بسم الله الرحمن الرحیم

فصل چهارم

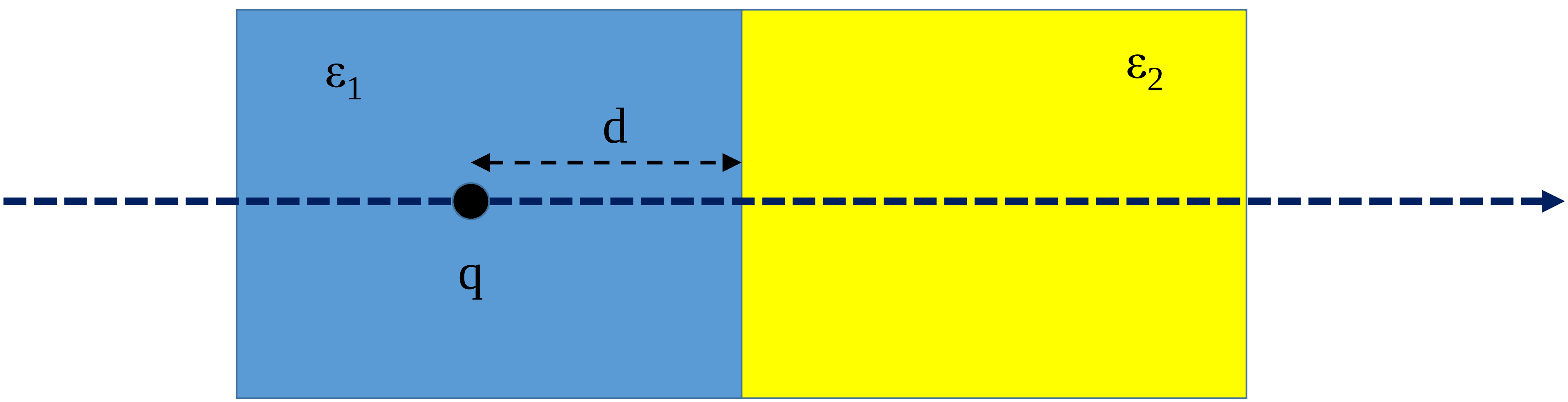
میدان الکتروستاتیک در محیط های دی الکتریک

بخش پنجم

Method of Images for Dielectric Interfaces

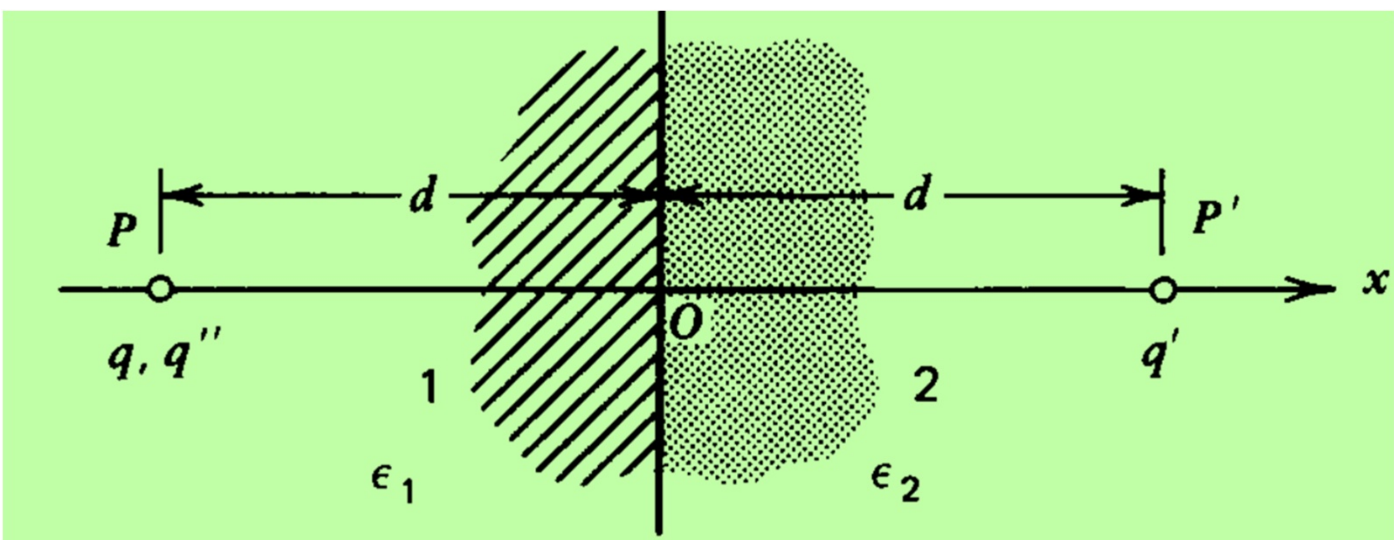
point charge q is placed at a point P, which is at a distance d from the boundary of two semi-infinite homogeneous dielectrics of permittivities ε_1 and ε_2.

