Reducing Stress Concentration Around a Hole in a Thin-Wall Cylinder Subjected to Internal Pressure Using Piezoelectric Patches

S. M. R. Allahyari & S. Golabi

Iranian Journal of Science and Technology, Transactions of Mechanical Engineering

ISSN 2228-6187

Iran J Sci Technol Trans Mech Eng
DOI 10.1007/s40997-019-00319-7

Iranian Journal of Science and Technology, Transactions of Mechanical Engineering
https://doi.org/10.1007/s40997-019-00319-7

RESEARCH PAPER

Reducing Stress Concentration Around a Hole in a Thin-Wall Cylinder Subjected to Internal Pressure Using Piezoelectric Patches

S. M. R. Allahyari[1] · S. Golabi[1]

Received: 20 April 2019 / Accepted: 25 September 2019

Abstract

Decreasing stress in a part or piece of equipment especially in a stress concentration point without increasing thickness or using stronger material is always an interesting issue for design engineers. In this paper, piezoelectric patches were used for stress concentration reduction around various hole sizes in thin-walled cylindrical shells subjected to internal pressure. Applied voltage to piezoelectric patches induces strain on both patch and part and alters stress flow on the part, leading to new distribution of stresses. Three dimensionless effective parameters that include hole and shell diameters as well as shell and piezo patch thicknesses were selected in this investigation. Ant colony optimization was utilized, and attachment of optimal arrangements of piezoelectric patches was derived for various numbers of piezoelectric patches. Using finite element analysis, it was shown that various combinations of these dimensionless parameters (R/t, tp/t and d/D) have different effects on reducing the maximum stress around the hole and up to 20% reduction on maximum stress was achieved during this research. Experimental tests were finally performed to verify finite element results.

Keywords Ant colony optimization (ACO) · Cylindrical shell · Finite element · Piezoelectric patch · Stress concentration

1 Introduction

The problem of stress concentration in pressurized cylinder with hole or opening has become crucially important due to stress concentration in a small area around the hole. This problem is observed in several engineering products such as aircraft fuselage, ship and submersible hull, boilers and pressure vessels (Mirzagoltabar 2014; Ramsey 1975; Lind 1968). These holes are used for passing stacks, piping, ducts, electrical wiring, etc. (Hsu et al. 2005).

Many researchers have focused on stress concentration analysis in pressure vessels with hole and studied various methods of reducing these stresses (Fridman 1986; Kornecki 1966). Xue et al. (2010) presented an analytical method for nozzle placed on cylindrical vessel under internal pressure and external loads Xue et al. (2010). Kharat and Kulkarni (2013) investigated stress concentration around openings in pressure vessels. Mirzagoltabar (2014) studied various methods of stress reduction around holes and openings in shells and plates. He suggested a new method of reinforcement using the rules for construction of pressure vessels (ASME Committee on Pressure Vessels 2015). Lu et al. (2014) used a 2D axisymmetric model to determine the stress intensity at the intersection of nozzle to pressure vessel under internal pressure. FEA of a cylindrical shell with a large circular hole under torsion and axial compression was presented by Kamalarajah et al. (2015). FEA of cylindrical vessel with internal pressure and radial nozzle was reported by Mukhtar and Al-Gahtani (2016). They determined the amount and location of stress concentration factor under the influence of various parameters.

Piezoelectrics have various applications in solid mechanics, including shape control (Nguyen and Tong 2007), vibration control (Mehrabian and Yousefi-Koma 2011), buckling (Przybylski 2008), crack propagation control (Wu and Wang 2011) and stress control. The use of piezoelectric patches for stress reduction is investigated by several researchers (Sensharma and Haftka 1996; Sensharma et al. 1993; Shah et al. 1994; Fesharaki 2014; Fesharaki and Golabi 2016a, b; Fesharaki and Madani 2016; Golabi and Fesharaki 2014; Fesharaki et al. 2018). Stress reduction in an isotropic plate with a hole subjected to induced strains by piezoelectric patches was investigated by Sensharma et al. (1993). Shah

✉ S. Golabi
golabi-s@kashanu.ac.ir

[1] Faculty of Mechanical Engineering, University of Kashan, Kashan, Iran

Published online: 03 October 2019

et al. (1994) focused on reducing stress concentration factor in an isotropic plate with hole. Comprehensive researches for determining various number of piezoelectric patches for stress reduction of plate with hole under unidirectional tension were studied by Golabi and Fesharaki (2014; Fesharaki and Golabi 2016a, b; Fesharaki and Madani 2016; Golabi and Fesharaki 2014). Golabi and Fesharaki (2014) determined the optimum pattern for pasting piezoelectric patches using particle swarm optimization algorithm for maximum possible stress concentration reduction. Fesharaki and Madani (2016) also showed the effect of thickness and stiffness of piezoelectric patches on stress reduction of plate with hole subjected to unidirectional tension. Fesharaki and Golabi (2016) suggested a reference line for best arrangement of piezoelectric patches considering different plate to piezo stiffness ratios which includes Young's module and thickness of plate and piezoelectric patches. Stress concentration reduction with piezoelectric patches in a classical plate with hole was investigated and improved by Fesharaki et al. (2018).

All previous researches regarding application of piezoelectric patches for stress reduction were mainly focused on a flat plate with hole subjected to unidirectional tensile. In this research, the possibility of stress reduction in thin cylindrical shell with hole subjected to internal pressure has been addressed. Internal pressure in cylindrical shells generates circumferential as well as longitudinal stresses on shell, and it was not clear whether piezo patches could also have any effect on stress reduction when there are bidirectional stresses. Beside bidirectional loading in a cylindrical shell, radius and thickness of shell are other parameters that may affect the effectiveness of patches and are needed to be studied. Reinforcement plates are generally used around the openings in pressure vessels to compensate the strength lost because of material removed for making an opening. Stress concentration is another consequence of generating an opening which is not considered during determining reinforcement plate dimensions. On the other hand, there are many applications including fuselage of submarine, airplane, missiles, many other thin cylindrical pressure parts whereby reinforcement plates are not good suggestions for stress reduction and using piezo patches could be a very good solution for reducing stress concentration in critical points. On the other hand, the results of using piezo patches on plates subjected to unidirectional tension could not be used for this objective and a new research was required to cover this issue which was reported in this paper.

In the following sections after determining the main problem and its constraints, FEA of the models and application of ACO technique for determining the optimum location of patches were described. The results of using various number of patches on stress reduction were determined and discussed at the end of this paper.

2 Problem Definition

As explained above, the effect of piezoelectric patches on reducing stress concentration of plates with hole under unidirectional tension was studied and addressed before; however, in cylindrical shells subjected to internal pressure there are many more affecting parameters that needed to be considered, i.e., *D*, *R*, *d*, *tp* and *t* which represent cylinder diameter, cylinder radius, hole diameter, piezoelectric thickness and shell thickness, respectively. The cylindrical shell is subjected to internal pressure (*P*) that generates both axial (or longitudinal) ($\sigma_1 = PR/2t$) and circumferential tension ($\sigma_\theta = PR/t$). By increasing the number of variables, the number of analysis increases enormously. On the other hand, by selecting suitable dimensionless parameters, without losing the generality of analysis, the number of variables (parameters) would be considerably reduced. Therefore, after considering various important conditions for choosing dimensionless parameters, the following ones were proposed in this research: *R*/*t* or *D*/*t*, *tp*/*t* and *d*/*D*. Because of large number of combinations of these dimensionless parameters, it was necessary to employ Taguchi's method to choose 16 cases out of these three dimensionless parameters and each in four levels. Then, the optimum arrangements of pasting 1 to 9 piezoelectric patches were proposed to be investigated for each case using ACO. Thus, totally 144 optimization problems needed to be solved and for each case the amount of stress reduction should be calculated and shown in relevant diagrams.

3 Analysis Methodology

As described earlier, four levels were considered for each dimensionless parameter (Table 1). Real and more common range of parameters were considered in determining these levels, and Taguchi method was used to specify 16 cases for research simulation as shown in Table 2. The flowchart of solution methodology is shown in Fig. 1. In each case, a cylinder with a hole was modeled using ABAQUS software. To reduce the effect of cylinder ends, the length of all cylinders was considered to be 3 m with 32 mm hole radius in the mid-length of the cylinder. During finite element analyses of various cases, the hole diameter was kept constant and

Table 1 Selected amount of different parameters

Parameter	Level 1	Level 2	Level 3	Leve4
R/*t*	10	40	70	100
d/*D*	0.11	0.18	0.25	0.33
tp/*t*	0.1	0.23	0.36	0.5

Table 2 Determined analysis cases according to Taguchi method

Case no.	*R*/*t*	*d*/*D*	*tp*/*t*	*r* = *d*/2	*R* = *D*/2	*t* (mm)	t_p (mm)	Pressure (Pa)
1	10	0.11	0.1	32	290.9	29.1	2.9	100,000
2	10	0.18	0.23	32	177.8	17.8	4.1	100,000
3	10	0.25	0.36	32	128	12.8	4.6	100,000
4	10	0.33	0.5	32	97	9.7	4.8	100,000
5	40	0.11	0.23	32	290.9	7.3	1.7	25,000
6	40	0.18	0.1	32	177.8	4.4	0.4	25,000
7	40	0.25	0.5	32	128	3.2	1.6	25,000
8	40	0.33	0.36	32	97	2.4	0.9	25,000
9	70	0.11	0.36	32	290.9	4.2	1.5	14,285.71
10	70	0.18	0.5	32	177.8	2.5	1.3	14,285.71
11	70	0.25	0.1	32	128	1.8	0.2	14,285.71
12	70	0.33	0.23	32	97	1.4	0.3	14,285.71
13	100	0.11	0.5	32	290.9	2.9	1.5	10,000
14	100	0.18	0.36	32	177.8	1.8	0.6	10,000
15	100	0.25	0.23	32	128	1.3	0.3	10,000
16	100	0.33	0.1	32	97	1	0.1	10,000

