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# Effect of stiffness ratio of piezoelectric patches and plate on stress concentration reduction in a plate with a hole

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## Abstract

The effect of piezoelectric patches and plate stiffness on placement and power of piezoelectric actuators for reduction the stress concentration factor around a hole in a plate under tension is investigated. For this purpose, two conditions are considered: patches are softer than plate and the plate is softer than patches. Patches are located at top/bottom and left/right of hole. Results show that the stiffness ratio of patches and plate can affect the best location to reduce the stress concentration factor. Next, by comparing the results, some suggestions are presented for locating the patches. Results are validated by some experimental tests.

**Keywords:** Piezoelectric patch, plate with central hole, stress concentration, stiffness, strain induce

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## 1. Introduction

In the recent years, there have been many developments on high-performance structures known as smart structures. The researchers have investigated on many subjects in this field such as control the buckling, crack, vibration, shape and stress in structures. Sridharan and Kim [1] worked on stiffened panels subjected to interactive buckling to control the buckling load with piezoelectric patches. Using simulated annealing algorithm, Correia et al. [2] investigated on optimizing the piezoelectric patches location to increase the buckling loads in a composite laminated plate. Chee et al. [3] and Lin and Nien [4] studied on shape control of composite laminated plates. They developed a finite element formulation for modeling the static and dynamic response of a plate embedded with piezoelectric patches. Nguyen et al. [5, 6] presented an iterative technique for optimizing the voltage and piezoelectric surface and location to control the shape of smart plates. They used linear least square method to find the applied voltage for control the shape of smart composite plate. Mukherjee and Joshi [7] developed an iterative technique to control the shape of a structure by piezoelectric patches. Luo and Tong [8] worked on shape control of bending and twisting plate with piezoelectric actuators. They developed finite element formulation to simulate a plate with piezoelectric patches and used linear least square for electric potential to achieve desired shapes for plate. Kang et al. [9, 10] investigated the topology optimization for static shape control of a plate integrated with piezoelectric patches. They used error value between desired and actuated shape as the objective function and by induced the required voltages for actuators, the best position for piezoelectric patch has been presented. Song et al. [11] presented a review on vibration control of smart structures with piezoelectric sensor and actuators. Mehrabian and Yousefi-Koma [12] suggested a novel technique to control the vibration of a flexible fin as a smart structure with optimum placement of piezoelectric patches as actuators. Qing et al. [13] developed a semi-analytical solution for dynamic and static analysis of plates with piezoelectric actuators. Using genetic algorithm, Roy and Chakraborty [14] studied on optimal vibration control of a smart composite shell. They formulated finite elements layered shell for coupled electromechanical analysis of a composite structures integrated with piezoelectric patches. Wu and Wang [15] worked on repair of delaminated vibrating beam with piezoelectric sensors and actuators. The piezoelectric patches are employed to induce local force on delamination area to remove the stress singularity. Kumar et al. [16] developed a finite element formulation for dynamic and static response of a laminated composite shell, integrated with piezoelectric patches subjected to mechanical, thermal and electrical loads. Wang and Wu [17] presented a review on repair the structure with piezoelectric patches and shape memory alloys. Using piezoelectric sensors and actuators, Platz et al. [18] obtained an approach to evaluate the crack propagation reduction in an aluminum panels. They discussed on proper placement and voltage to reduce the crack propagation in the panel. Wu and Wang [19] studied on repair of a notched beam with piezoelectric patch subjected to dynamic loads. They located a piezoelectric patch on notch as a sensor to monitor the stress concentration by measuring the output charge, and a piezoelectric patch is placed around the notch as an actuator to reduce the stress concentration by induced prospered voltage. Sensharma et al. [20, 21] worked on reduction the stress concentration around a hole using applied induced strains such as piezoelectric patches. They proposed that piezoelectric patches and shape memory alloys may be used to control the stress and strain concentration factor by applied induced strains. Shah et al. [22] used piezoelectric actuators to reduce the stress concentration factor around a hole in a plate subjected to tension load. They demonstrate that if piezoelectric actuators located at high stress area and induced negative strains to the host plate, the stress in the plate is decreased but the stress in piezoelectric patches are increased. They proposed that by located the actuators in the compression area and induced positive strain to the host plate, the stress flow in host plate is smooth and the stress around the hole is reduced indirectly.

This idea that proposed strain induced instruments such as piezoelectric actuators can be used to reduce the stress concentration factor is investigated in previous works [21, 22]. But researchers don't consider the relation between the stiffness of host plates and piezoelectric patches in the stress control analysis and there is no experimental test in this area. In this paper, by considering the relation between the stiffness of piezoelectric patches and host plate, the best placements of actuators for reduction stress concentration factor around a hole in a plate under tension is presented. The results are presented for two conditions for stiffness relations and by comparing the results, the best placement of piezoelectric patches for reduction the stress concentration factor around a hole is investigated. The results are validating by some experimental test.

## 2. Problem definition and mathematical formulation

Using first order shear deformation theory, the displacements components u, v and w at any point of plate elements are assumed as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\theta_x(x,y) \\ v(x,y,z) &= v_0(x,y) + z\theta_y(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \quad (1)$$

Where, $u_0, v_0$ and $w_0$ are the midplane displacements components and $\theta_x$ and $\theta_y$ are rotations of the normal to the midplane about x and y direction respectively.

For using finite element method to solve the problem, using isoparametric relationships, the displacements and coordinates inside the element are defined as

$$u=\sum_{i=1}^{n} N_i \delta_i, \qquad x=\sum_{i=1}^{n} N_i x_i, \qquad y=\sum_{i=1}^{n} N_i y_i \tag{2}$$

Where, n is the number of nodes and $N_i$ are the element shape functions.