The relocation of the charge breaks the symmetry, and therefore simple methods as mentioned above are not very useful; the fields become dependent on distances and angles.



We will take the approach of first choosing a reasonable number of image charges with reasonable locations. These choices can then be tested by requiring that the produced fields satisfy the boundary conditions.

We take the potential in region 1 to be represented by the charge *q and an image charge of* magnitude *q' located in region 2 at distance d on the x axis. The potential in region* 2, however, will be only that of an image charge *q'' located at P (that is, at the* location of the original charge *q)*



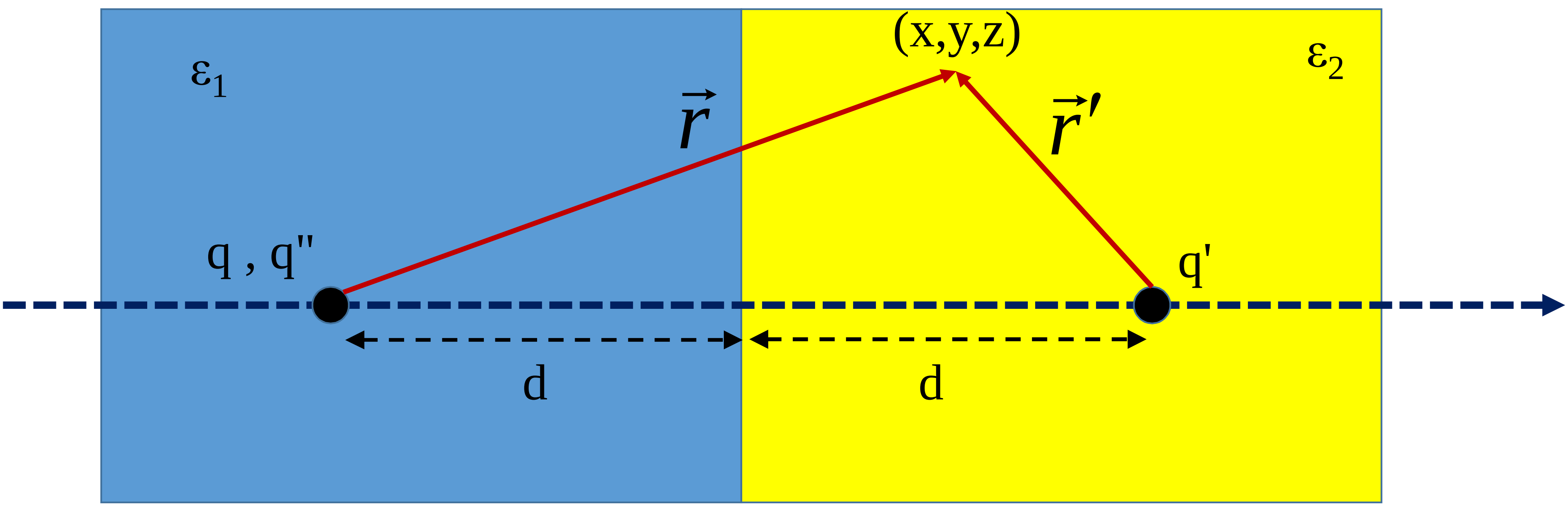
$$\Phi_1(\mathbf{r}) = \frac{1}{4\pi\varepsilon_1}\left(\frac{q}{r} + \frac{q'}{r'}\right) \qquad \text{for } x < 0$$