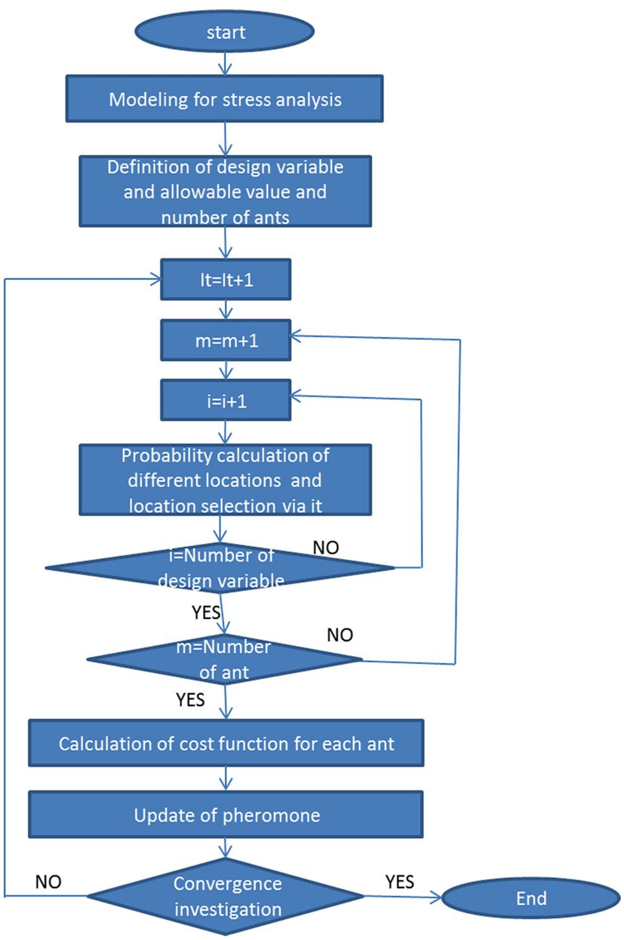

Fig. 1 Optimization flowchart

other parameters including cylinder radius and thickness were changed, because determining the suitable mesh size during FEA requires convergence test and, in this research, the main affecting parameter on convergence of the mesh size was the radius of hole. Consequently, 8040 elements in cases 1, 5, 9 and 13, 11,240 elements in cases 2, 6, 10 and 14, 13,640 elements in cases 3, 7, 11 and 15 and 16,100 elements in cases 4, 8, 12 and 16 were proposed for the models after convergence test.

3D linear hexahedral elements (C3D8R) and 8-node linear piezoelectric brick (C3D8E) were selected for cylindrical shell with hole and piezoelectric patches. Piezoelectric patches were installed on cylindrical shell with tie constraint. Electrical potential was applied as boundary condition on two side of piezoelectric patches. Pressure was applied on inner side of cylindrical shell as shown in Table 2. Displacement of both ends of cylinders was confined in all 3 directions.

To find suitable locations for attaching the patches, a symmetric 20 × 20 grid was considered around the hole. The size of each square was equal to the size of provided piezoelectric patches, i.e., 8 × 8 mm and to reduce the calculation time only one quarter of this area was considered in each analysis. The developed program did not consider and would not attach any patch on the hole location. Up to 9 patches were considered to be pasted in this 10 × 10 grid (quarter of whole cylinder) area, and their effects on stress reduction were determined. In other words, when the analysis was performed for determining the optimum location of one piezoelectric patch, in fact, 4 patches were attached in a 20 × 20 grids in similar locations in each quarter.

The cylinder is made of steel with Young modulus and Poison ratio of 210 GPa and 0.3, respectively. PZT4 piezoelectric patches polarized across the thickness were chosen to induce strain on cylinder with mechanical properties defined as follows:

PZT4:

$$\text{Elastic (GPa)} = \begin{bmatrix} 115.4 & 74.28 & 74.28 & 0 & 0 & 0 \\ 74.28 & 139 & 77.84 & 0 & 0 & 0 \\ 74.28 & 77.84 & 139 & 0 & 0 & 0 \\ 0 & 0 & 0 & 25.64 & 0 & 0 \\ 0 & 0 & 0 & 0 & 25.64 & 0 \\ 0 & 0 & 0 & 0 & 0 & 25.64 \end{bmatrix} \quad (1)$$

$$\text{Piezoelectric (Columb/m2)} = \begin{bmatrix} 0 & 0 & -5.27 \\ 0 & 0 & -5.27 \\ 0 & 12.71 & 15.08 \\ 0 & 0 & 0 \\ 12.71 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (2)$$

$$\text{Dielectric (Farad/m)} = \begin{bmatrix} 5.872 & 0 & 0 \\ 0 & 6.752 & 0 \\ 0 & 0 & 6.752 \end{bmatrix} * 10^{-9} \quad (3)$$

Starting from one piezoelectric patch, ACO technique was used to determine the most suitable location for attaching the patches. The analysis continues up to attachment of 9 patches in each quarter for each case using ACO algorithm.

4 Mathematical Model

The model consists of a thin cylinder with a hole around which the piezoelectric patches were attached. Using thin shell theory, displacement field was considered as:

$$u_{\text{tot}} = u_0(x,\theta) + z\beta_x(x,\theta) \quad (4)$$

$$v_{\text{tot}} = v_0(x,\theta) + z\beta_\theta(x,\theta) \quad (5)$$

$$w_{\text{tot}} = w_0(x,\theta) \quad (6)$$

Herein, u_0, v_0 and w_0 are displacement of mid-plane of shell thickness where $z = 0$. u_{tot}, v_{tot} and w_{tot} are displacement in the (x, z, θ) point, and β_x and β_θ are rotation of perpendicular plane around the x and θ axis. In strain tensor:

$$\varepsilon_r = \varepsilon_{zx} = \varepsilon_{z\theta} = 0 \quad (7)$$

By ignoring temperature and pyroelectric effect, coupled piezoelectric linear constitutive equations are:

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{kij}E_k \quad (8)$$

$$D_i = e_{ikl}\varepsilon_{kl} + \alpha_{ij}E_j \quad (9)$$

where σ_{ij} and ε_{kl} are stress and strain tensors, E_k and D_i are electric field and displacement vectors, and c_{ijkl}, e_{kij} and α_{ij} represent elastic, piezoelectric and dielectric coefficients, respectively. Elastic and dielectric coefficients were considered in constant electric field and constant strain, respectively.

In order to derive equations for cylinder with hole and piezoelectric patches under internal pressure, the minimum total potential energy principle was used (Uchino and Debus 2012).

$$P = U - W = \frac{1}{2}\int_V \left(\varepsilon^T\sigma - E^T D\right)\mathrm{d}V - \sum_{i=1}^{k} X^T F \quad (10)$$

$$\delta P = 0 \quad (11)$$

where P, U and W are the total potential energy, strain energy and work of external forces. X, F and k are displacement vector, external force and number of external forces. Because of the square shape of piezoelectric patches and considering the fact that these patches are attached as tile shape and are discontinuous, numerical and finite element method must be implemented to solve these equations to determine stress distribution in various locations around the hole in the shell.

5 Optimization

In any optimization problem, design variables, design constraints, objective function(s), type of problem and proposed solution method must be specified. In this research, the main objective was to minimize the maximum stress around a hole on a cylindrical shell subjected to internal pressure. Finite element technique using ABAQUS software was implemented to find the maximum stress around the hole. To minimize the maximum stress, it was paste piezoelectric patches around the hole. Therefore, the design variables were the location of patches on a 10×10 grid (quarter of whole cylinder) (Fig. 2). Changing the location of patches alters the value and location of maximum stress around the hole. Since a very large number of analyses were needed to find the best location for pasting up to nine patches, ACO technique was implemented to find the best locations and optimum solutions. The flowchart of this analysis is shown in Fig. 1.

ACO is a heuristic optimization algorithm inspired from the nature and is based on behavior of ants that live together in large groups. This method is an example of social intelligence which enhances the efficiency of weak individual members. This algorithm was firstly developed by Dorigo et al. (1996). The outcome of this algorithm is to determine the shortest path between nest and food.

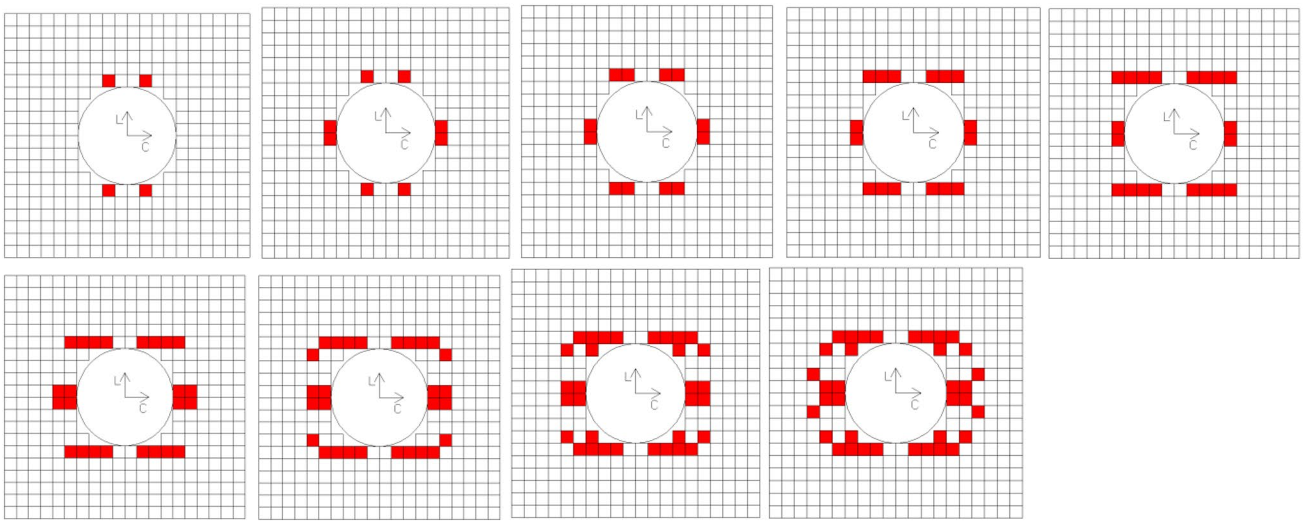

Fig. 2 Optimal patch locations for case 1

As mentioned earlier, the number of piezoelectric patches varies from 1 to 9 for each case listed in Table 2. Therefore, the number of design variables was equal to the number of patches used for stress reduction. In order to determine the optimal arrangement of piezoelectric patches in each case, the following steps were performed:

(1) Modeling the cylindrical shell for stress analysis considering geometrical dimensions, material properties, loading, boundary conditions, etc.
(2) Selection of ant numbers. Number of ants varies from 100 to 150 for various number of piezoelectric patches. Each ant consists of the coordinates of m ($m = 1, 2 \ldots, 9$) patches.
(3) Assume equal amounts of pheromone τ_{ij} for all locations in 10×10 grid.
(4) For each ant select m positions from permissible locations in 10×10 grid (N) of piezoelectric patches via probability Eq. (12).

$$P_{ij}^{k} = \begin{cases} \frac{(\tau_{ij})^{\alpha}}{\sum_{ij \in N} (\tau_{ij})^{\alpha}} & \text{if } ij \in N \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

where P_{ij}^{k} is probability of selection of each location in N for the kth ant. τ_{ij} stands for the pheromone amount of ij location in two-dimensional set N. After selection of each location by the kth ant, pheromone of the selected location is assigned to zero for not allowing double selection of any point for each ant. Power α determines the weight of τ_{ij} and set to 1 in this research.