The linear piezoelectric constitutive equations for coupling the electric field and the elastic field can be written as

$$\sigma = Q\varepsilon - eE, \qquad D = e^T \varepsilon + pE \tag{3}$$

Where, $\sigma$ and $\varepsilon$ are the elastic stress and strain vector respectively. Also Q, D, e and p are elastic constant matrix, electric displacement vector, piezoelectric stress coefficient matrix and dielectric matrix respectively. The electric field vector E is defined as

$$E = -\nabla\Phi \tag{4}$$

Where, $\Phi$ is the electric voltage applied across the thickness of piezoelectric patches.

To drive the equation of motion for a plate containing piezoelectric patches, Hamilton's principle was used. For electromechanical coupled system, the principle is

$$\int_{a_1}^{a_2} \delta(T - U + W_{ext})\,da = 0 \tag{5}$$

Where $a_1$ and $a_2$ are arbitrary instants and T, U and $W_{ext}$ are kinetic energy, potential energy and the work done by external forces respectively. These parameters are defined as

$$T = \int_V \frac{1}{2} \rho \{\dot{k}\}^T \{\dot{k}\} dV$$

$$U = \int_V \frac{1}{2} \left[ \{\varepsilon\}^T \{\sigma\} - \{E\}^T \{D\} \right] dV \qquad (6)$$

$$W_{ext} = \sum_{i=1}^{n_f} \{k\}^T \{F_c\}$$

Where $\dot{k}, \rho, F_c$ and $n_f$ are velocity vector, mass density, external force vector and number of applied forces respectively and V is the volume of the structure.

To control the stress concentration around a hole in a plate, two proposed methods are existed in previous works [22]. One method, proposed that locating the piezoelectric patches at high stress concentration zone (obviously top and bottom of hole, for a plate under tension) and by induced compressive strain to the host plate, the stress concentration factor is reduced directly. Second method proposed that by changing the stress flow line in host plate, the stress concentration factor is reduced indirectly. This method explain that for control the stress flow line in a plate with hole under tension, the piezoelectric patches must locate at left and right of the hole, and by induced tensile strains to the host plate the stress concentration factor is reduced. This method declares that locating the piezoelectric actuators at top and bottom of hole (in high stress concentration area), can decrease the stress in host plate, but increase the stress in piezoelectric patches. Two mentioned method are shown in figure 1.

In proposed methods in previous works the effect of stiffness ratio of host plate and piezoelectric patches are not considered but the stiffness ratio, affects the location and capability of piezoelectric patches to reduce the stress concentration factors. The stiffness ratio (Rs) of a piezoelectric patches and plate are defined as:

$$Stiffness\ ratio\ (Rs) = \frac{(E/t)_{Piezo}}{(E/t)_{Plate}} \qquad (7)$$

To investigate two proposed method and analysis the effect of stiffness ratio of host plate and piezoelectric patches on locating the piezoelectric actuators and reducing the stress concentration around hole, considered four conditions listed in table 1.

For geometry of the problem, a thin rectangular aluminum plate with 0.2 meter length, 0.1 meter width, and with 20mm diameter central hole under 1 MPa tension is considered throughout this study. The piezoelectric actuators made of PZT-4 material are extending around the hole elliptically to a radial distance 20mm and within 45° along the circumference direction. The geometry of the problem is shown in figure 1. To simulate and analyze the problem definition, a python code is developed for Abaqus software. In order to maintain a relationship between the applied mechanical load and applied electric field $E_0$, the value of applied electric field was computed such that the stress in an infinitive piezoelectric patch because of applied electric field $E_0$ and applied mechanical load $S_0$ are to be equal[23].

## 3. Effect of piezoelectric location and stiffness ratio

### *3.1. Piezoelectric patch at top/bottom of hole & Rs = 2 (condition 1)*

As the first condition, locating the piezoelectric patches at top/bottom of hole and considering the stiffness ratio of piezoelectric patches and host plate is equal to 2 (Rs=2). The induced voltage contracts the piezoelectric patches and caused to induced compression strain to the host plate.

Figure 2 shows longitudinal stress in width of the host plate by applies various voltages. It can be seen that by applied voltage, the stress at top/bottom of hole is reduced efficiently, and increasing in induced voltage increase stress concentration reduction. Also the curves are broken at the end of locating piezoelectric patches.

Figure 3 shows longitudinal stress around hole in host plate. According to this figure it can be seen that the curves are met together at end of locating piezoelectric patches at 45°. But it should be mentioned that by increasing the voltage, location of maximum stress around hole is changed. Also it can be seen that, approximately near value of $-2E_0$ for electric field the stress at top/bottom of hole and at the location with maximum value is equal. It means that although increasing the voltage can reduce the stress concentration around hole, but increasing the voltages can changes the stress concentration location around hole and if the voltages increase enough, the stress concentration will increase again.

Figure 4 shows stress in piezoelectric patch. It shows that by increasing the voltage, the stress in piezoelectric patches is increased.

### *3.2.Piezoelectric patch at left/right of hole & Rs = 2 (condition 2)*

The second condition is similar to first condition with difference in locating piezoelectric patches. In this condition, the piezoelectric patches are located at left/right of hole. The induced voltages expand the piezoelectric patches and caused to induced extension strain to the host plate.

Figure 5 shows longitudinal stress around hole in plate. It can be seen that by inducing extension strain at left/right of hole, the stress concentration at top/bottom of hole is reduced. Also by increasing in voltage the stress at piezoelectric position is increased. The curves meet together at 45° around the hole at the edge of piezoelectric patches.

Figure 6 shows circumferential stress for host plate around hole. It shows that induced expansion strains at left/right of hole can decrease the stress at high concentration area and increase the stress at location of piezoelectric patches.

Figure 7 shows stress in piezoelectric actuators due to various electric fields. Notice that, expansion strain caused compression stress in the piezoelectric patches and by increasing the voltage, the stress is increasing in negative values.

