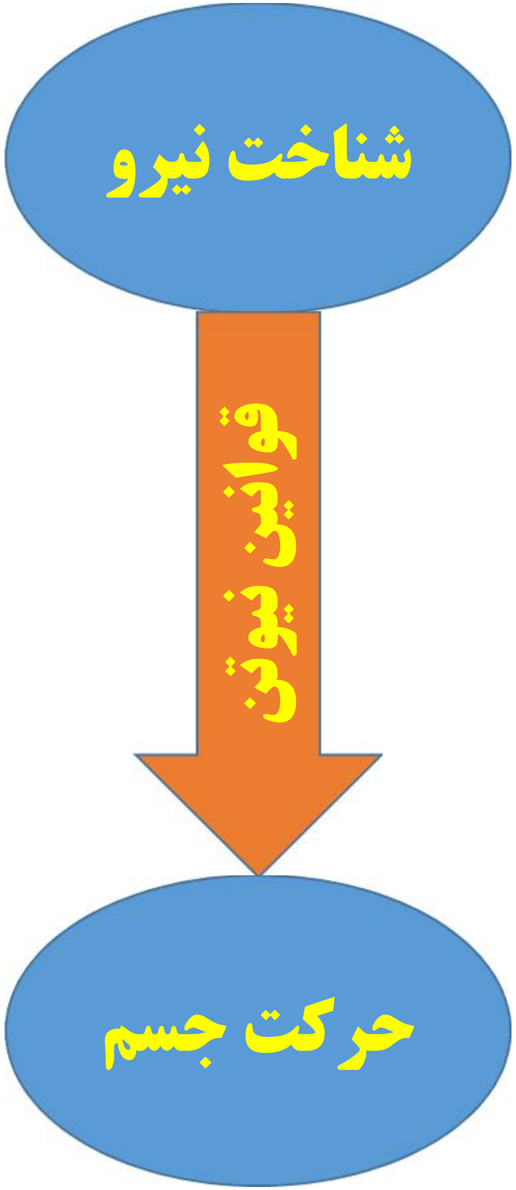
فصل ششم

تحلیل تکانه در سیستم های سیال جاری

قوانین بقا و قوانین نیوتن



در حرکت خطی

$$\vec{F} = m\vec{a} \quad \rightarrow \quad \vec{F} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} \quad \rightarrow \quad \vec{F} = \frac{d\vec{P}}{dt}$$

$$\vec{F} = 0 \quad \rightarrow \quad \vec{P} = cons\tan t$$

در حرکت دورانی

$$\vec{M} = I\vec{\sim} \quad \rightarrow \quad \vec{M} = I\frac{d\vec{\mu}}{dt} = \frac{dI\vec{\mu}}{dt} \quad \rightarrow \quad \vec{M} = \frac{d\vec{H}}{dt}$$

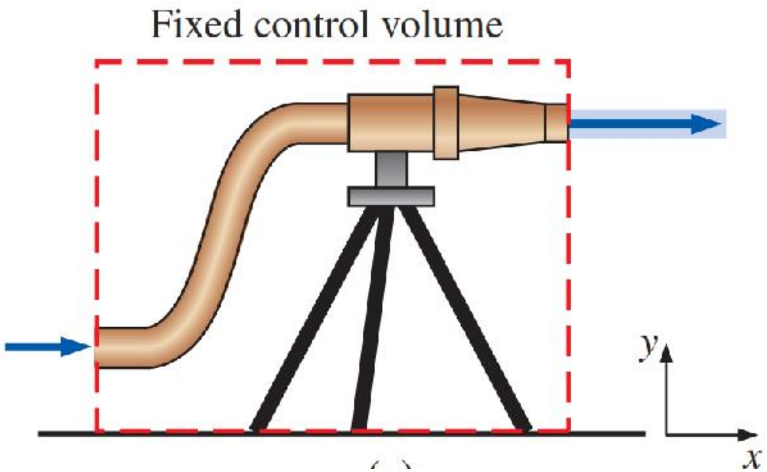
$$\vec{M} = 0 \quad \rightarrow \quad \vec{H} = cons\tan t$$

