The instantaneous torque is pulsating. The average value of the torque is

$$T_{\text{avg}} = -\frac{L_{\text{sm}}L_{\text{rm}}M}{4}\sin(\alpha + \delta) \tag{3.56}$$

This is the basic principle of operation of an *induction machine,* in which the stator winding is excited by an ac current and ac current is induced in the rotor winding. Note that the single-phase induction machine is also not self-starting, because at $\omega_m = 0$ no average torque is developed. The machine is brought up to the speed $\omega_m = \omega_s - \omega_r$ so that it can produce an average torque. To eliminate pulsating torque, polyphase induction machines are used for high-power applications.

The mechanism of torque production in electromagnetic systems producing both translational and rotary motions has been discussed in this chapter. In rotating machines torque can be produced by variation in the reluctance of the magnetic path or mutual inductance between the windings.

Reluctance machines are simple in construction, but torque developed in these machines is small. Cylindrical machines, although more complex in construction, produce larger torques. Most electrical machines are of the cylindrical type. The performance of the various rotating electrical machines is discussed in more detail in the following chapters.

PROBLEMS

3.1 In a translational motion actuator, the λ–i relationship is given by

$$i = \lambda^{3/2} + 2.5\lambda(x - 1)^2$$

for $0 < x < 1$ m, where i is the current in the coil of the actuator. Determine the force on the moving part at $x = 0.6$ m.

3.2 The λ–i relationship for an electromagnetic system is given by

$$\lambda = \frac{1.2i^{1/2}}{g}$$

where g is the air gap length. For current $i = 2$ A and $g = 10$ cm, determine the mechanical force on the moving part

(a) Using the energy of the system.

(b) Using the coenergy of the system.

3.3 An actuator system is shown in Fig. P3.3. All dimensions are in centimeters. The magnetic material is cast steel, whose magnetization characteristic is shown in Fig. 1.7. The magnetic core and air gap have a square cross-sectional area. The coil has 500 turns and 4.0 ohms resistance.

(a) The gap is $d = 1$ mm.

(i) Determine the coil current and supply voltage (dc) required to establish an air gap flux density of 0.5 tesla.

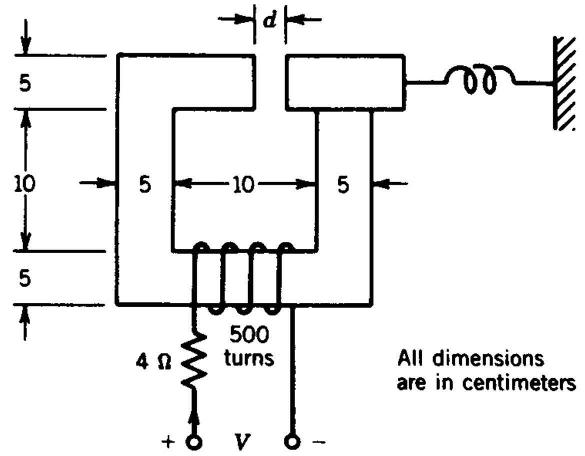

FIGURE P3.3

- **(ii)** Determine the stored energy in the actuator system.
- **(iii)** Determine the force of attraction on the actuator arm.
- **(iv)** Determine the inductance of the coil.

(b) The actuator arm is allowed to move and finally the air gap closes.

- **(i)** For zero air gap determine the flux density in the core, force on the arm, and stored energy in the actuator system.
- **(ii)** Determine the energy transfer (excluding energy loss in the coil resistance) between the dc source and the actuator. Assume that the arm moved slowly. What is the direction of energy flow? How much mechanical energy is produced?

3.4 Fig. P3.4 shows an electromagnet system for lifting a section of steel channel. The coil has 600 turns. The reluctance of the magnetic material can be neglected up to a flux density of 1.4 tesla.

(a) For a coil current of 15 A (dc) determine the maximum air gap g for which the flux density is 1.4 tesla.

(b) For the air gap in part (a), determine the force on the steel channel.