(5) Generating a new model using the coordinates proposed by each ant and determining the maximum stress utilizing PYTHON coding in ABAQUS software.
(6) Updating the pheromone contents on each location by considering pheromone drop and its evaporation for each ant according to Eq. (13).

$$\tau_{ij} \leftarrow (1 - \rho)\tau_{ij} + \rho \Delta \tau_{ij} \ \forall \rho \in (0, 1] \tag{13}$$

ρ is the rate of pheromone evaporation. In this study, $\rho = 0.6$ and $\Delta \tau_{ij} = \frac{\min(\sigma \max)}{\max(\sigma \max)}$ where the nominator and denominator are the least and the most maximum stress of a population of ants found during FEA was added to the best ant.

(7) Investigating the terminating conditions which will be based on maintained pheromone in different nodes, otherwise go to step 4 and find new ants.

A 10×10 matrix was used for the location of each ant. The amount of pheromone was saved in another matrix. Pheromone amount of grid points located on hole were set to zero in all optimization processes.

All above steps were programed with Python coding in ABAQUS environment to save calculation time despite many researches in which the results of FEA were transferred to an optimization program such as MATLAB to generate new ants and data for new FEA.

6 Finite Element Results

6.1 Determining Optimal Arrangements of Piezoelectric Patches

As described above, 16 cases, shown in Table 2, were selected by Taguchi method and for each case; optimal arrangements of 1 to 9 piezoelectric patches were determined using ACO algorithm to reduce the maximum generated stress around an opening in a cylindrical shell subjected to internal pressure. The optimum arrangements of patches are shown in Fig. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16 and 17. In these figures, *L* represents axial direction of cylinder and *C* represents circumferential direction.

In each case, first piezoelectric patch is located in the up-down position or almost adjacent to it. Second to fourth piezoelectric patches are aligned on a straight line along the circumferential direction of the cylinder in all cases.

In a cylindrical shell, only membrane stresses are generated because of internal pressure, however, when a discontinuity like an opening is created on a shell, local stresses are also induced around the discontinuity (Mirzagoltabar 2014; Ramsey 1975; Mukhtar and Al-Gahtani 2016; Rajaiah and Kumar 1982, 1985). Due to high effects of these patches on