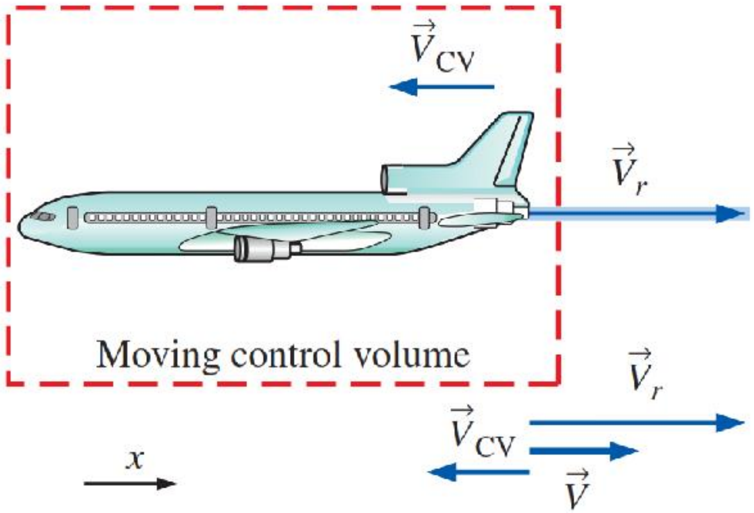
حجم معیار و نیروهای وارد بر آن

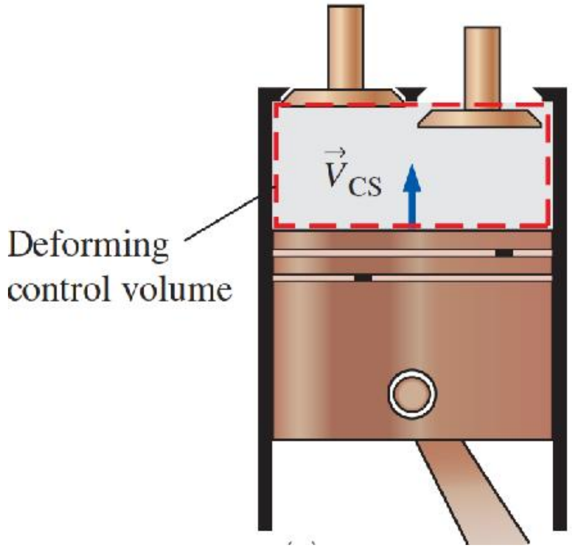
حجم معیار ثابت و ساکن

حجم معیار ثابت و متحرک

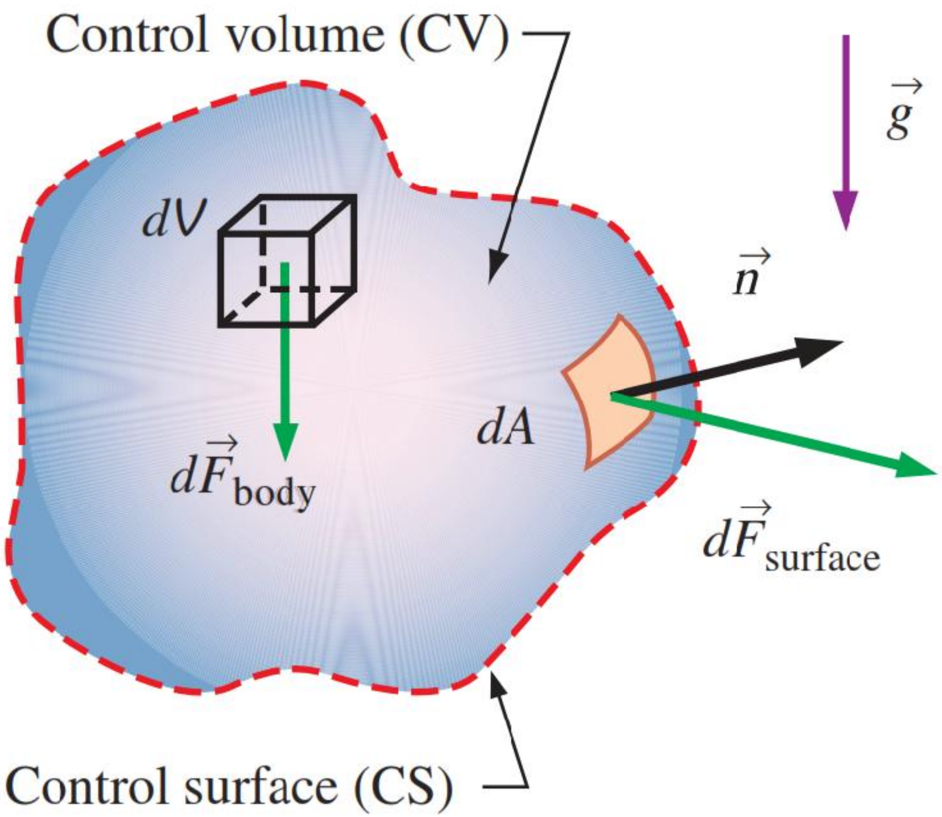
حجم معیار با تغییر شکل







$$\sum \vec{F} = \sum \vec{F}_{\text{body}} + \sum \vec{F}_{\text{surface}}$$



نیروی گرانش وارد بر حجم کنترل $\sum \vec{F}_{\text{body}}$

نیروی سطحی وارد بر حجم کنترل $\sum \vec{F}_{\text{surface}}$

نیروی گرانش وارد بر حجم کنترل

$$d\vec{F}_{\text{gravity}} = \rho\vec{g}\, dV$$

$$\sum \vec{F}_{\text{body}} = \int_{\text{CV}} \rho\vec{g}\, dV = m_{\text{CV}}\vec{g}$$