$$\Phi_2(\mathbf{r}) = \frac{1}{4\pi\varepsilon_2}\frac{q''}{r} \qquad \text{for } x > 0$$

q' and q'' magnitudes of the image charges

boundary conditions by determining physical values for charges ***q' and q''***

$$
\begin{cases} r = [(x+d)^2 + y^2 + z^2]^{1/2} \\ r' = [(x-d)^2 + y^2 + z^2]^{1/2} \end{cases}
$$



boundary conditions

The potential should be continuous at x = 0; that is,

$$\Phi_1(x = 0, y, z) = \Phi_2(x = 0, y, z).$$

$$\Phi_1(\mathbf{r}) = \frac{1}{4\pi\varepsilon_1}\left(\frac{q}{r} + \frac{q'}{r'}\right) \qquad \text{for } x < 0$$

$$\Phi_2(\mathbf{r}) = \frac{1}{4\pi\varepsilon_2}\frac{q''}{r} \qquad \text{for } x > 0$$

$$\frac{1}{\varepsilon_1}(q + q') = \frac{q''}{\varepsilon_2}$$

2. The normal component of the displacement vector is continuous on the boundary since it has no free charges

$$(\mathbf{D}_2 - \mathbf{D}_1) \cdot \mathbf{n}_2 = \sigma \quad \xrightarrow{\sigma = 0}$$

$$D_{1n}(x = 0, y, z) = D_{2n}(x = 0, y, z)$$

$$\varepsilon_1\, \partial\Phi_1/\partial x = \varepsilon_2\, \partial\Phi_2/\partial x \text{ at } x = 0.$$

$$\Rightarrow \quad q'' = q - q'$$

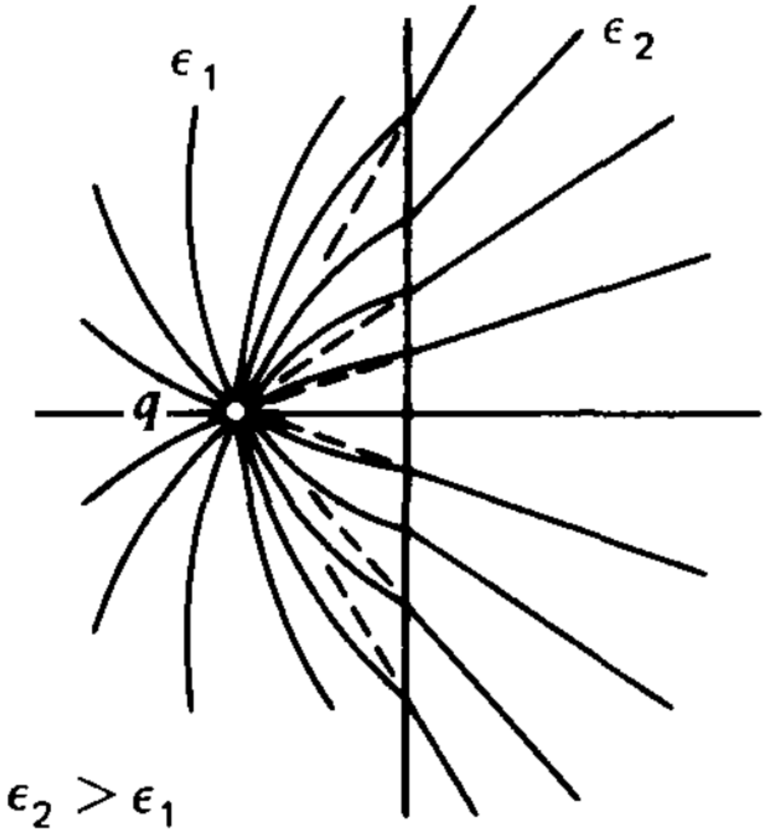
$$\begin{cases} \dfrac{1}{\varepsilon_1}(q + q') = \dfrac{q''}{\varepsilon_2} \\ q'' = q - q' \end{cases}$$