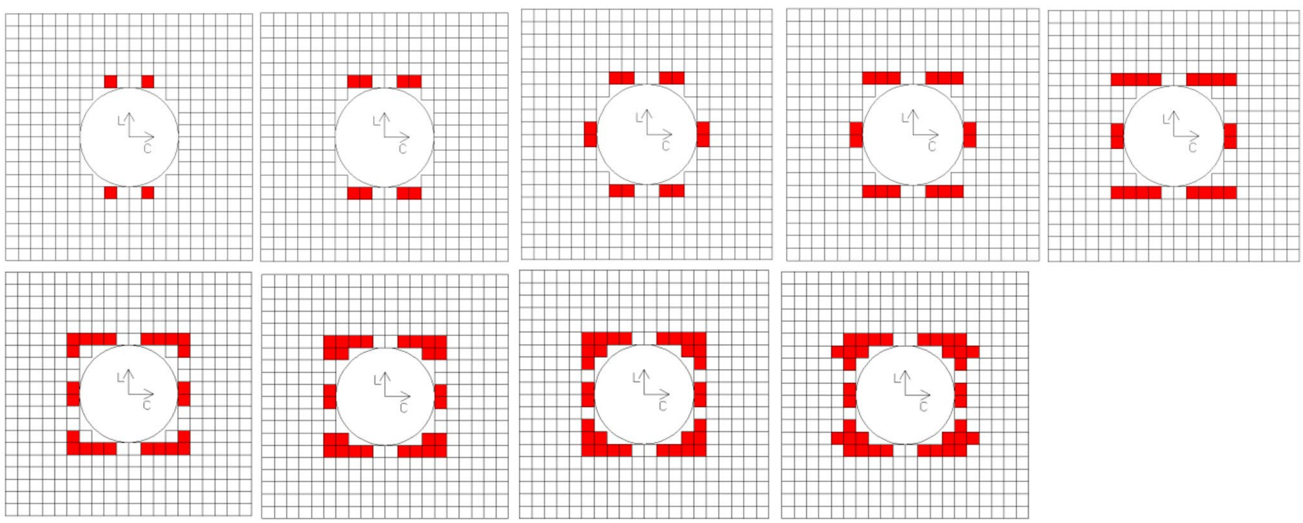

Fig. 3 Optimal patch locations for case 2

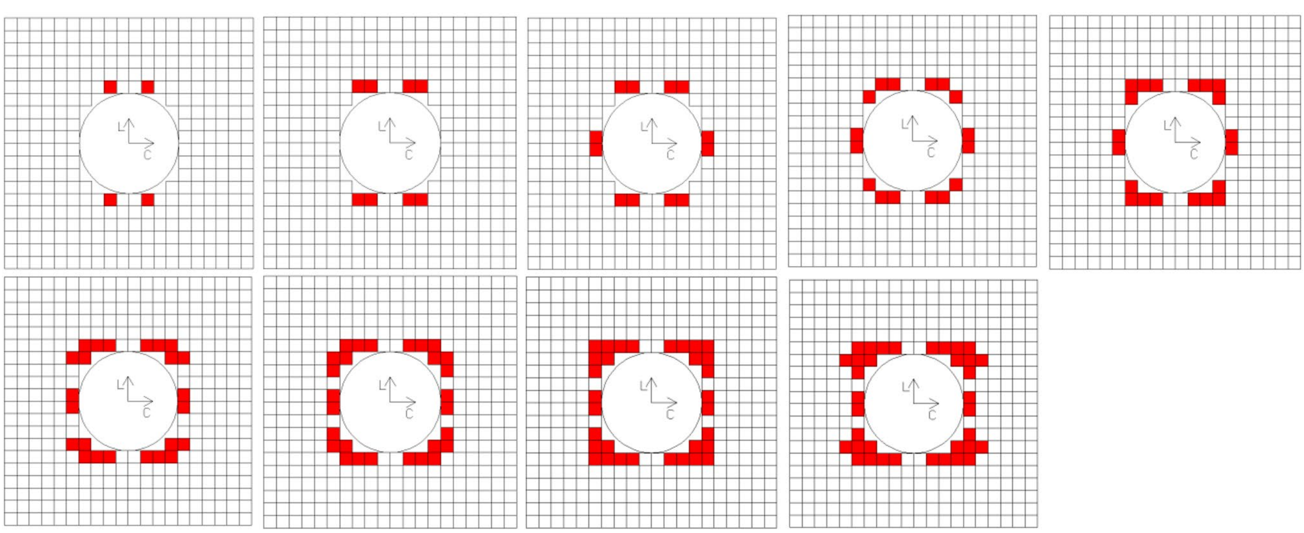

Fig. 4 Optimal patch locations for case 3

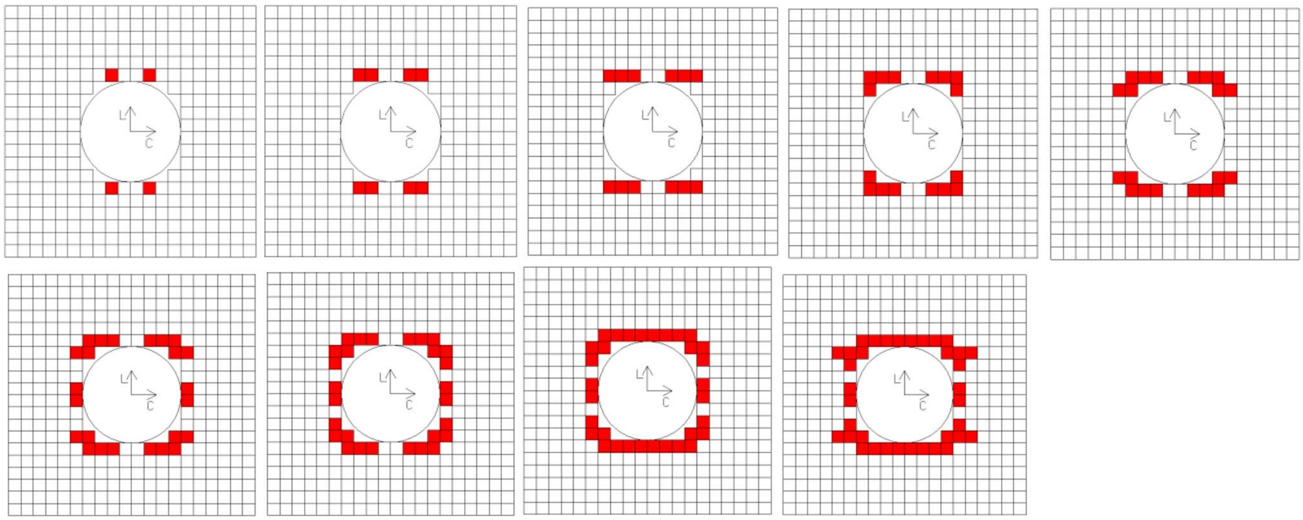

Fig. 5 Optimal patch locations for case 4

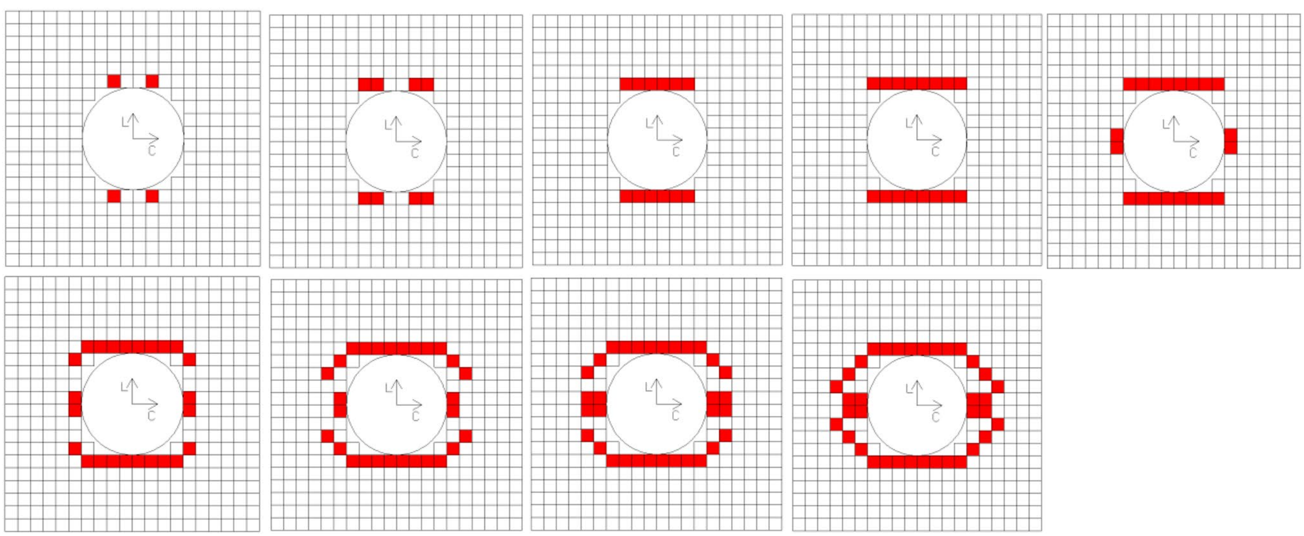

Fig. 6 Optimal patch locations for case 5

stress reduction, alignment of these patches in circumferential direction can be the best way to alleviate the bending stresses.

Except case 1 to case 3, in all other cases, by increasing the number of piezoelectric patches, the circumferential grids are filled first. Interestingly, it is observed that case 1 to case 3 exhibit lowest stress concentration reduction. Cases 1 to 3 corresponds to low values of R/t. By increasing the number of piezoelectric patches, the position of the patches is firstly in the straight line, top and bottom of the hole along the circumferential directions. Locating piezoelectric patches on this line are effective for creating a bending moment and stress concentration reduction. On the other hand, low value of R/t stands for a relatively thick shell in which the patches do not have considerable effect on stress reduction.

Excessive patches filled the longitudinal directions around the hole but not on a straight line. In majority of cases, right-left position of hole was also filled. In some cases, area around the hole was completely filled with patches to move the maximum stresses far from the hole by redistributing the stress flow. It should also be noted that last few piezoelectric patches do not have significant effect on stress concentration reduction.

The influence of different parameters is discussed in the next section. It is worthy to mention that the arrangements of piezoelectric patches were almost the same for similar R/t parameter. It was proved that R/t parameter was more

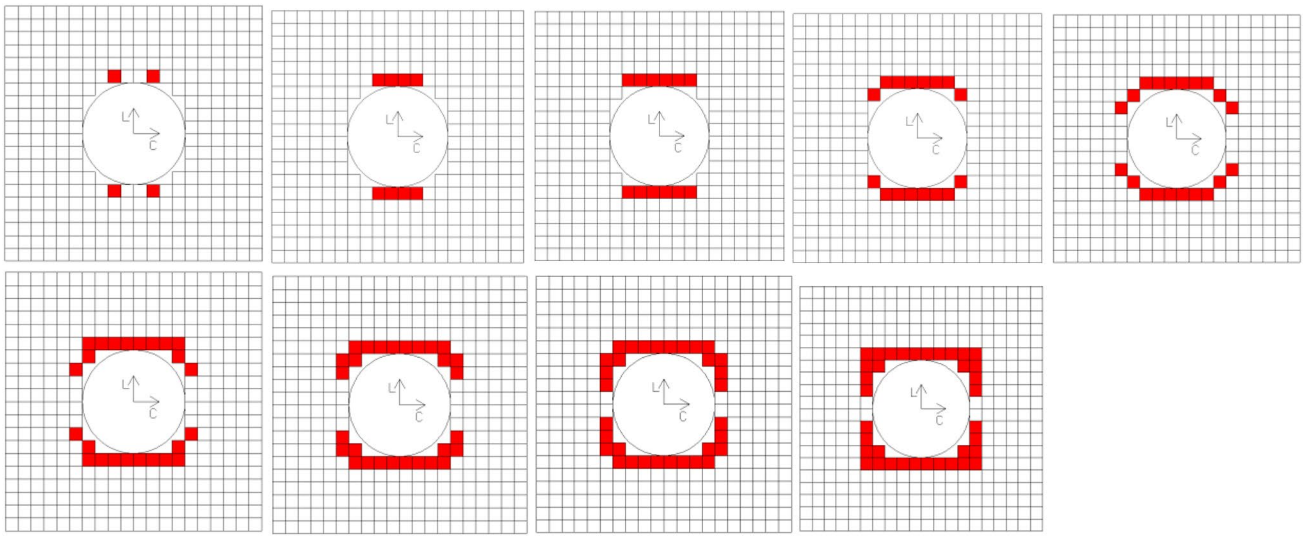

Fig. 7 Optimal patch locations for case 6

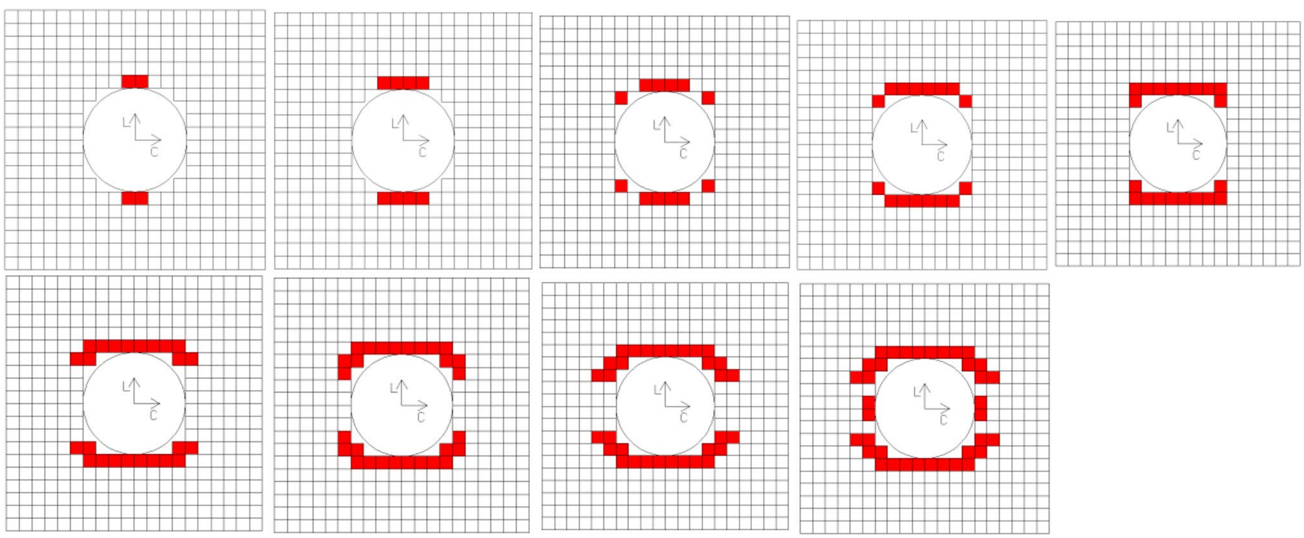

Fig. 8 Optimal patch locations for case 7

effective than other parameters on stress concentration reduction.

6.2 Influence of Different Parameters on Reducing Stress Concentration

In Figs. 18, 19, 20, 21, 22, 23, 24 and 25, effect of different parameters (*d/D, R/t* and t_p/t) on stress concentration reduction are shown. *R/t* is the most important parameter in classification and design of pressure vessels (ASME Committee on Pressure Vessels 2015; Budynas and Nisbett 2008). *R/t* has significant influence on bending stiffness of cylindrical shell. *d/D* is a common dimensionless parameter for investigation of hole effect in cylindrical shells (Van Dyke 1965; Ryu et al. 2004). t_p/t is also presented as significant dimensionless parameter in stress reduction in plate with hole via piezoelectric patches (Fesharaki and Golabi 2016a, b).

In Figs. 18, 19, 20 and 21 for constant amount of *d/D*, the effects of other parameters on stress reduction are shown. In Fig. 18 ($d/D = 0.11$), the amount of stress concentration reduction is shown by curves with $R/t = 10$, 40, 70, 100 and $tp/t = 0.1$, 0.23, 0.36, 0.5. Increasing *R/t* and *d/D* lead to more stress concentration reduction by patches, because, large *R/t* resembles relatively thinner shell and is more affected by piezo patches. By increasing