نیروی سطحی وارد بر حجم کنترل

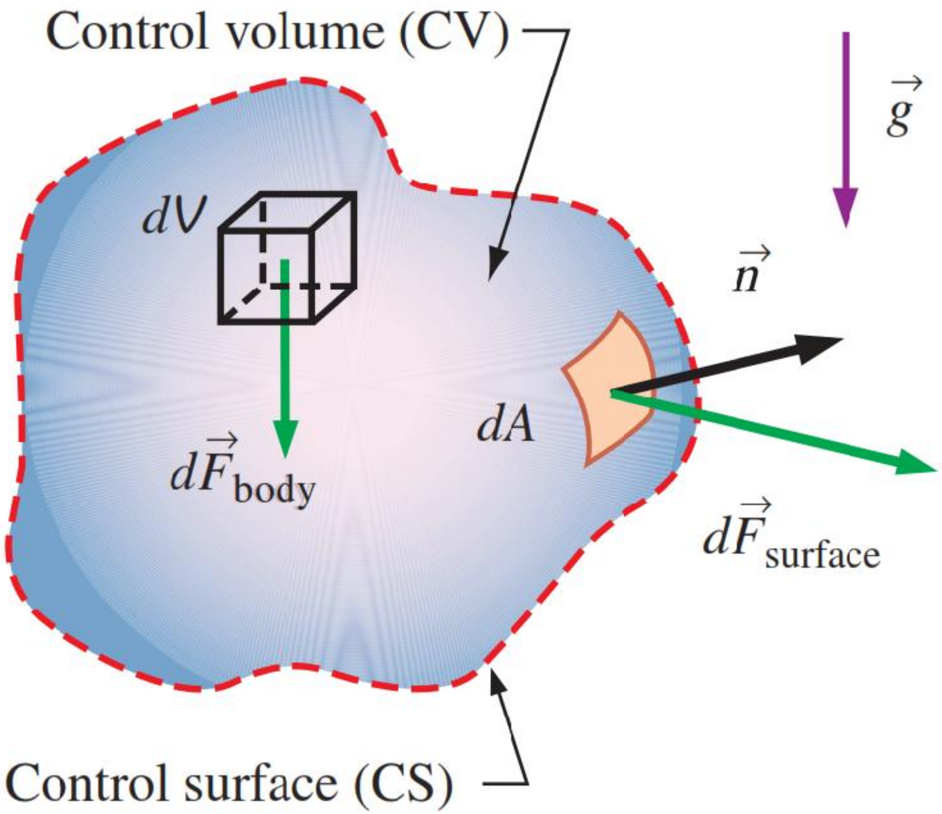
تانسور تنش

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

فشار

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

تنش برشی



$$d\vec{F}_{\text{surface}} = \sigma_{ij}\cdot\vec{n}\,dA$$

$$\sum\vec{F}_{\text{surface}} = \int_{\text{CS}} \sigma_{ij}\cdot\vec{n}\,dA$$

$$\sum \vec{F} = \sum \vec{F}_{\text{body}} + \sum \vec{F}_{\text{surface}} = \int_{\text{CV}} \rho \vec{g}\, dV + \int_{\text{CS}} \sigma_{ij} \cdot \vec{n}\, dA$$

نیروی کل وارد بر حجم کنترل

$$\underbrace{\sum \vec{F}}_{\text{total force}} = \underbrace{\sum \vec{F}_{\text{gravity}}}_{\text{body force}} + \underbrace{\sum \vec{F}_{\text{pressure}} + \sum \vec{F}_{\text{viscous}} + \sum \vec{F}_{\text{other}}}_{\text{surface forces}}$$

معادله تکانه خطی

قضیه رینولدز برای حجم کنترل
RTT

$$\frac{dN_{Sys}}{dt} = \int_{CV} \frac{\partial}{\partial t}(\beta \rho)\, dV + \int_{CS} \beta \rho\, \vec{v}.\hat{n}\, dA$$

$$N = \vec{P} = m\vec{V} \quad , \quad \beta = \frac{m\vec{V}}{m} = \vec{V}$$

$$\frac{d(m\vec{V})_{\text{sys}}}{dt} = \frac{d}{dt}\int_{\text{CV}} \rho\vec{V}\, dV + \int_{\text{CS}} \rho\vec{V}\,(\vec{V}_r \cdot \vec{n})\, dA$$