$$q' = -\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2} q$$

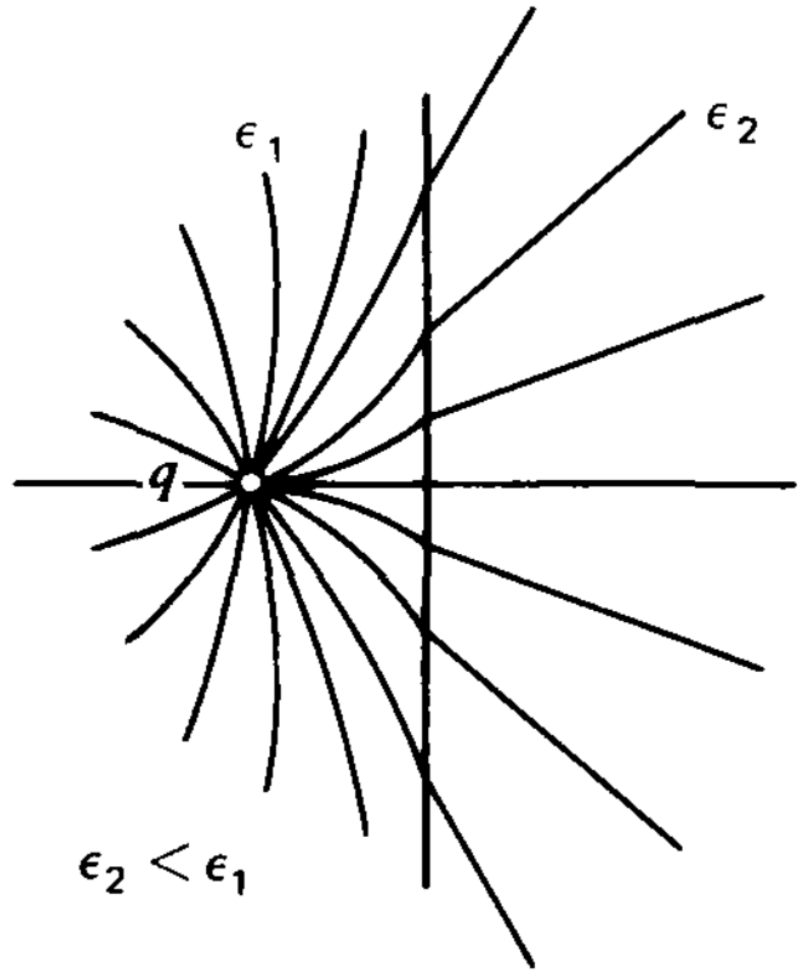
$$q'' = \frac{2\varepsilon_2}{\varepsilon_1 + \varepsilon_2} q$$

$$\Phi_1 = \frac{q}{4\pi\varepsilon_1}\left[\frac{1}{r} - \left(\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2}\right)\frac{1}{r'}\right]$$