### *3.3.Piezoelectric patch at top/bottom of hole & Rs = 0.5 (condition 3)*

Third condition appertains to locating the piezoelectric patches at top/bottom of hole with stiffness ratio equal to 0.5. In this condition, similar to first condition, electric fields contract the piezoelectric patches and caused to induce contraction strain to the host plate. Figure 8 shows longitudinal stress in width of host plate due to electric fields. It shows that induced strains can reduce the stress concentration at top/bottom of hole efficiently.

Figure 9 shows longitudinal stress around hole in host plate. It can be seen that applying $-3E_0$ can decrease the stress at top/bottom of hole from 3.13 to 1.12 but in another location around hole, the stress is increased. It means that there is a limitation for voltage to decrease the stress concentration around hole. Figure 10 shows longitudinal stress in piezoelectric actuators similar to figure 4.

### *3.4.Piezoelectric patch at left/right of hole & Rs = 0.5 (condition 4)*

Fourth condition appertains to locating the piezoelectric patches at left/right of hole with stiffness ratio equal to 0.5. In this condition, similar to second condition, electric fields expand the piezoelectric patches and caused to induce extension strain to the host plate.

Figure 11 shows longitudinal stress in host plate around hole. It can be seen that by apply electric field, the stress at top/bottom of hole is decreased and stress at left/right of hole is increased rapidly and this is a difference between figure 11 for condition 4 and figure 5 for condition 2.

Circumferential stress around hole in host plate and stress in piezoelectric patches are shown in figures 12 and 13 respectively. It can be seen that by increasing the voltage, the stress in host plate is decreased and in piezoelectric patches is increased.

## 4. Validating results

To validate the results, some experimental test is considered. The geometry of plate and piezoelectric patches are same as mentioned before. The plate is made of aluminum with 1mm thickness and piezoelectric patches are made of PZT-4 with 0.5mm thickness and the stiffness ratio of piezoelectric patches and plate are Rs=2. The piezoelectric patches are glued on bottom and top face of the host plate by “Uho” glue with graphite powder for conducting. The host plate is subjected to 1MPa in tension. And piezoelectric patches are located at left/right of hole. To specify the effect of voltage, two strain gauges are bonded at top/bottom of hole in high stress concentration area. Figure 14 shows the schematic of experimental test.

The strain obtained from data logger in experimental test and FE analysis is presented in table 2. It can be seen that the difference between results from test and FE analysis is in acceptance ranges. The results show that in all voltages, strains from experimental test have less value than strains from FE results. One reason could be the effect of the adhesive in gluing piezoelectric patches to the host plate.

## 5. Results and conclusion

From previous sections it can be seen that location piezoelectric patches at top/bottom or left/right of hole in a plate under tension can reduced the stress concentration factor. Also for all conditions by increasing the voltage, the stress in piezoelectric patches are increased by apply electric fields. But notice that increasing stress in piezoelectric patches is not desirable. Now consider conditions 1 and 2 together. Figure 15 shows the stresses in piezoelectric patches and host plate due to various voltages. It can be seen that if piezoelectric patches locate at top/bottom of hole with Rs=2, the voltage can decrease stress concentration in plate, but it makes increasing the stress in piezoelectric patches and it is not desirable. Also by locating piezoelectric patches at left/right of hole, the curves of stress for patches and plate meet together with 7% reduction in stress concentration factor.

For second discussion consider conditions 3 and 4 together for stiffness ratio equals to 0.5 (Rs=0.5). Figure 16 shows the stresses for piezoelectric patches and plate. It shows that both locating patches at top/bottom and left/right of hole, the patches can decrease the stress concentration factor. But locating patches at top/bottom of hole has a greater effect. This condition can reduce the stress concentration factor about 31.1%.

By considering four conditions together, it can be deduced that for reduction the stress concentration factor with piezoelectric patches, stiffness ratio of patches and plate has an important effect on locating the patches around the stress concentration area. According to presented results and discussions, it is suggested three points:

- If selected piezoelectric patches are more rigid than plate, the patches must use to change the stress flow and control the stress concentration indirectly. For this condition, patches must not locate at high stress concentration area.
- If selected piezoelectric patches are softer than plate, the patches may locate at high stress concentration area and control the stress concentration directly.
- To further reduce the stress concentration in plate, selecting patches with softer properties than host plate is better choice.

## 6. Conclusion

In this paper, by considering the relation between the stiffness of piezoelectric patches and host plate, the placements of the actuators for reduction the stress concentration factor around a hole in a plate under tension is investigated. The results are presented for two conditions for stiffness relation. Results show that stiffness ratio of patches and plate can affect the position of patches for reduction the stress concentration factor. It was shown that if selected piezoelectric patches are more rigid than plate, the patches must use to change the stress flow and control the stress concentration factor indirectly. For this condition, patches must not locate at high stress concentration area and must locate at left/right of hole. Also if selected piezoelectric patches are softer that plate, locating the patches at high stress concentration area is the best location for piezoelectric patches. The presented results in this paper are validating by some experimental tests.

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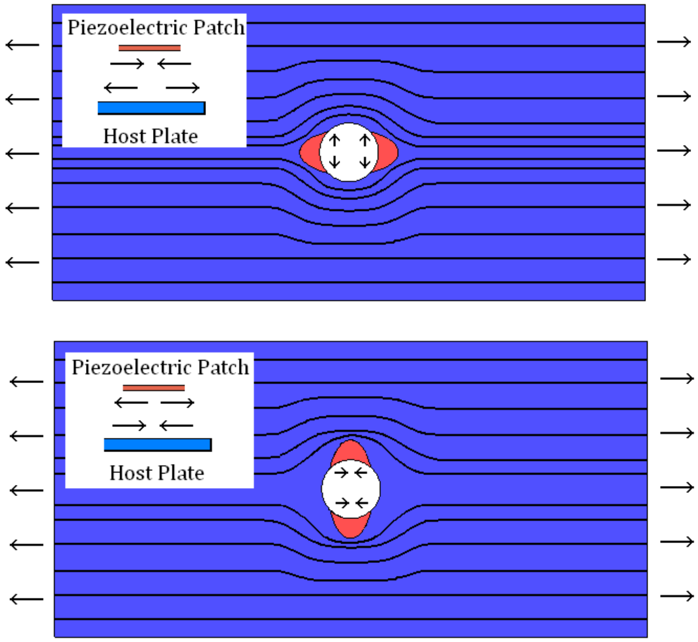