$$\sum \vec{F} = \frac{d}{dt}\int_{\text{CV}} \rho\vec{V}\, dV + \int_{\text{CS}} \rho\vec{V}(\vec{V}_r \cdot \vec{n})\, dA$$

$$\sum \vec{F} = \frac{d}{dt}\int_{\mathrm{CV}} \rho \vec{V}\, d\mathcal{V} + \int_{\mathrm{CS}} \rho \vec{V}(\vec{V}_r \cdot \vec{n})\, dA$$

$$\begin{pmatrix}\text{The sum of all}\\ \text{external forces}\\ \text{acting on a CV}\end{pmatrix} = \begin{pmatrix}\text{The time rate of change}\\ \text{of the linear momentum}\\ \text{of the contents of the CV}\end{pmatrix} + \begin{pmatrix}\text{The net flow rate of}\\ \text{linear momentum out of the}\\ \text{control surface by mass flow}\end{pmatrix}$$

Fixed CV: $$\sum \vec{F} = \frac{d}{dt}\int_{\mathrm{CV}} \rho \vec{V}\, d\mathcal{V} + \int_{\mathrm{CS}} \rho \vec{V}(\vec{V} \cdot \vec{n})\, dA$$

تحلیل تکانه در جریان های دائم

$$\frac{d}{dt}\int_{CV} \rho\vec{V}\,dV = 0$$

$$\sum \vec{F} = \int_{CS} \rho\vec{V}\,(\vec{V}_r\cdot\vec{n})\,dA$$

تعریف دبی جرمی

$$\dot{m} = \int_{A_c} \rho(\vec{V}\cdot\vec{n})\,dA_c = \rho V_{avg} A_c$$

در ورودی و خروجی یکنواخت

$$\sum \vec{F} = \int_{A_c} \rho\vec{V}(\vec{V}\cdot\vec{n})\,dA_c = \rho V_{avg} A_c \vec{V}_{avg} = \dot{m}\vec{V}_{avg}$$