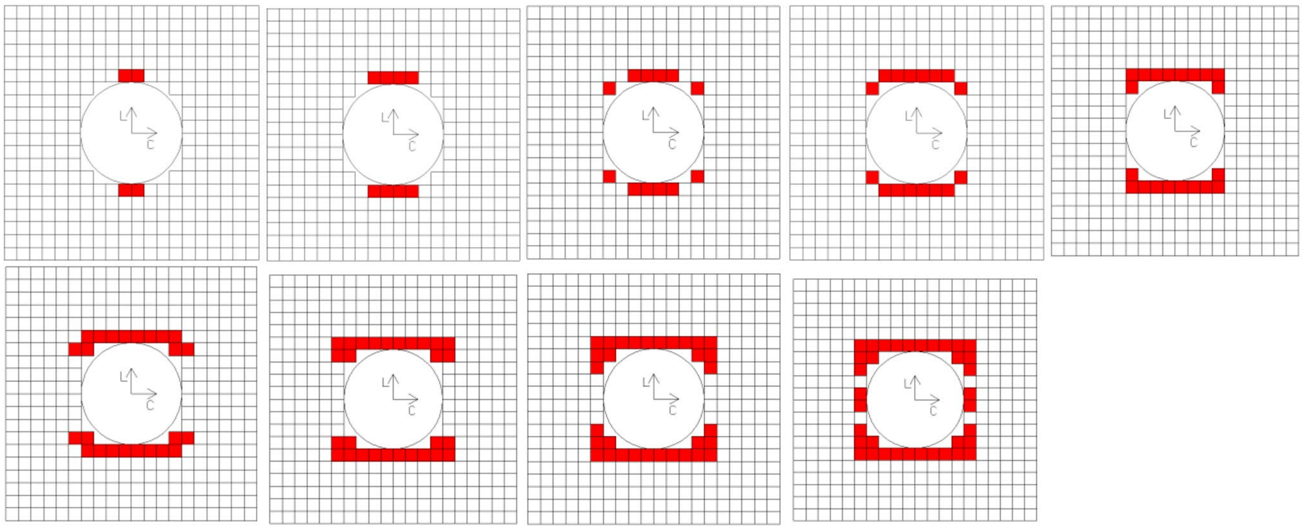

Fig. 9 Optimal patch locations for case 8

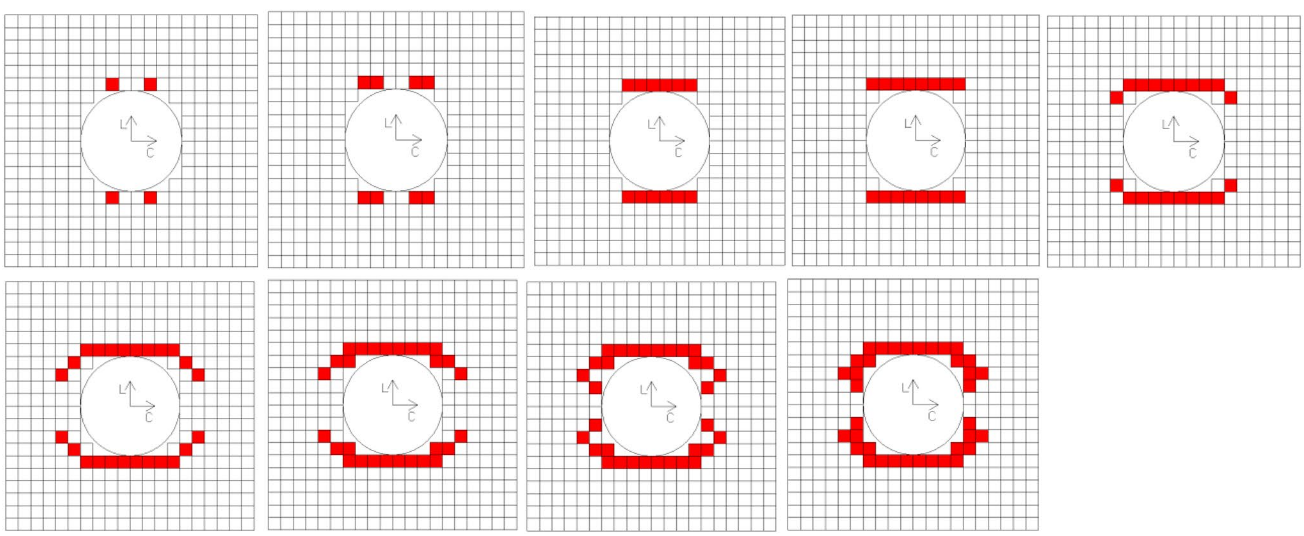

Fig. 10 Optimal patch locations for case 9

tp/t, influence of piezoelectric patches on stress reduction increases because, thicker piezo patches could generate more strain and consequently more variation on stress flow and movement of maximum stress. Maximum stress reduction of 18.44% was achieved with $tp/t = 0.5$ and $R/t = 100$ (maximum proposed amount of *R/t* and *tp/t*). The figures show that maximum amount of stress concentration reduction occurs after mounting the first few patches and adding more patches has less effect on stress reduction. Last piezoelectric patches are not located on straight line upside-downside of hole and are not effective in creating a bending moment and stress concentration reduction.

In case 1, i.e., $tp/t = 0.1$ and $R/t = 10$, the amount of stress concentration reduction is negligible. This is due to the fact that piezo thickness is relatively thin and cannot produce considerable strain in shell and therefore do not have considerable influence on varying the stress flow and stress reduction.

Curves in Fig. 19 show stress reduction for $d/D = 0.18$, $R/t = 10$, 40, 70, 100 and $tp/t = 0.23$, 0.1, 0.5, 0.36. With escalation of *R/t*, stress reduction increases as well, while in each case *tp/t* decreases. Maximum stress reduction of 21.91% is achieved with $tp/t = 0.36$ and $R/t = 100$. The amount of stress concentration reduction occurs after

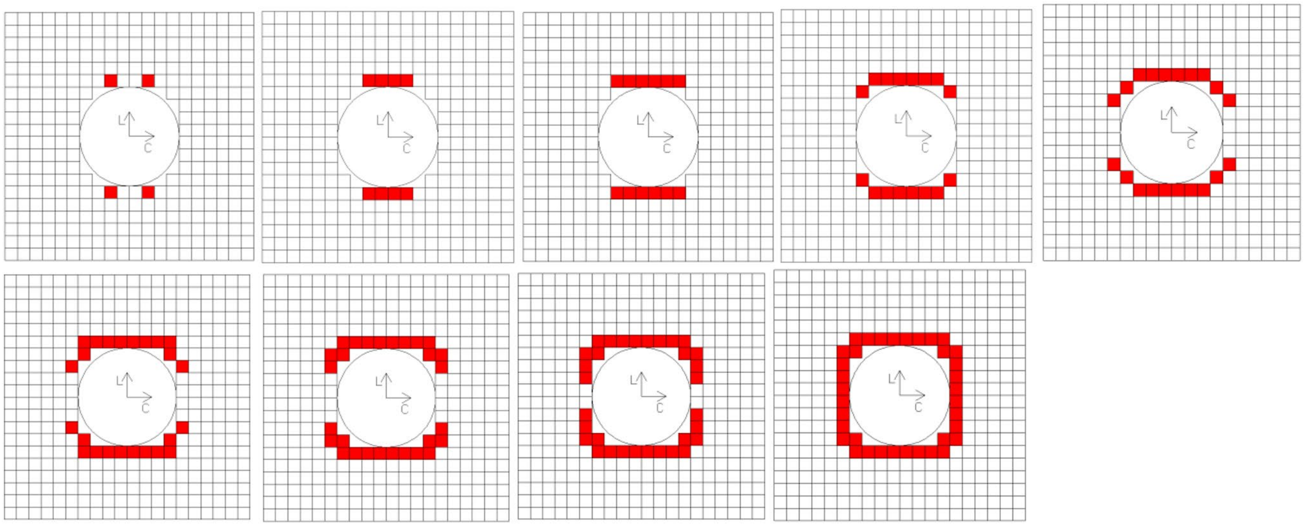

Fig. 11 Optimal patch locations for case 10

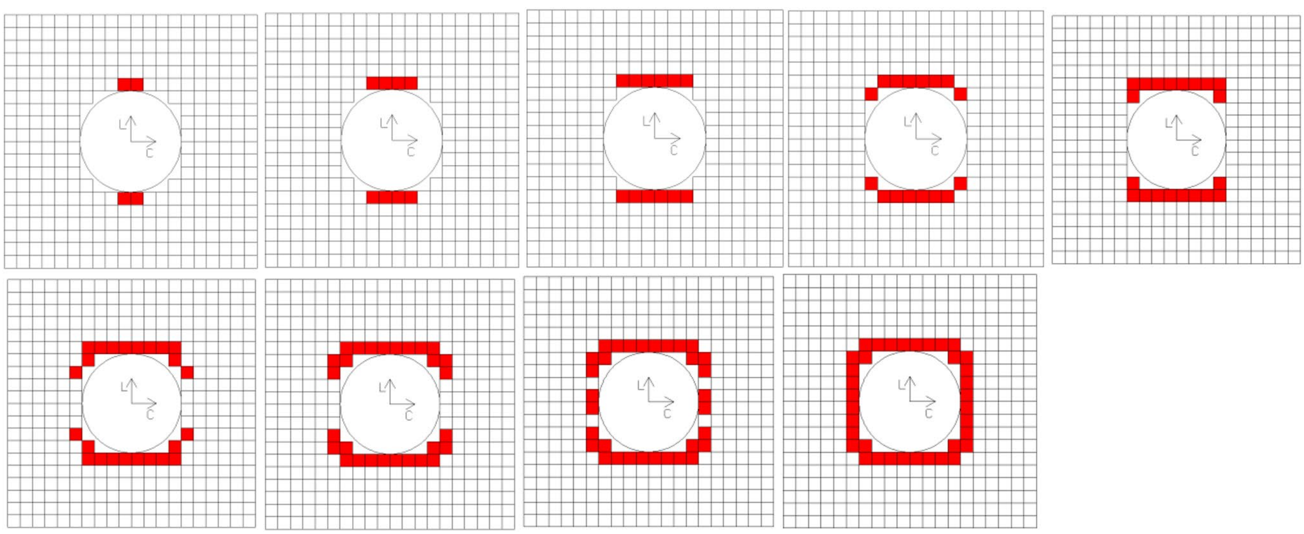

Fig. 12 Optimal patch locations for case 11

mounting the first few piezoelectric patches, and adding more patches has less effect on stress reduction. Because of low amount of *R/t* in case 2 (*tp/t* = 0.23 and *R/t* = 10), stress reduction is not significant. Contrary to previous figure, in this one, curve with R/t = 40 (case 6) has very little impact on stress reduction because of small amount of *tp/t* or patch thickness.

Curves in Fig. 20 show stress reduction for *d/D* = 0.25, *R/t* = 10, 40, 70, 100 and *tp/t* = 0.36, 0.5, 0.1, 0.23. Unlike two past figures, larger amount of *tp/t* has more impact on stress reduction in larger amount of *R/t* (*R/t* = 40). With large amount of *R/t* (*R/t* = 70) in case 10, higher amount of stress reduction is expected, but because of small magnitude of *tp*/*t*, small influence on stress reduction is observed. In this case, in spite of relatively high amount of *R/t* that leads to small bending strength, stress reduction is not much. *R/t* = 10 still has little effect on stress reduction.

Curves in Fig. 21 show stress reduction for *d/D* = 0.33. As explained before, this figure shows that thicker patches have more effect on stress reduction. In cases 11 and 15 (*R/t* = 70,100), small effect on stress reduction can be observed which is due to small magnitude of *tp/t*. With larger amount of *tp/t*, small amount of *R/t* (*R/t* = 10) still shows very small effect on stress reduction and similar to stress reduction of *R/t* = 100, stress reduction is more than previous figures.