$$\Phi_2 = \frac{2q}{4\pi(\varepsilon_1 + \varepsilon_2)}\frac{1}{r}$$



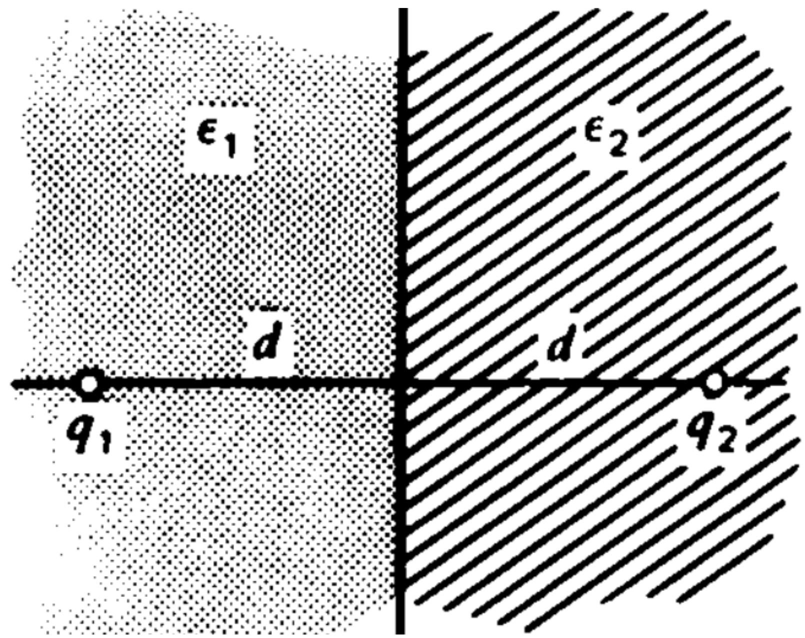
q' has the opposite sign of q



q' has the same sign as q

Example: Forces Between Charges Embedded in Dielectric Materials

two semi-infinite homogeneous dielectrics 1 and 2 with permittivitics ε_1 and ε_2, *respectively. Two charges* q_1 *and* q_2 are placed in media 1 and 2, respectively, each at a distance d from the interface and with the line between the charges normal to the interface.



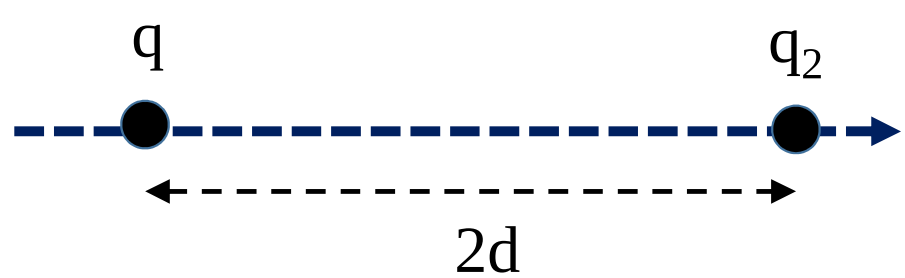
The charge q_2 experiences two types of forces; one type is due to the presence of charge q_1 and the other is due to its proximity to the interface:

1- the first force: $$F_{1\to 2} = q_2 E_1$$

E_1 is the electric field caused by q_1 at the site of q_2.
the field E_1 caused by charge q_1 *in region 2 is produced by a charge q located at the position of* q_1

$$q = 2\varepsilon_2 q_1/(\varepsilon_1 + \varepsilon_2)$$

$$F_{1\to 2} = \frac{q_1 q_2}{8\pi(\varepsilon_1 + \varepsilon_2)d^2}\hat{n}$$



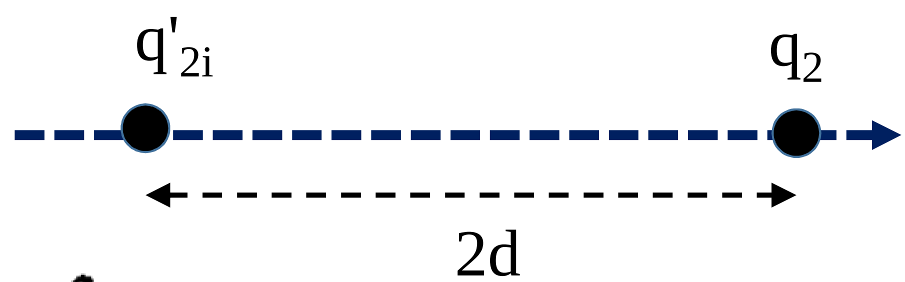
Where **n** is a unit vector along the line joining the charges.

2- the second force:

produced by the induced charge at the interface. This force can alternatively be calculated from the image charges.

$$q'_{2i} = -\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2} q$$

image charge the induced charge at the interface



$$\mathbf{F}_{2i} = \frac{1}{16\pi\varepsilon_2} \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2} \frac{q_2^2}{d^2} \hat{\mathbf{n}}$$

$$\mathbf{F}_2 = \mathbf{F}_{21} + \mathbf{F}_{2i}$$

$$\mathbf{F}_2 = \left[\frac{1}{16\pi\varepsilon_2}\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2}\frac{q_2^2}{d^2} + \frac{1}{8\pi(\varepsilon_1 + \varepsilon_2)}\frac{q_1 q_2}{d^2}\right]\hat{\mathbf{n}}$$

$$\mathbf{F}_1 = -\left[\frac{1}{16\pi\varepsilon_1}\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}\frac{q_1^2}{d^2} + \frac{1}{8\pi(\varepsilon_1 + \varepsilon_2)}\frac{q_1 q_2}{d^2}\right]\hat{\mathbf{n}}$$

A point charges near spherical dielectric interfaces.

?