Figure 1. Two method for reduction in stress concentration around a hole by applied induced strains

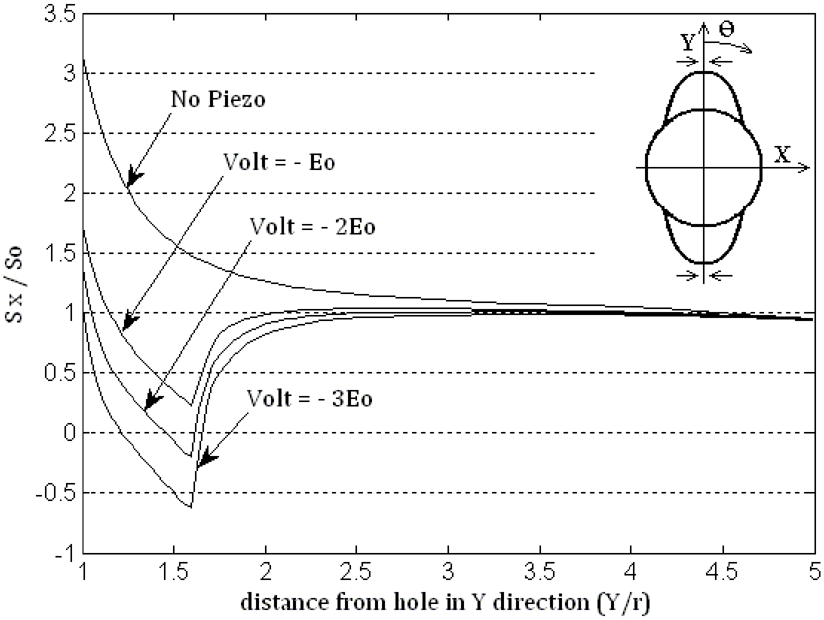

Figure 2. longitudinal stress in width of the host plate by applies various voltages (Rs = 2, condition 1)

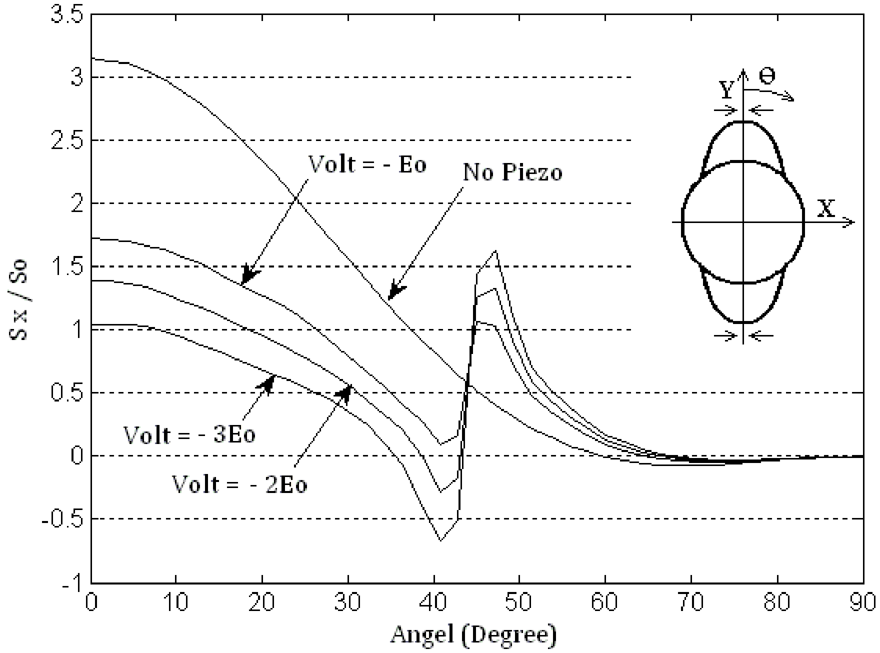

Figure 3. longitudinal stress around hole in host plate (Rs = 2, condition 1)

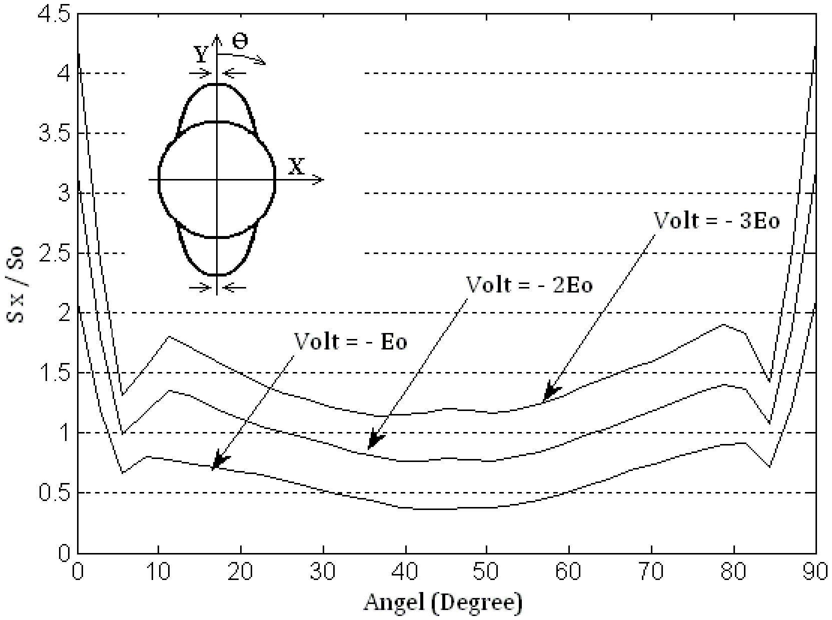

Figure 4. Stress in piezoelectric patch (Rs = 2, condition 1)

