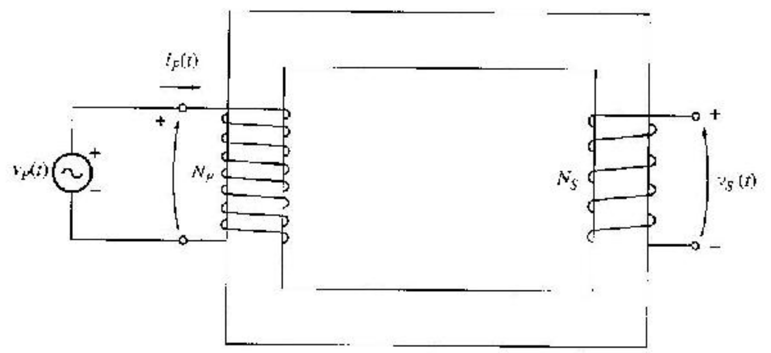
$$\lambda = \sum_{i=1}^{N} \phi_i$$

This relation is true provided on the assumption that the flux induced at each turn is at the same magnitude and direction. But in reality, the flux value at each turn may vary due to the position of the coil it self, at certain positions, there may be a higher flux level due to combination of other flux from other turns of the primary winding.

Hence the most suitable approach is to actually average the flux level as $\bar{\phi} = \frac{\lambda}{N}$

Hence Faraday's law may be rewritten as: $e_{ind} = N \frac{d\bar{\phi}}{dt}$

The voltage ratio across a Transformer



f the voltage source is $v_p(t)$, how will the transformer react to this applied voltage?
Based upon Faraday's Law, looking at the primary side of the transformer, we can determine the average flux level based upon the number of turns; where,

$$\bar{\phi} = \frac{1}{N_P} \int v_P(t)\, dt$$

This relation means that the average flux at the primary winding is proportional to the voltage level at the primary side divided by the number of turns at the primary winding. This generated flux will travel to the secondary side hence inducing potential across the secondary terminal.

For an ideal transformer, we assume that 100% of flux would travel to the secondary windings. However, in reality, there are flux which does not reach the secondary coil, in this case the flux leaks out of the transformer core into the surrounding. This leak is termed as **flux leakage**.

Taking into account the leakage flux, the flux that reaches the secondary side is termed as **mutual flux**.

Looking at the secondary side, there are similar division of flux; hence the overall picture of flux flow may be seen as below:

Primary Side:

$$\overline{\phi}_P = \phi_M + \phi_{LP}$$

$\overline{\phi}_P$ = total average primary flux

ϕ_M = flux component linking both rpimary and secondary coils

ϕ_{LP} = primary leakage flux

For the secondary side, similar division applies.

Hence, looking back at Faraday's Law,

$$v_P(t) = N_P \frac{d\overline{\phi}_P}{dt} = N_P \frac{d\overline{\phi}_M}{dt} + N_P \frac{d\overline{\phi}_{LP}}{dt}$$

Or this equation may be rewritten into:

$$v_P(t) = e_P(t) + e_{LP}(t)$$

The same may be written for the secondary voltage.

The primary voltage due to the mutual flux is given by

$$e_P(t) = N_P \frac{d\phi_M}{dt}$$

And the same goes for the secondary (just replace 'P' with 'S')

From these two relationships (primary and secondary voltage), we have

$$\frac{e_P(t)}{N_P} = \frac{d\phi_M}{dt} = \frac{e_S(t)}{N_S}$$

Therefore,

$$\frac{e_P(t)}{e_S(t)} = \frac{N_P}{N_S} = a$$

<u>Magnetization Current in a Real transformer</u>

Although the output of the transformer is open circuit, there will still be current flow in the primary windings. The current components may be divided into 2 components:

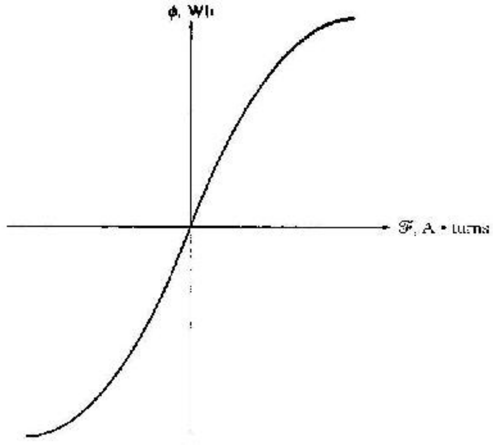
1) Magnetization current, i_M – current required to produce flux in the core.
2) Core-loss current, i_{h+e} – current required to compensate hysteresis and eddy current losses.