ورودی و خروجی غیر یکنواخت (تصحیح شار تکانه)

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V}\, dV + \sum_{out} \beta \dot{m} \vec{V}_{avg} - \sum_{in} \beta \dot{m} \vec{V}_{avg}$$

ضریب تصحیح شار تکانه

در جریان های دائم

$$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$

همان سرعت متوسط در مقطع

مثال: در غیاب نیروهای خارجی مانند وزن و فشار و نیروهای عکس العملی در جهت حرکت

$$\textit{No external forces:} \qquad 0 = \frac{d(m\vec{V})_{CV}}{dt} + \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$

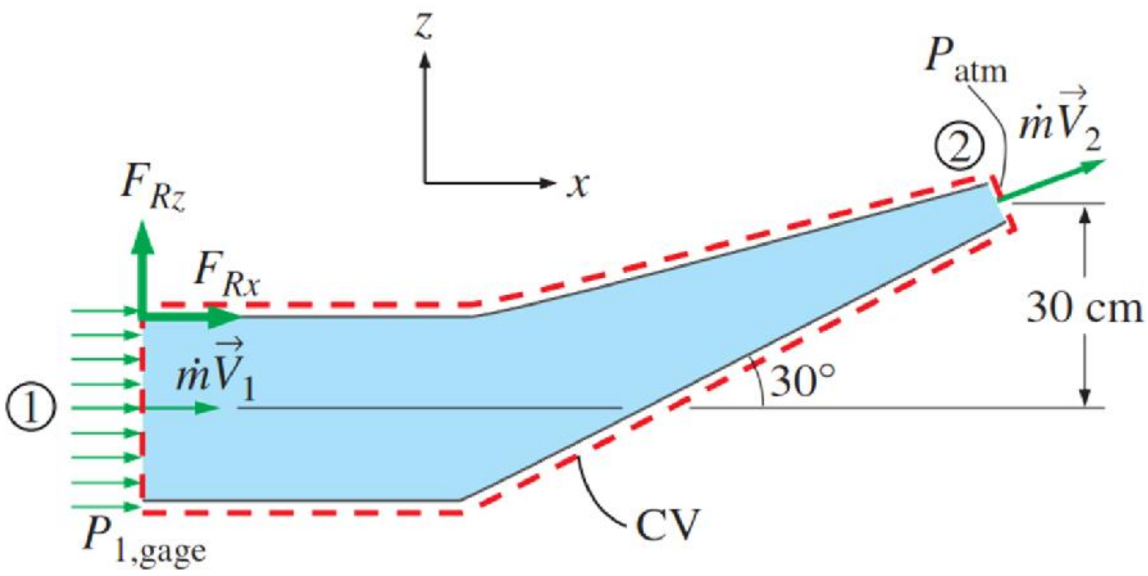
در لحظه ای کوتاه جرم حجم معیار ثابت

$$\frac{d(m\vec{V})_{CV}}{dt} = m_{CV} \frac{d\vec{V}_{CV}}{dt} = (m\vec{a})_{CV} = m_{CV}\vec{a}$$

$$\vec{F}_{thrust} = m_{body}\vec{a} = \sum_{in} \beta \dot{m} \vec{V} - \sum_{out} \beta \dot{m} \vec{V}$$

Example: The Force to Hold a Deflector Elbow in Place

A reducing elbow is used to deflect water flow at a rate of 14 kg/s in a horizontal pipe upward 30° while accelerating it (Fig. 6?0). The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is 113 cm^2 at the inlet and 7 cm^2 at the outlet. The elevation difference between the centers of the outlet and the inlet is 30 cm. The weight of the elbow and the water in it is considered to be negligible. Determine (a) the gage pressure at the center of the inlet of the elbow and (b) the anchoring force needed to hold the elbow in place.