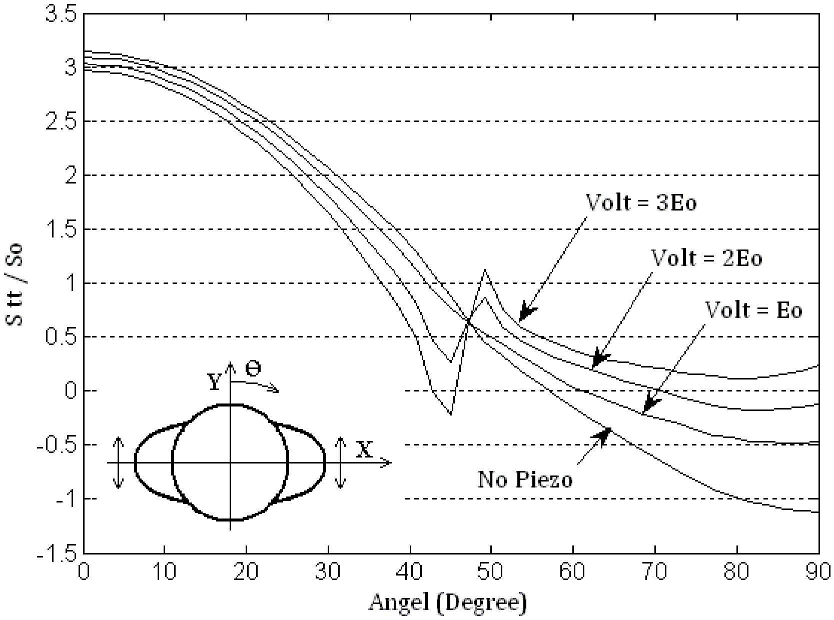

Figure 5. Longitudinal stress around hole in plate (Rs = 2, condition 2)

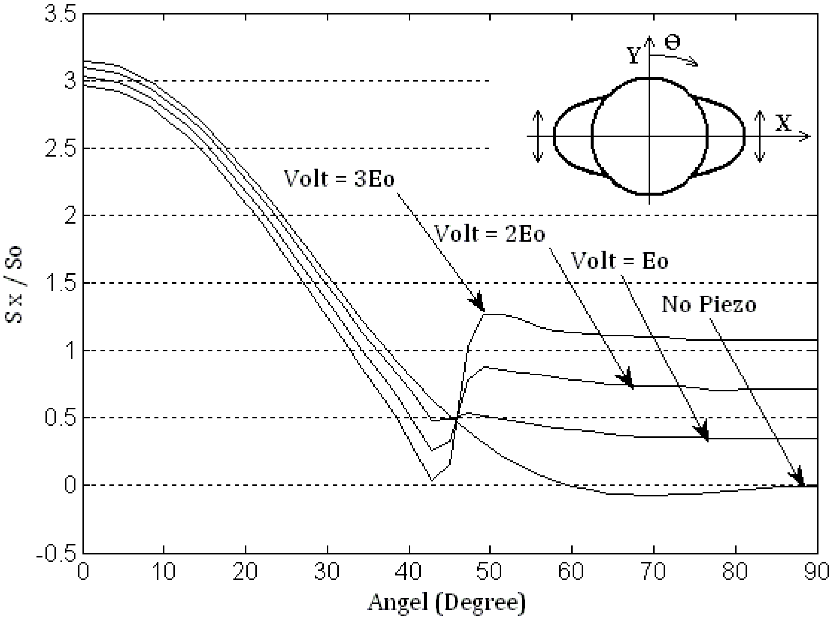

Figure 6. circumferential stress for host plate around hole (Rs = 2, condition 2)

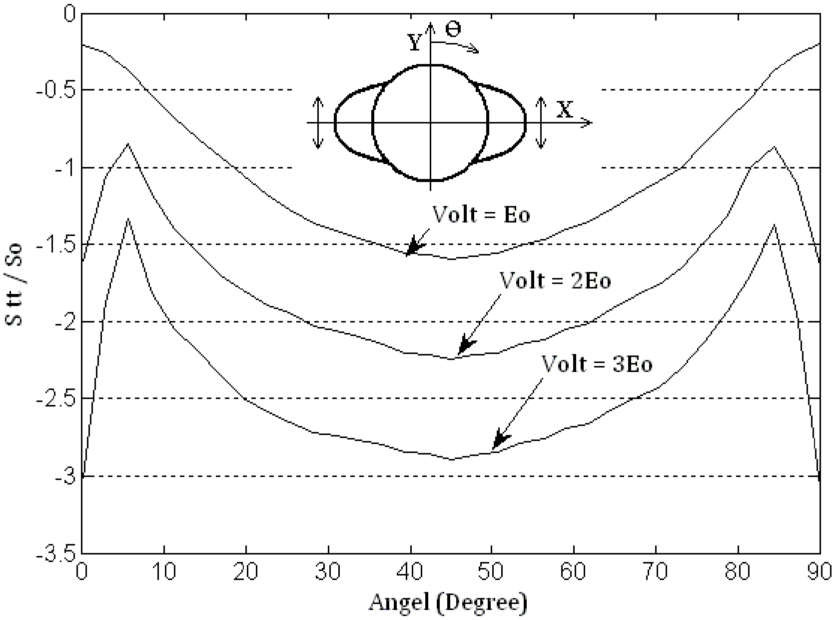

Figure 7. Stress in piezoelectric actuators (Rs = 2, condition 2)