We know that the relation between current and flux is proportional since,

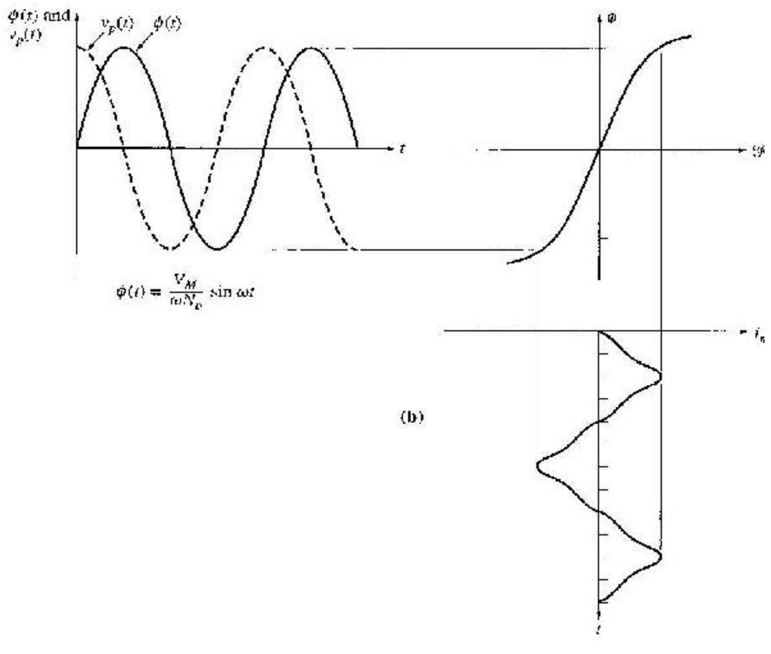
$$F = Ni = \phi R$$
$$\therefore i = \frac{\phi R}{N}$$

Therefore, in theory, if the flux produce in core is sinusoidal, therefore the current should also be a perfect sinusoidal. Unfortunately, this is not true since the transformer will reach to a state of near saturation at the top of the flux cycle. Hence at this point, more current is required to produce a certain amount of flux.

If the values of current required to produce a given flux are compared to the flux in the core at different times, it is possible to construct a sketch of the magnetization current in the winding on the core. This is shown below:



(a)



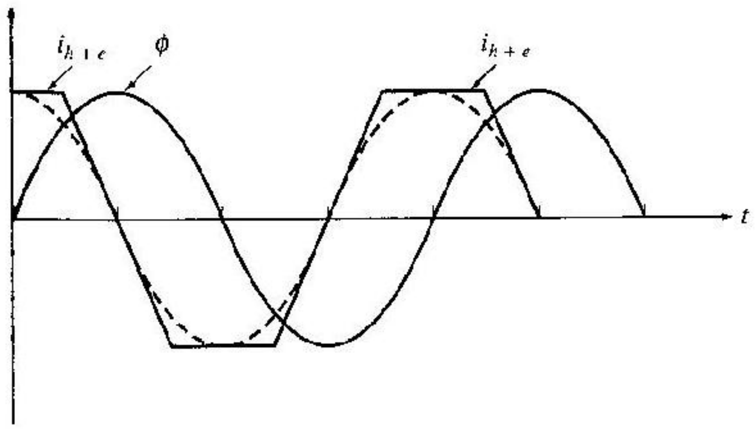
(b)

Hence we can say that current in a transformer has the following characteristics:

1. It is not sinusoidal but a combination of high frequency oscillation on top of the fundamental frequency due to magnetic saturation.
2. The current lags the voltage at 90^o
3. At saturation, the high frequency components will be extreme as such that harmonic problems will occur.

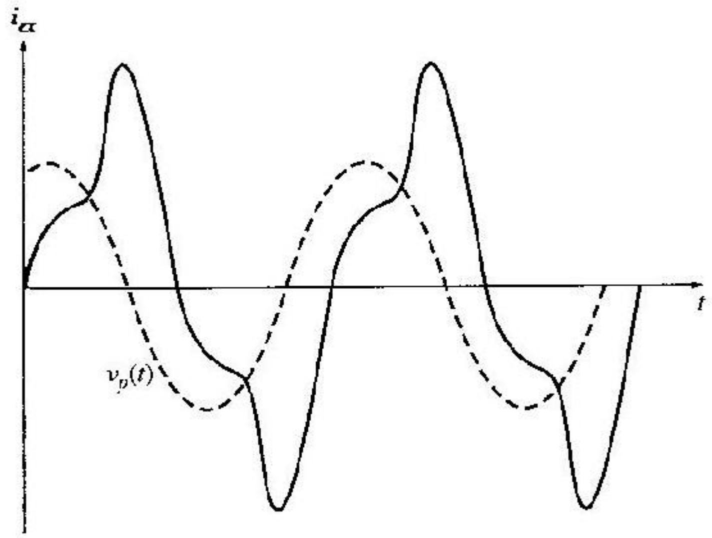
Looking at the core-loss current, it again is dependent upon hysteresis and eddy current flow. Since Eddy current is dependent upon the rate of change of flux, hence we can also say that the core-loss current is greater as the alternating flux goes past the 0 Wb. Therefore the core-loss current has the following characteristics:

a) When flux is at 0Wb, core-loss current is at a maximum hence it is in phase with the voltage applied at the primary windings.
b) Core-loss current is non-linear due to the non-linearity effects of hysteresis.



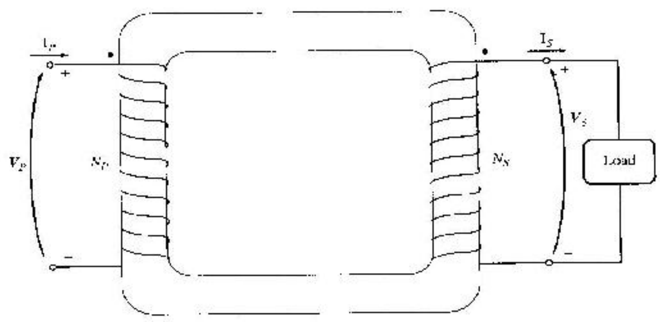
Now since that the transformer is not connected to any load, we can say that the total current flow into the primary windings is known as the **excitation current**.

$$i_{ex} = i_m + i_{h+e}$$



Current Ratio on a Transformer and the Dot Convention.

Now, a load is connected to the secondary of the transformer.



The dots help determine the polarity of the voltages and currents in the core withot having to examine physically the windings.

A current flowing into the dotted end of a winding produces a positive magnetomotive force, while a current flowing into the undotted end of a winding produces a negative magnetomotive force.

In the figure above, the net magnetomotive force is $F_{net} = N_P i_P - N_S i_S$

This net magnetomotive force must produce the net flux in the core, so

$F_{net} = N_P i_P - N_S i_S = \phi R$

Where R is the reluctance of the core. The relationship between primary and secondary current is approx

$F_{net} = N_P i_P - N_S i_S \approx 0$ as long as the core is unsaturated.

Thus,

$N_P i_P \approx N_S i_S$

$$\frac{i_P}{i_S} = \frac{N_S}{N_P} = \frac{1}{a}$$

In order for the magnetomotive force to be nearly zero, current must flow into one dotted end and out of the other dotted end.

As a conclusion, the major differences between an ideal and real transformer are as follows:

a) An ideal transformer's core does not have any hysteresis and eddy current losses.
b) The magnetization curve of an ideal transformer is similar to a step function and the net mmf is zero.
c) Flux in an ideal transformer stays in the core and hence leakage flux is zero.
d) The resistance of windings in an ideal transformer is zero.

4. The equivalent circuit of a transformer

Taking into account real transformer, there are several losses that has to be taken into account in order to accurately model the transformer, namely:

i) **Copper (I^2R) Losses** – Resistive heating losses in the primary and secondary windings of the transformer.
ii) **Eddy current Losses** – resistive heating losses in the core of the transformer. They are proportional to the square of the voltage applied to the transformer.
iii) **Hysteresis Losses** – these are associated with the rearrangement of the magnetic domains in the core during each half-cycle. They are complex, non-linear function of the voltage applied to the transformer.
iv) **Leakage flux** – The fluxes ϕ_{LP} and ϕ_{LS} which escape the core and pass through only one of the transformer windings are leakage fluxes. They then produced self-inductance in the primary and secondary coils.

The exact equivalent circuit of a real transformer

The Exact equivalent circuit will take into account all the major imperfections in real transformer.

i) Copper loss

They are modeled by placing a resistor R_P in the primary circuit and a resistor R_S in the secondary circuit.

ii) Leakage flux

As explained before, the leakage flux in the primary and secondary windings produces a voltage given by:

$$e_{LP}(t) = N_P \frac{d\phi_{LP}}{dt} \qquad e_{LS}(t) = N_S \frac{d\phi_{LS}}{dt}$$

Since flux is directly proportional to current flow, therefore we can assume that leakage flux is also proportional to current flow in the primary and secondary windings. The following may represent this proportionality:

$$\phi_{LP} = (PN_P)i_P$$

$$\phi_{LS} = (PN_S)i_S$$

Where P = permeance of flux path
N_P = number of turns on primary coils
N_S = number of turns on secondary coils