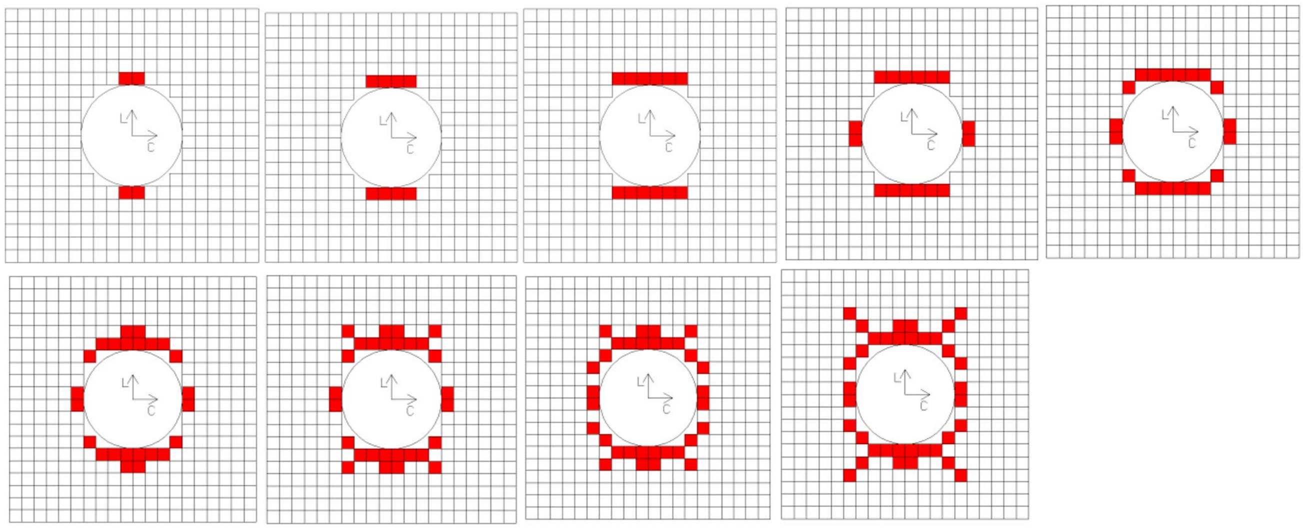

Fig. 13 Optimal patch locations for case 12

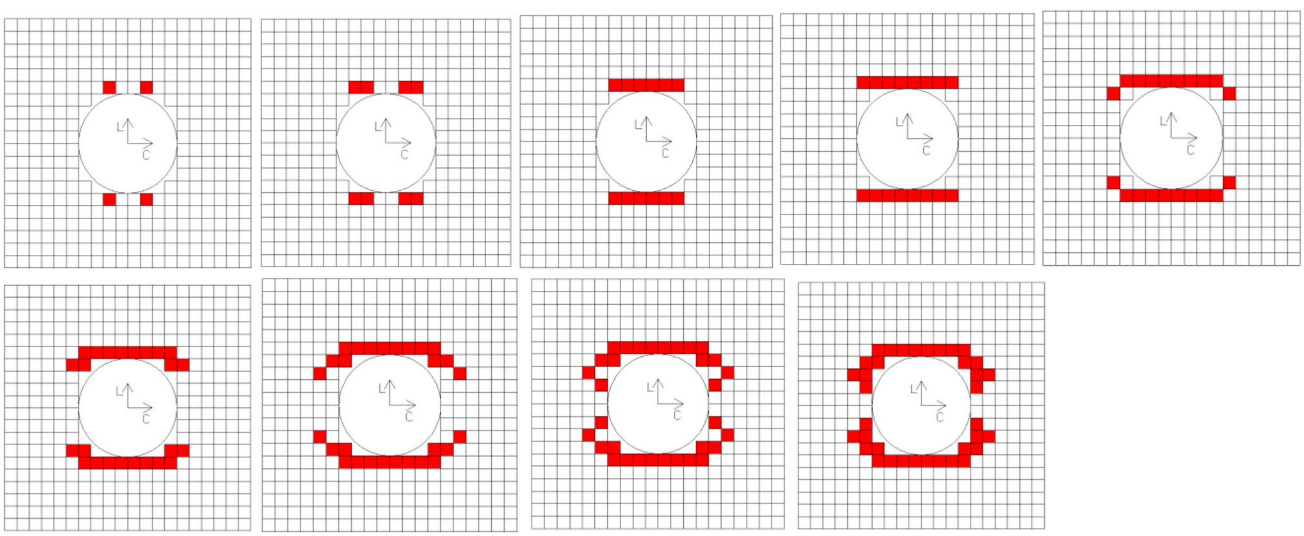

Fig. 14 Optimal patch locations for case 13

From these last four figures, it can be concluded that the amount of stress reduction increases via increase of *R/t* and *tp/t*. *R/t* has more effect on stress reduction than *tp/t*, and this stress reduction method is not effective for thick cylinders ($R/t \leq 10$).

In Figs. 22, 23, 24 and 25, the impact of other parameters on stress reduction is shown for constant amount of *R/t*.

In Fig. 22 ($R/t = 10$), the curves show the variation of stress reduction for $d/D = 0.11$, 0.18, 0.25, 0.33 that correspond to $tp/t = 0.1$, 0.23, 0.36, 0.5, respectively. With simultaneous increase of both parameters, stress reduction increases. However, as shown before, stress reduction is negligible for thick shells ($R/t = 10$).

The curves in Fig. 23 ($R/t = 40$) show the variation of stress reduction for $d/D = 0.11$, 0.18, 0.25, 0.33 that correspond to $tp/t = 0.23$, 0.1, 0.5, 0.36, respectively. Curves for $d/D = 0.25$ and 0.33 show higher amount of stress reduction than curves for $d/D = 0.11$ and 0.18. $d/D = 0.25$ has higher stress reduction than $d/D = 0.33$, and $d/D = 0.11$ has larger stress reduction than $D/d = 0.18$ because of increase of *tp/t*.

The curves in Fig. 24 ($R/t = 70$) show the variation of stress reduction for $d/D = 0.11$, 0.18, 0.25, 0.33 that correspond to $tp/t = 0.36$, 0.5, 0.1, 0.23, respectively. With increase of *tp/t*, stress reduction increases, while *d/D* does not have much effect on it.

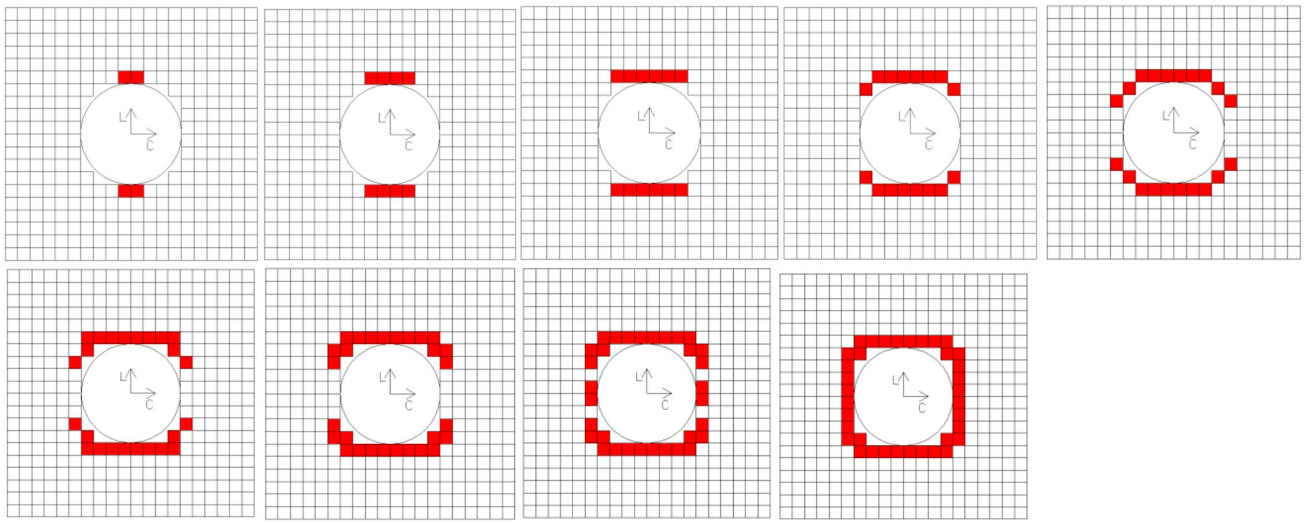

Fig. 15 Optimal patch locations for case 14

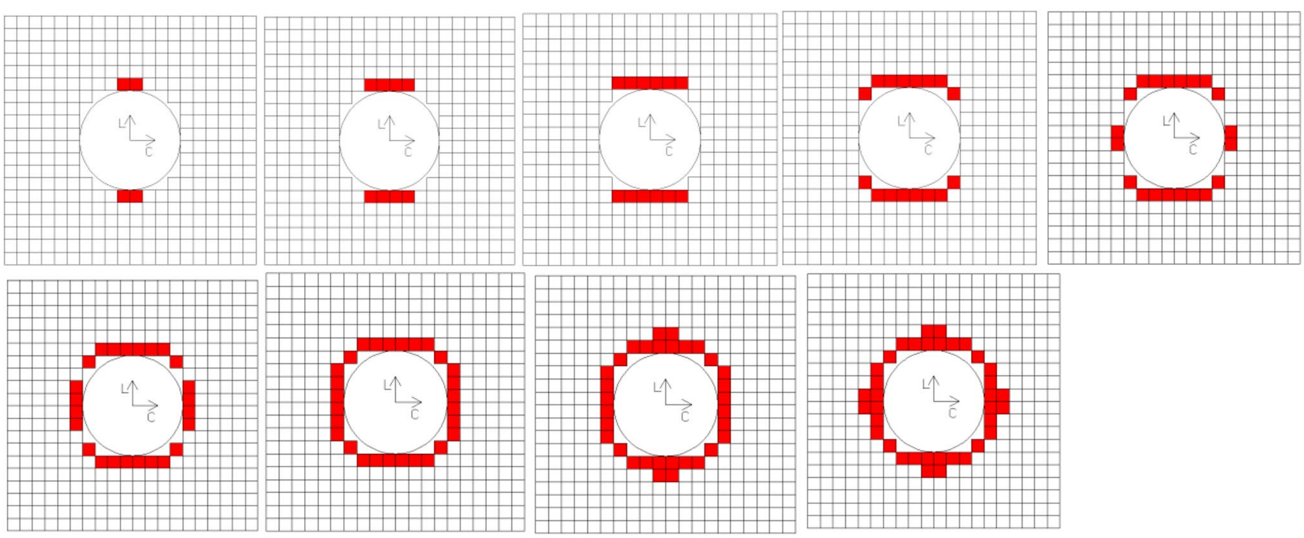

Fig. 16 Optimal patch locations for case 15

The curves in Fig. 25 ($R/t = 100$) show the variation of stress reduction for $d/D = 0.11, 0.18, 0.25, 0.33$ that correspond to $tp/t = 0.5, 0.36, 0.23, 0.1$, respectively. As above, with increase of *tp/t*, stress reduction mostly increases and *d/D* does not have considerable effect on it.