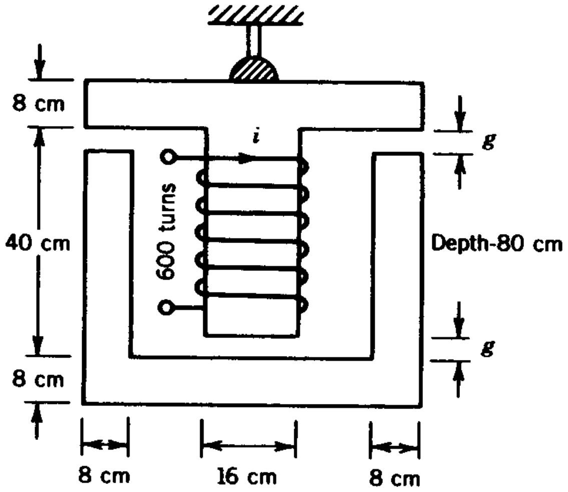

FIGURE P3.4

(c) The steel channel has a mass of 1000 kg. For a coil current of 15 A, determine the largest gap at which the steel channel can be lifted magnetically against the force of gravity (9.81 m/sec^2).

3.5 An electromagnet lift system is shown in Fig. P3.5. The coil has 2500 turns. The flux density in the air gap is 1.25 T. Assume that the core material is ideal.

(a) For an air gap, $g = 10$ mm:

- **(i)** Determine the coil current.
- **(ii)** Determine the energy stored in the magnetic system.
- **(iii)** Determine the force on the load (sheet of steel).
- **(iv)** Determine the mass of the load (acceleration due to gravity = 9.81 m/sec^2).

(b) If the air gap is 5 mm, determine the coil current required to lift the load.

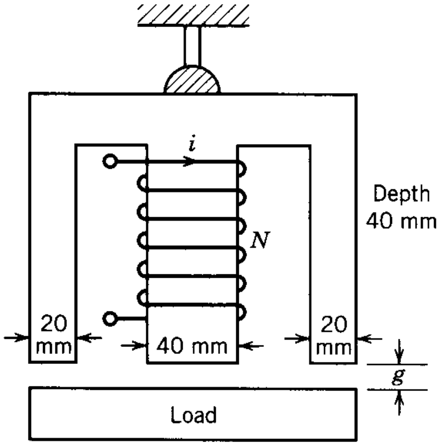

FIGURE P3.5

3.6 The cross section of a cylindrical magnetic actuator is shown in Fig. P3.6. The plunger has a cross-sectional area 0.0016 m^2. The coil has 2500 turns and a resistance of 10 Ω. A voltage of 15 V (dc) is applied to the coil terminals. Assume that the magnetic material is ideal.

(a) Determine the air gap g in mm for which the flux density in the air gap is 1.5 T. Determine the stored energy for this condition.

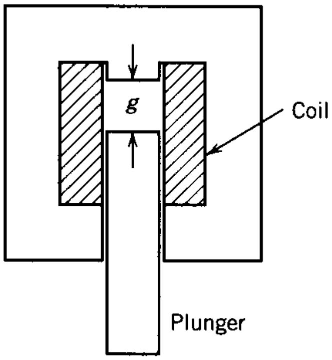

FIGURE P3.6

(b) Obtain an expression for the force on the plunger as a function of the air gap length g.

(c) Determine the force on the plunger for the condition of part (a).

(d) Suppose the plunger moves quickly from an initial gap of 5 mm to the fully closed position. The plunger moves so quickly that the flux linkage of the coil (and hence the flux density in the air gap) hardly changes during the motion.

  (i) Determine the force during the motion.

  (ii) Determine the amount of mechanical energy produced during the motion.

3.7 The electromagnet shown in Fig. P3.7 can be used to lift a sheet of steel. The coil has 400 turns and a resistance of 5 ohms. The reluctance of the magnetic material is negligible. The magnetic core has a square cross section of 5 cm by 5 cm. When the sheet of steel is fitted to the electromagnet, air gaps, each of length $g = 1$ mm, separate them. An average force of 550 newtons is required to lift the sheet of steel.

(a) For dc supply,

  (i) Determine the dc source voltage.

  (ii) Determine the energy stored in the magnetic field.

(b) For ac supply at 60 Hz, determine the ac source voltage.

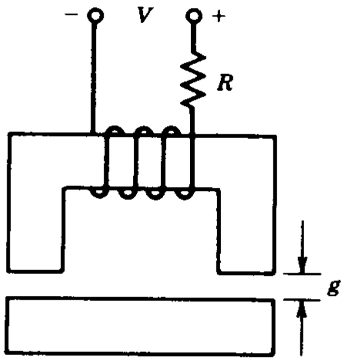

FIGURE P3.7

3.8 The features of a moving-iron ammeter are shown in Fig. P3.8. When current flows through the curved solenoid coil, a curved ferromagnetic rod is pulled into the solenoid against the torque of a restraining spring. The inductance of the coil is $L = 4.5 + 18\theta\ \mu$H, where θ is angle of deflection in radians. The spring constant is 0.65×10^{-3} N · m/rad.