Thus,

$$e_{LP}(t) = N_P \frac{d}{dt}(PN_P)i_P = N_P{}^2 P \frac{di_P}{dt}$$

$$e_{LS}(t) = N_S \frac{d}{dt}(PN_S)i_S = N_S{}^2 P \frac{di_S}{dt}$$

The constants in these equations can be lumped together. Then,

$$e_{LP}(t) = L_P \frac{di_P}{dt}$$

$$e_{LS}(t) = L_S \frac{di_S}{dt}$$

Where $L_P = N_P^2 P$ is the self-inductance of the primary coil and $L_S = N_S^2 P$ is the self-inductance of the secondary coil.

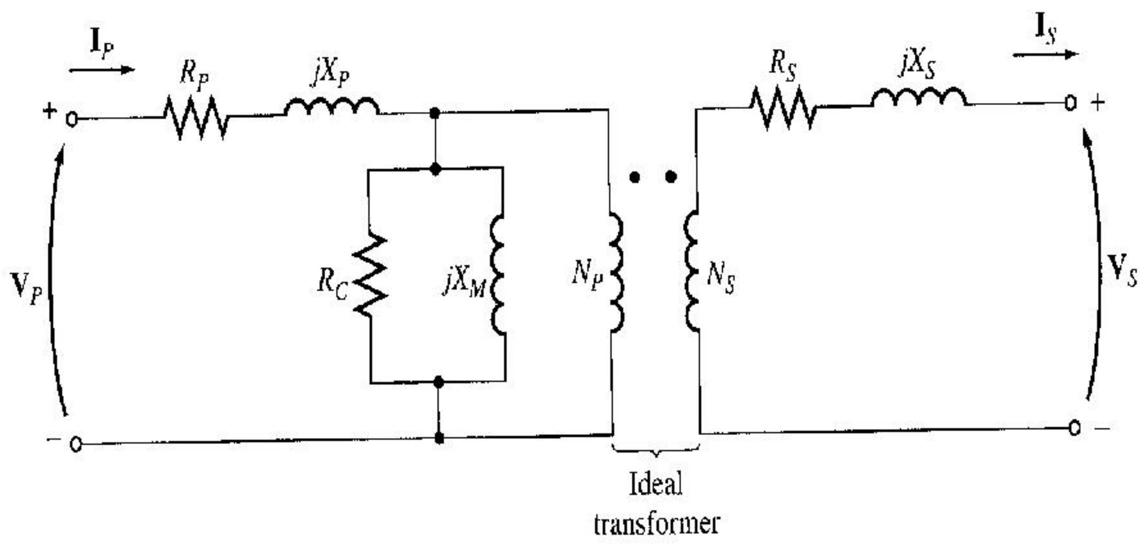
Therefore the **leakage element may be modelled as an inductance** connected together in series with the primary and secondary circuit respectively.

iii) Core excitation effects – magnetization current and hysteresis & eddy current losses

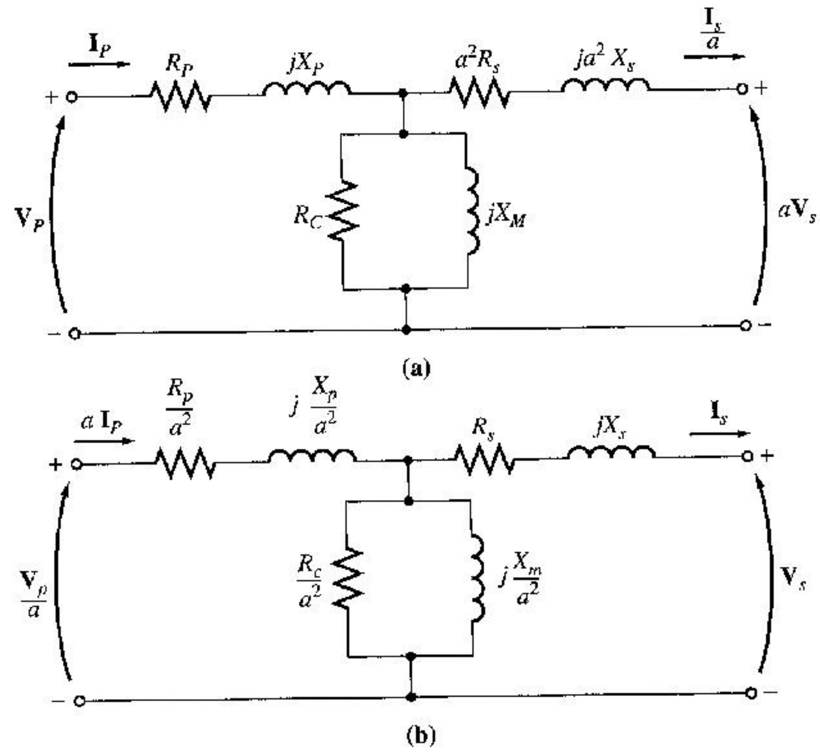
The magnetization current i_m is a current proportional (in the unsaturated region) to the voltage applied to the core and lagging the applied voltage by 90° - modeled as reactance X_m across the primary voltage source.