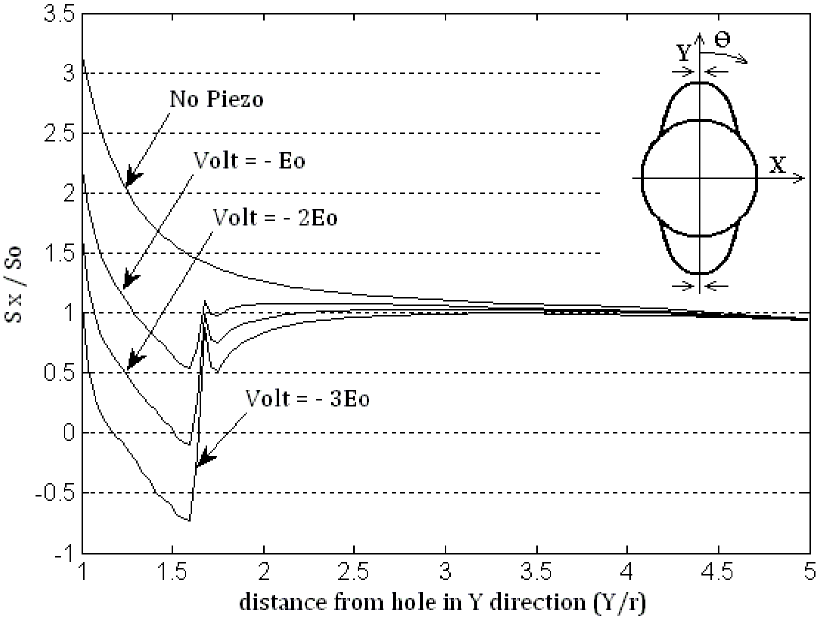

Figure 8. Longitudinal stress in width of host plate due to electric fields (Rs = 0.5, condition 3)

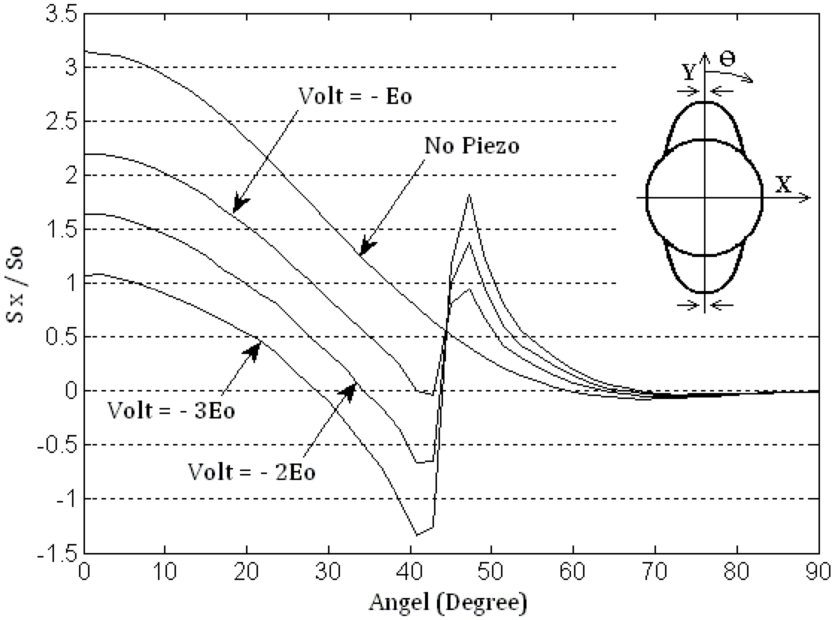

Figure 9. Longitudinal stress around hole in host plate (Rs = 0.5, condition 3)

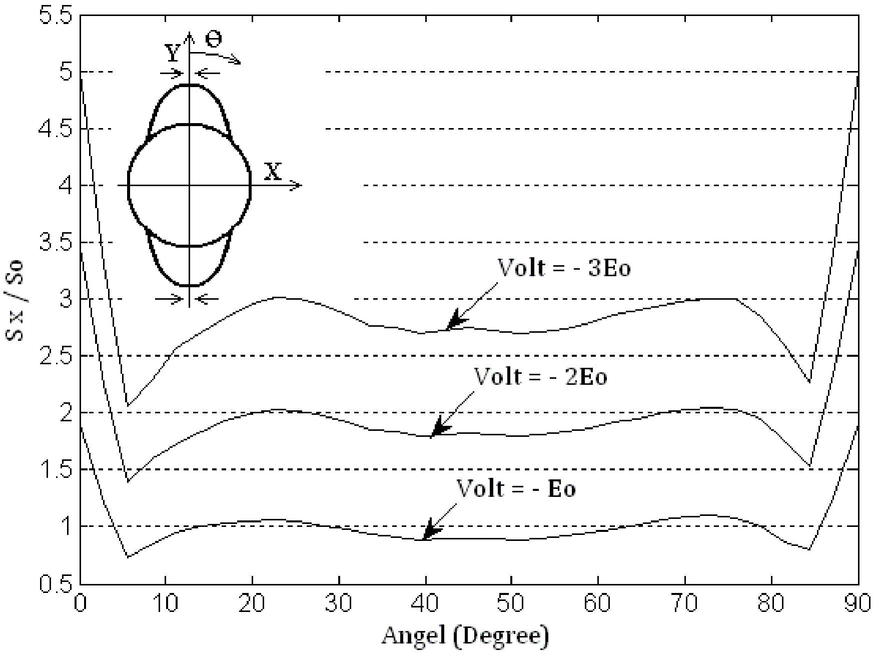

Figure 10. Longitudinal stress in piezoelectric actuators (Rs = 0.5, condition 3)