استفاده از معادله پایستگی برای بدست آوردن سرعت در مقاطع

$$\dot{m}_1 = \dot{m}_2 = \dot{m} = 14 \text{ kg/s}$$

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{14 \text{ kg/s}}{(1000 \text{ kg/m}^3)(0.0113 \text{ m}^2)} = 1.24 \text{ m/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{14 \text{ kg/s}}{(1000 \text{ kg/m}^3)(7 \times 10^{-4} \text{ m}^2)} = 20.0 \text{ m/s}$$

استفاده از معادله برنولی برای بدست آوردن فشار

$$z_1 = 0 \qquad P_2 = P_{atm}$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$P_1 - P_2 = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2 - z_1\right)$$

$$P_1 - P_{atm} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)$$

$$\times\left(\frac{(20\ \text{m/s})^2 - (1.24\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0.3 - 0\right)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$P_{1,\ gage} = 202.2\ \text{kN/m}^2 = \mathbf{202.2\ kPa} \quad \text{(gage)}$$

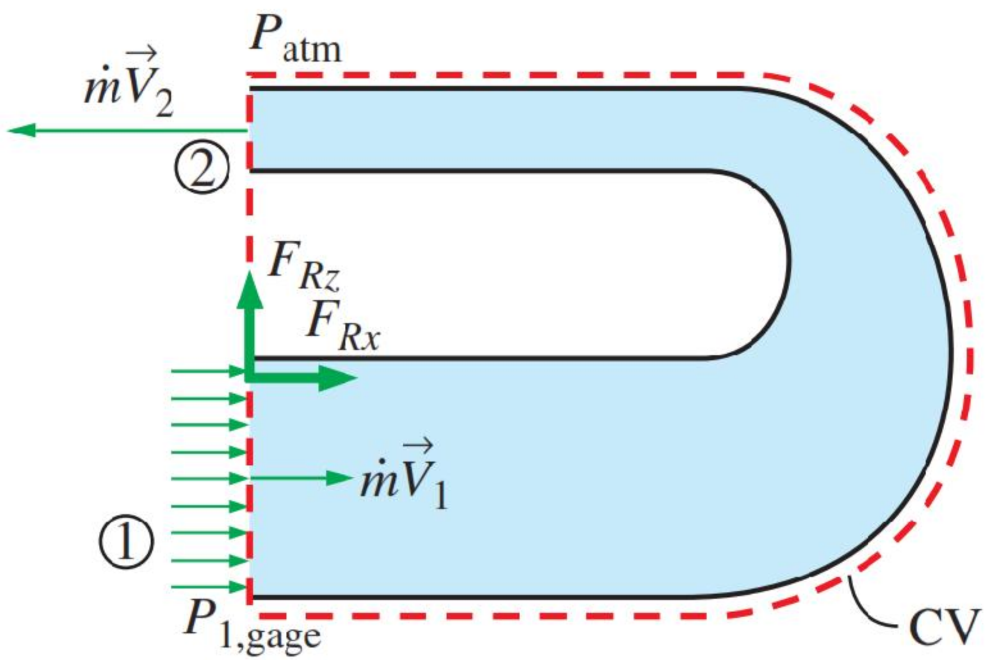
$$\begin{cases} F_{Rx} + P_{1,\,\text{gage}} A_1 = \beta \dot{m} V_2 \cos\theta - \beta \dot{m} V_1 \\ F_{Rz} = \beta \dot{m} V_2 \sin\theta \end{cases} \qquad \beta = \beta_1 = \beta_2$$

$$\begin{aligned} F_{Rx} &= \beta \dot{m}(V_2 \cos\theta - V_1) - P_{1,\,\text{gage}} A_1 \\ &= 1.03(14\ \text{kg/s})[(20 \cos 30^\circ - 1.24)\ \text{m/s}]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) \\ &\quad - (202{,}200\ \text{N/m}^2)(0.0113\ \text{m}^2) \\ &= 232 - 2285 = \mathbf{-2053\ N} \end{aligned}$$

$$F_{Rz} = \beta \dot{m} V_2 \sin\theta = (1.03)(14\ \text{kg/s})(20 \sin 30^\circ\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{144\ N}$$

EXAMPLE 6–3 **The Force to Hold a Reversing Elbow in Place**

The deflector elbow in Example 6–2 is replaced by a reversing elbow such that the fluid makes a 180° U-turn before it is discharged, as shown in Fig. 6–21. The elevation difference between the centers of the inlet and the exit sections is still 0.3 m. Determine the anchoring force needed to hold the elbow in place.