From the last four figures, it can be concluded that stress reduction increases via increase of *d/D* and *tp/t*. Influence of *tp/t* is more than *d/D* since the patches reinforce the shell beside their strain induction effects. This method for stress reduction has small effect on thick cylinders with $R/t \leq 10$. These figures also show that *R/t* has more effect than other two parameters on stress reduction.

The above figures show that relatively thicker piezo patches, i.e., larger *tp/t* ratios, have more effect on stress reduction. The amount of stress reduction is more in relatively thinner shells, i.e., larger *R/t* ratios and the effect of patches for $R/t < 10$ on stress concentration reduction is negligible.

As can be seen from all stress concentration charts, stress concentration is more around relatively smaller holes; therefore, piezo patches have more effect on stress reduction for smaller amount of *d/D*.

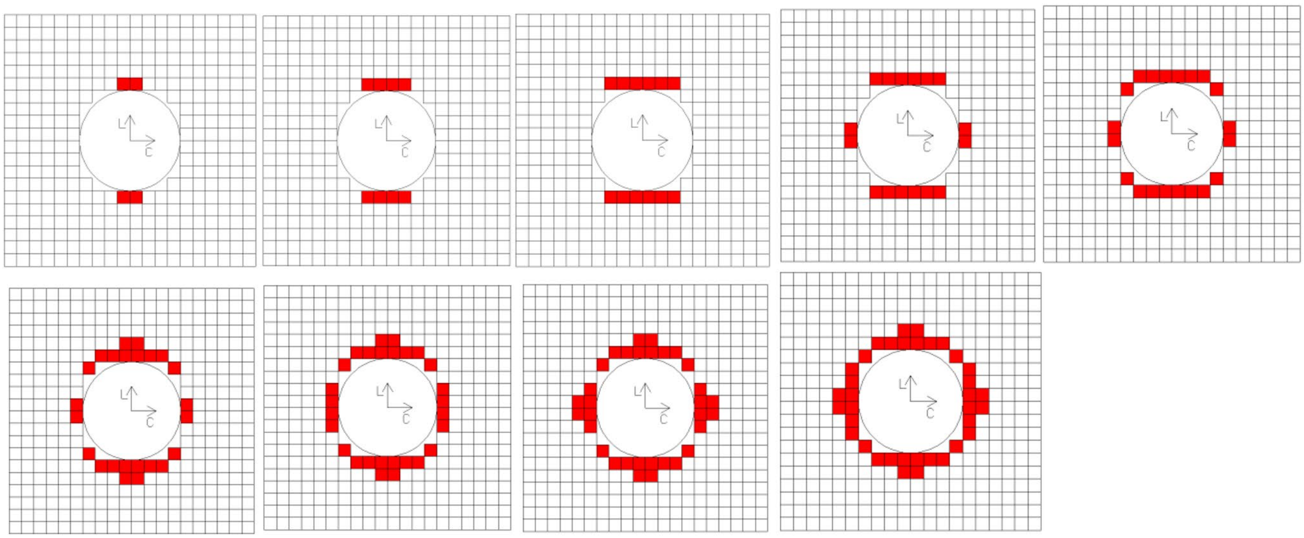

Fig. 17 Optimal patch locations for case 16

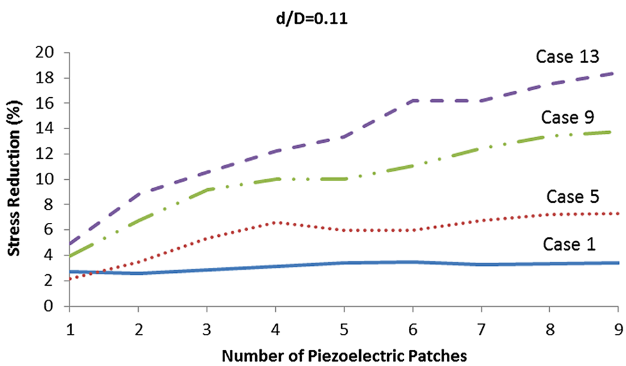

Fig. 18 Stress reduction for $d/D = 0.11$

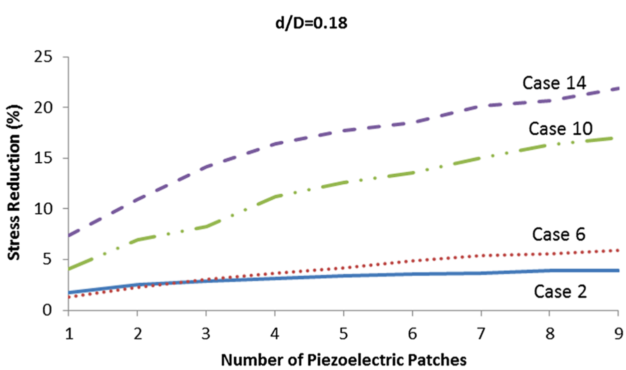

Fig. 19 Stress reduction for $d/D = 0.18$

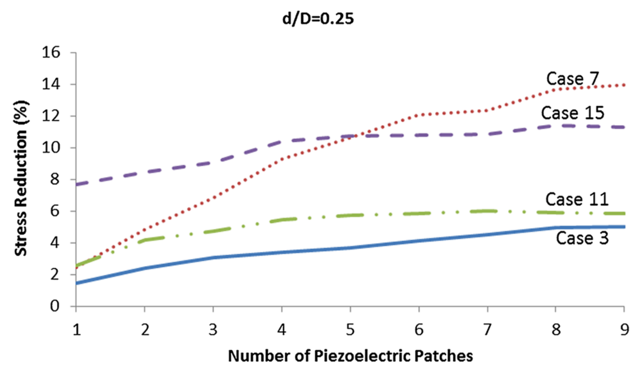

Fig. 20 Stress reduction for $d/D = 0.25$

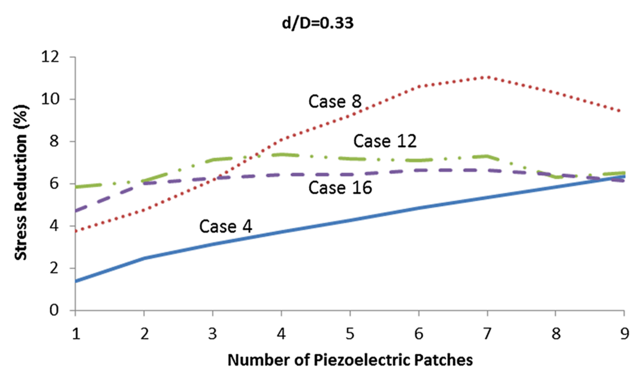

Fig. 21 Stress reduction for $d/D = 0.33$

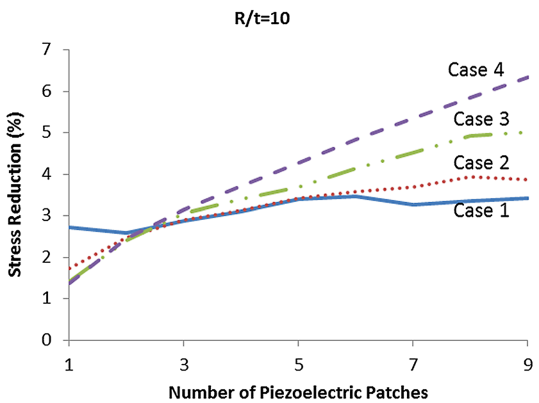

Fig. 22 Stress reduction for $R/t=10$

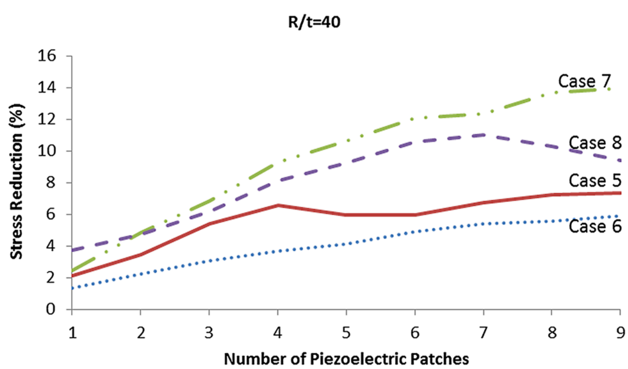

Fig. 23 Stress reduction for $R/t=40$

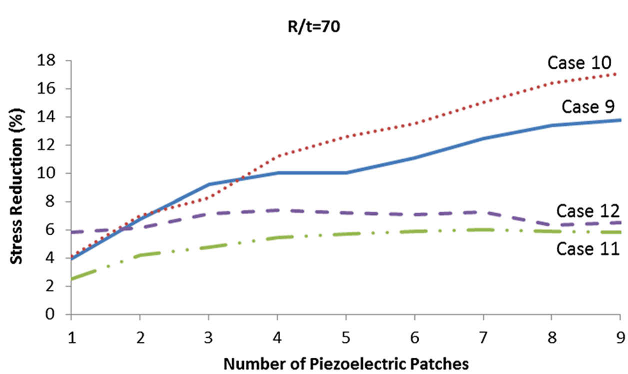

Fig. 24 Stress reduction for $R/t=70$

7 Experimental Results

Three tests have been performed for validation of this research. A 3-mm-thickness cylinder with 166.3 mm outer diameter and 2.1 m length with flat heads was made and subjected to 90 psi (6.2 bar) internal pressure. Three