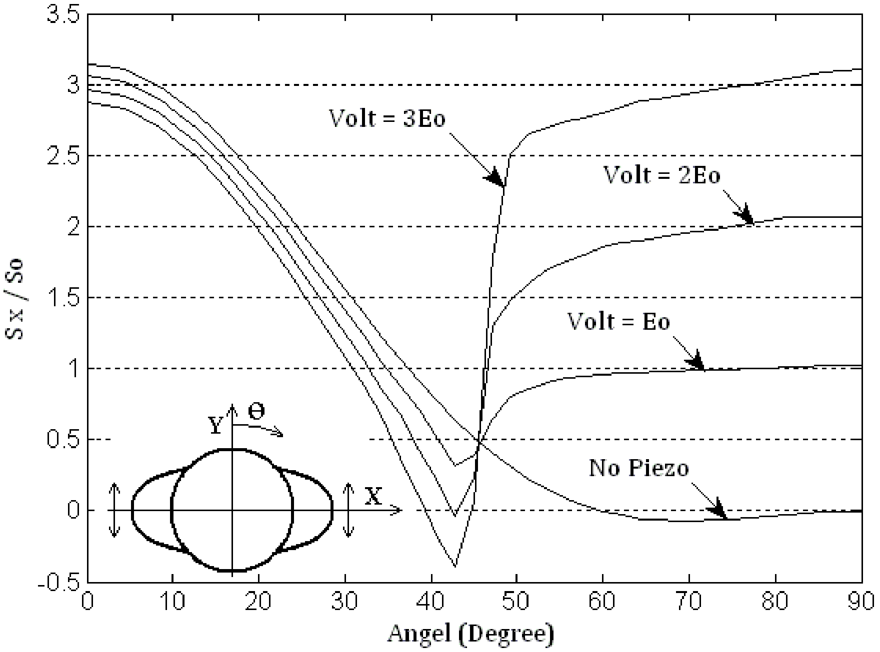

Figure 11. Longitudinal stress in host plate around hole (Rs = 0.5, condition 4)

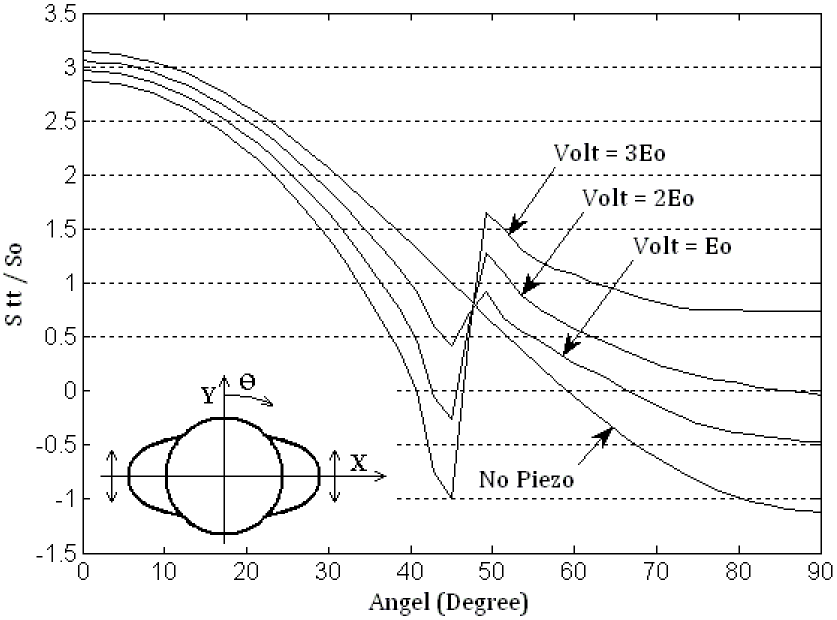

Figure 12. Circumferential stress around hole in host plate (Rs = 0.5, condition 4)

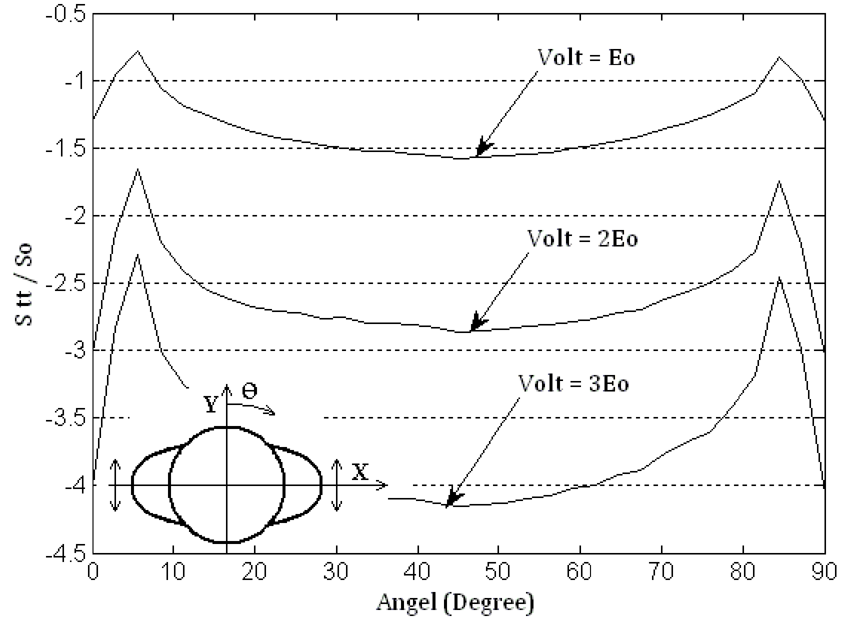

Figure 13. Circumferential stress around hole in piezoelectric patch (Rs = 0.5, condition 4)