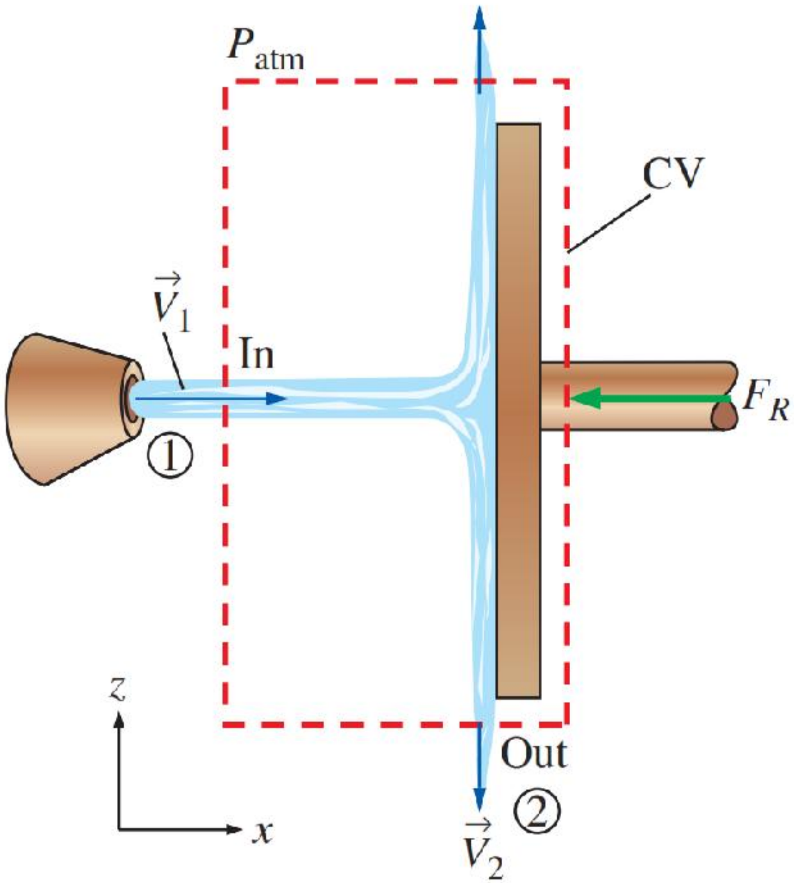


$$F_{Rx} + P_{1,\text{ gage}}A_1 = \beta_2\dot{m}(-V_2) - \beta_1\dot{m}V_1 = -\beta\dot{m}(V_2 + V_1)$$

$$\begin{aligned}
F_{Rx} &= -\beta\dot{m}(V_2 + V_1) - P_{1,\text{ gage}}A_1 \\
&= -(1.03)(14\text{ kg/s})[(20 + 1.24)\text{ m/s}]\left(\frac{1\text{ N}}{1\text{ kg·m/s}^2}\right) - (202{,}200\text{ N/m}^2)(0.0113\text{ m}^2) \\
&= -306 - 2285 = \mathbf{-2591\ N}
\end{aligned}$$

Example: Water Jet Striking a Stationary Plate

Water is accelerated by a nozzle to an average speed of 20 m/s, and strikes a stationary vertical plate at a rate of 10 kg/s with a normal velocity of 20 m/s. After the strike, the water stream splatters off in all directions in the plane of the plate. Determine the force needed to prevent the plate from moving horizontally due to the water stream.



$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$

$$V_{1,x} = V_1 \text{ and } V_{2,x} = 0$$

$$-F_R = 0 - \beta \dot{m} V_1$$

$$F_R = \beta \dot{m} V_1 = (1)(10\ \text{kg/s})(20\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{200\ N}$$