(a) Show that the ammeter measures the root-mean-square value of the current.

(b) Determine the deflection in degrees for a current of 10 amperes (rms).

(c) Determine the voltage drop across the ammeter terminal when 10 A (rms) at 60 Hz flows through the ammeter. The coil resistance is 0.015 Ω.

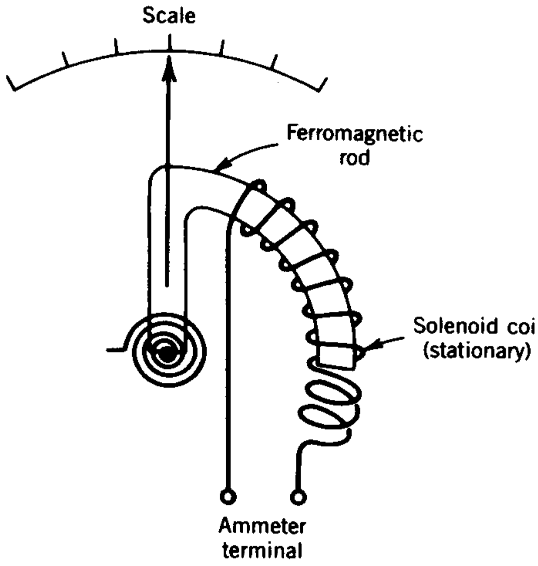

FIGURE P3.8

3.9 A reluctance machine of the form shown in Fig. 3.6 has no rotor winding. The inductance of the stator winding is

$$L_{ss} = 0.1 - 0.3\cos 2\theta - 0.2\cos 4\theta\,\text{H}$$

A current of 10 A (rms) at 60 Hz is passed through the stator coil.

(a) Determine the values of speed (ω_m) of the rotor at which the machine will develop an average torque.

(b) Determine the maximum torque and power (mechanical) that could be developed by the machine at each speed.

(c) Determine the maximum torque at zero speed.

3.10 A reluctance motor with four rotor poles is shown in Fig. P3.10. The reluctance ($\mathcal{R}$) of the magnetic system can be assumed to be a sinusoidally varying function of θ and is given by

$$\mathcal{R}(\theta) = 2 \times 10^5 - 10^5\cos(4\theta)$$

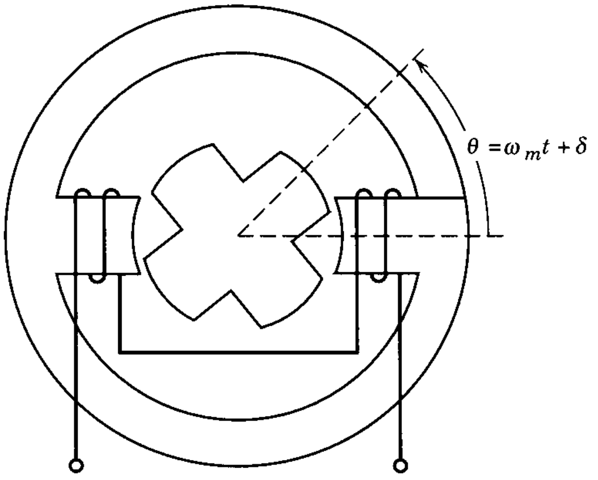

FIGURE P3.10

The coil has 200 turns and negligible resistance and is connected to a 120 V, 60 Hz, single-phase supply.

(a) Obtain an expression for the flux (Φ) as a function of time. (*Hint*: $v = N\,d\Phi/dt$.)

(b) Show that the torque developed is

$$T = \frac{1}{2}\Phi^2 \frac{d\mathcal{R}}{d\theta}$$

(c) Determine the values of speed (ω_m) of the rotor at which the machine will develop an average torque.

(d) Determine the maximum torque and power (mechanical) that could be developed by the machine at each speed.

3.11 The rotating machine of Fig. 3.7 has the following parameters.

$$L_{ss} = 0.15\ \text{H}$$

$$L_{rr} = 0.06\ \text{H}$$

$$L_{sr} = 0.08 \cos\theta\ \text{H}$$

(a) The rotor is driven at 3600 rpm. If the stator winding carries a current of 5 A (rms) at 60 Hz, determine the instantaneous voltage and rms voltage induced in the rotor coil. Determine the frequency of the rotor induced voltage.

(b) Suppose the stator and rotor coils are connected in series and a current of 5 A (rms) at 60 Hz is passed through them. Determine the speeds at which the machine will produce an average torque. Also determine the maximum torque that the machine will produce at each speed.