The core loss current i_{h+e} is a current proportional to the voltage applied to the core that is in phase with the applied voltage – modeled as a resistance R_C across the primary voltage source.

The resulting equivalent circuit:



Based upon the equivalent circuit, in order for mathematical calculation, this transformer equivalent has to be simplified by referring the impedances in the secondary back to the primary or vice versa.

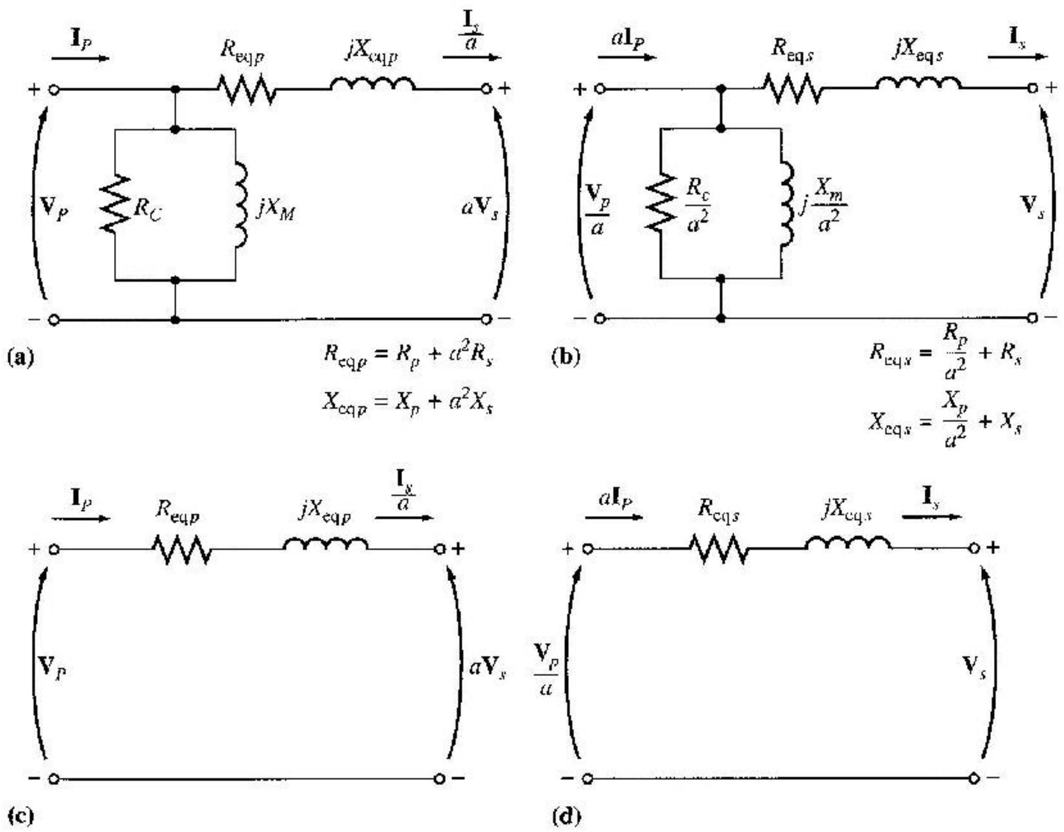


(a) Equivalent transformer circuit referring to the primary
(b) Equivalent transformer circuit referring to the secondary

Approximate Equivalent circuits of a Transformer

The derived equivalent circuit is detailed but it is considered to be too complex for practical engineering applications. The main problem in calculations will be the excitation and the eddy current and hysteresis loss representation adds an extra branch in the calculations.

In practical situations, the excitation current will be relatively small as compared to the load current, which makes the resultant voltage drop across Rp and Xp to be very small, hence Rp and Xp may be lumped together with the secondary referred impedances to form and equivalent impedance. In some cases, the excitation current is neglected entirely due to its small magnitude.



(a) Referred to the primary side
(b) Referred to the secondary side
(c) With no excitation branch, referred to the primary side
(d) With no excitation branch, referred to the secondary side