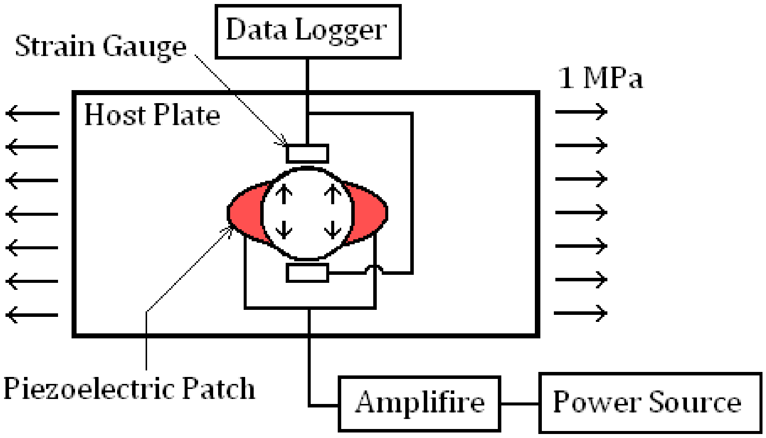

Figure 14. Schematic of experimental test

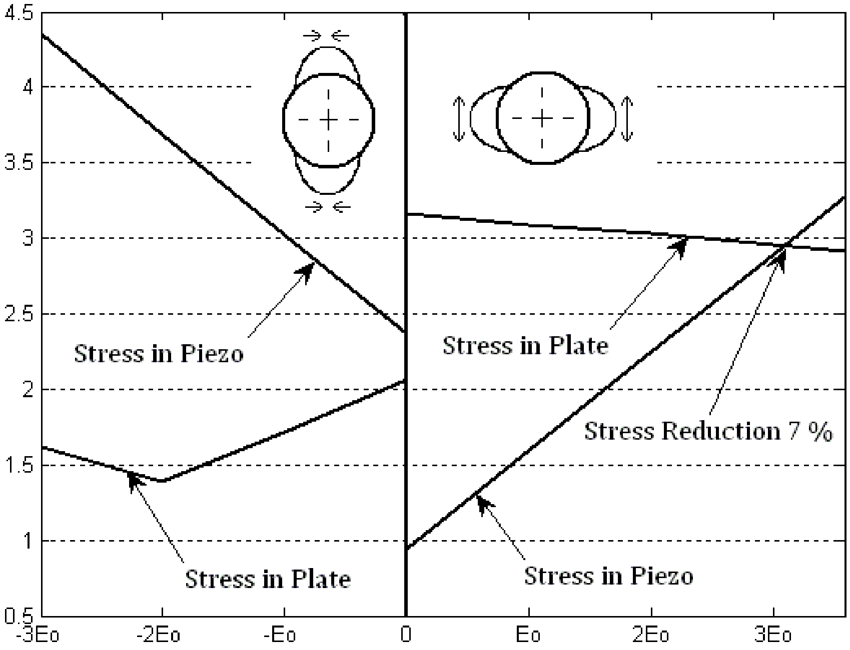

Figure 15. Stresses in piezoelectric patches and host plate due to various voltages (Rs = 2)

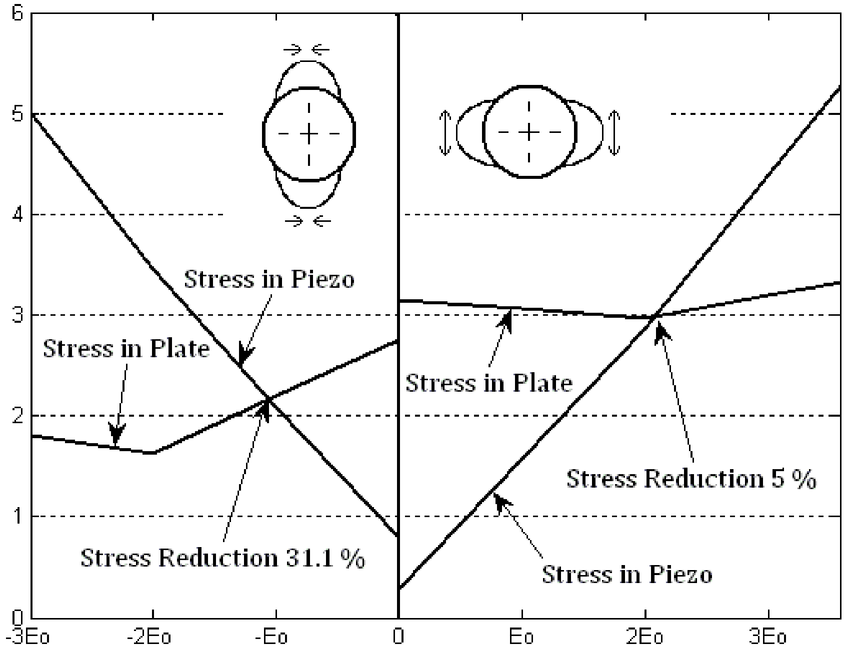

Figure 16. Stresses in piezoelectric patches and host plate due to various voltages (Rs = 0.5)

Table 1. Four condition for stiffness ratio and piezoelectric patches location.

| Condition Number: | No. 1 | No. 2 | No. 3 | No. 4 |
|---|---|---|---|---|
| Stiffness ratio (Rs) | Rs = 2 | Rs = 2 | Rs = 0.5 | Rs = 0.5 |
| Piezoelectric position | Top - Bottom | Left – Right | Top - Bottom | Left - Right |

Table 2.  Comparison the results of test and FE analysis

| Voltage | Strain from FE ($\times 10^{-6}$) | Strain from Test( $\times 10^{-6}$) | Error |
|---|---|---|---|
| 0 | 45.116 | 43.074 | 4.52 % |
| 16.75 ($E_0$) | 44.184 | 41.972 | 5 % |
| 33.5 ($2E_0$) | 43.251 | 40.791 | 5.69 % |
| 55.25 ($3E_0$) | 42.042 | 39.383 | 6.32 % |
| 72 ($4E_0$) | 41.110 | 38.021 | 7.51 % |