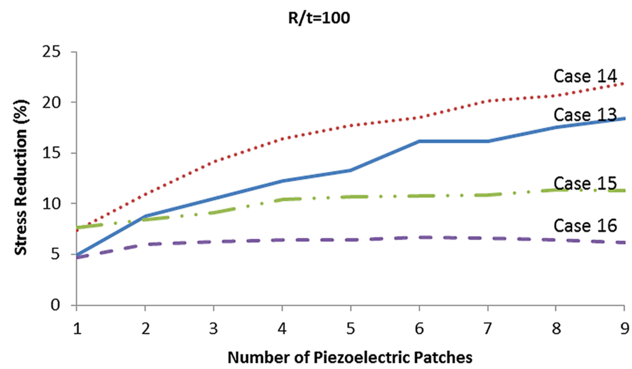

Fig. 25 Stress reduction for $R/t=100$

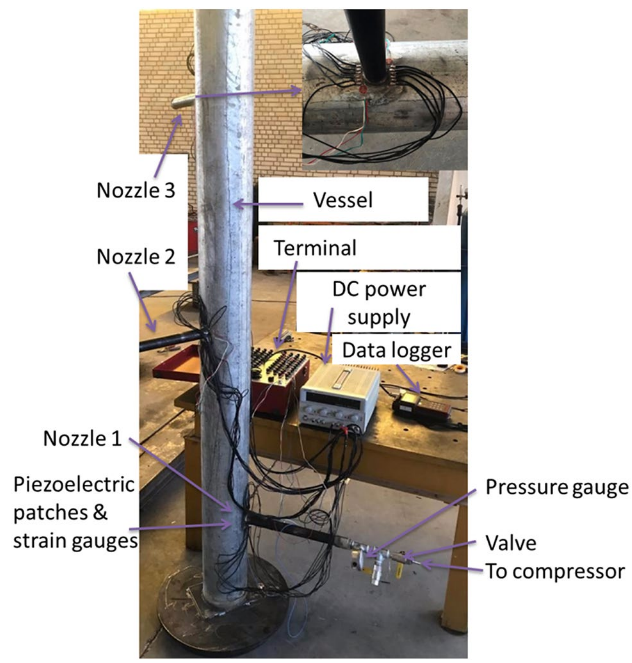

Fig. 26 Setup of experimental test

nozzles, each with 33.7 mm outer diameter, 3 mm thickness, 200 mm length, 0.7 m distance from each other, 0.35 m from both ends and not in line (with 120° discrepancies in circumferential view), were welded in the axial direction of the proposed shell with 2.5 mm weld leg (Fig. 26).

Piezoelectric patches and Rosette strain gauges have been attached around the nozzles as showed in Fig. 27. Strain gauges were pasted in up-down and right-left position of two nozzles. Each piezoelectric patch covered a 10×5 mm space. Two strain gauges were utilized in each test for higher accuracy. The material of cylinder and nozzles was SA106 carbon steel with 209 GPa Young modulus and 0.3 Poison ratio. PZT4 type piezoelectric patches with 0.5 mm thickness

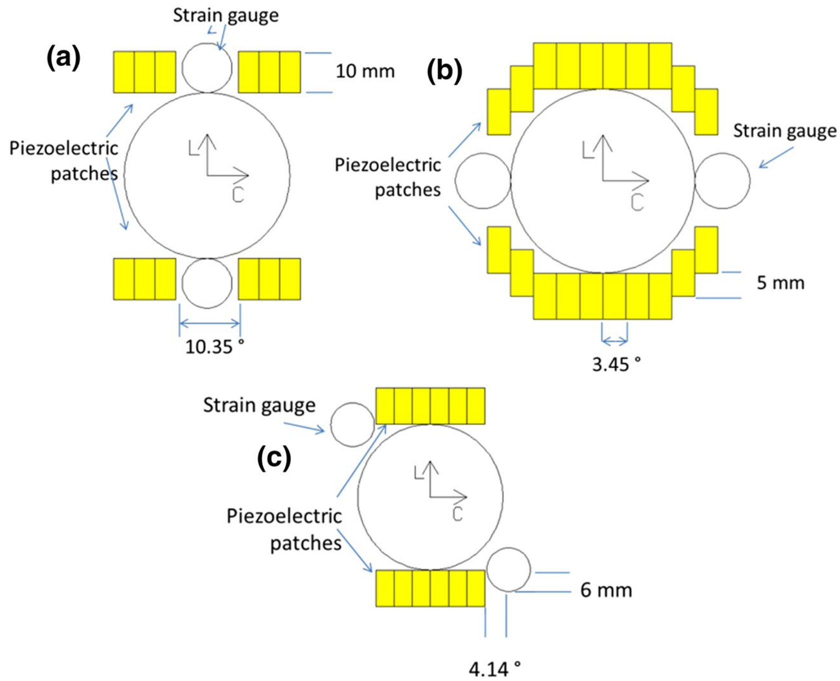

Fig. 27 Piezoelectric patches and strain gauge arrangement in (**a**) nozzle 1, **b** nozzle 2 and **c** nozzle 3

Table 3 Results of analysis and experiment

Row	Nozzle number	Potential difference (V)	Strain (%) analysis $\times 10^{-6}$	Strain (%) experiment $\times 10^{-6}$	Error (%)
1	1	0	119	110	7.6
2	1	50	117.2	106	9.6
3	1	100	115.4	106	8.2
4	1	200	113.3	103	9.1
5	2	0	46.3	42	9.3
6	2	50	45.9	41	10.6
7	2	100	44.9	39	13.1
8	2	200	43.5	40	8
9	3	0	120.3	112	6.5
10	3	50	119.8	110	8.2
11	3	100	116.5	109	6.4
12	3	200	114.3	102	10.8

were utilized in the experiments. The patches were polarized in thickness direction and pasted via electrically conductive silver adhesive (AA-DUCT 907 atom adhesives) on the cylinder according to three obtained arrangements (for each nozzle). A specific amount of electric potential was applied on piezoelectric patches by laboratory DC power supply. The experiment has been conducted with four different voltages (Table 3). Normal strains in circumferential direction $\left(\varepsilon_{cc}\right)$ determined from above analysis and this experiment were presented and compared in Table 3 for different cases. In most cases, the results of analysis were in good agreement with experimental ones.

8 Conclusions

This research has focused on reducing the amount of maximum stress around holes in pressure vessels subjected to internal pressure using piezoelectric patches instead of reinforcement plates. Since there are many affecting parameters on stress concentration around a hole, the most influencing dimensionless parameters were determined, i.e., *R/t, tp/t* and *d/D*. After determining the most applicable range for each parameter, four levels were defined for each dimensionless parameter and with implementing Taguchi method, 16 cases were selected for this investigation. Using ACO algorithm, the effect of optimum attachment of up to 9 piezoelectric patches around the holes was researched for each case. It was shown that:

R/t is the most influencing parameter on stress reduction, and *tp/t* and *d/D* are the second and third important ones, respectively.

It was shown that piezoelectric patches do not have considerable effects on stress reduction around a hole in thick cylinders, i.e., $R/t \leq 10$. Larger amount of *tp/t* has more effect on stress reduction because of both strain induced by piezoelectric effect and the reinforcement influence of patch. The strain induced by thicker patches and larger *tp/t* could vary the stress flow more than the thinner ones and can consequently move the location of the maximum stress and distribute the concentrated stress on a wider area.

Since there are both tangential and axial tension in a cylindrical shell, the piezo patches, despite unidirectional tension, have limited potential for stress reduction and the pattern of patches is not similar to that in unidirectional tension. Consequently, depending on the cylinder parameters up to 22% stress reduction could be achieved by attaching piezoelectric patches around holes in thin cylinders.

In all cases, the attachment of piezoelectric patches moves the location of maximum stress from the points around the hole away from the hole. This new maximum stress at new location could sometimes be reduced by adding more patches in some cases.

The above data are intended to be used to train a neural network to predict the effect of various combinations of the above dimensionless parameters on reducing stress concentration around a hole in cylindrical pressure vessel by attaching piezoelectric patches.

The experimental tests showed good compliance of the results with finite element simulation and previous studies.

Acknowledgements The authors wish to thank the University of Kashan for supporting this research by Grant No. 682570.

References

ASME Committee on Pressure Vessels (2015) Rules for construction of pressure vessels: Sec VIII, Div 1. American Society of Mechanical Engineers

Budynas RG, Nisbett JK (2008) Shigley's mechanical engineering design. McGraw-Hill, New York

Dorigo M, Maniezzo V, Colorni A (1996) Ant system: optimization by a colony of cooperating agents. IEEE Trans Syst Man Cybern Part B Cybern 26:29–41

Fesharaki JJ (2014) Optimum pattern of piezoelectric actuator placement for stress concentration reduction in a plate with a hole using particle swarm optimization algorithm. In: Proceedings of the institution of mechanical engineers, part c: journal of mechanical engineering science, p 0954406214538617

Fesharaki JJ, Golabi SI (2016a) Effect of stiffness ratio of piezoelectric patches and plate on stress concentration reduction in a plate with a hole. Mech Adv Mater Struct 24:1–32

Fesharaki JJ, Golabi SI (2016b) A novel method to specify pattern recognition of actuators for stress reduction based on Particle swarm optimization method. Smart Struct Syst 17:725–742

Fesharaki JJ, Madani SG (2016) Effect of stiffness and thickness ratio of host plate and piezoelectric patches on reduction of the stress concentration factor. Int J Adv Struct Eng 8:229–242

Fesharaki JJ, Madani SG, Golabi S (2018) Best pattern for placement of piezoelectric actuators in classical plate to reduce stress concentration using PSO algorithm. Mech Adv Mater Struct. https://doi.org/10.1080/15376494.2018.1472332

Fridman A (1986) Asymptotic method of investigating the deformation of cylindrical shells with holes. Soviet Appl Mech 22:1181–1186

Golabi SI, Fesharaki JJ (2014) Reducing stress concentration factor around a hole on a plate by piezoelectric patch. Int J Adv Des Manuf Technol 6:59–64

Hsu C-Y, Liang C-C, Shiah S-W, Jen C-Y (2005) A study of stress concentration effect around penetrations on curved shell and failure modes for deep-diving submersible vehicle. Ocean Eng 32:1098–1121

Kamalarajah R, Stoffberg W, Bull JW, Chizari M (2015) Stress analysis of uniform circular cylindrical shells with large circular holes. In: Lecture notes in engineering and computer science: proceedings of the world congress on engineering

Kharat A, Kulkarni V (2013) Stress concentration at openings in pressure vessels-a review. Int J Innov Res Sci Eng Technol 2:670–677

Kornecki A (1966) Comments on stresses about a circular hole in a cylindrical shell. AIAA J 4:0765a–0765a

Lind N (1968) Stress concentration of holes in pressurized cylindrical shells. AIAA J 6:1397

Lu M-H, Yu J-S, Chen J-J (2014) The effect of analysis model on the stress intensity calculation for the nozzle attached to pressure vessel under internal pressure loading. Int J Press Vessels Pip 117:9–16

Mehrabian AR, Yousefi-Koma A (2011) A novel technique for optimal placement of piezoelectric actuators on smart structures. J Franklin Inst 348:12–23

Mirzagoltabar M (2014) Studying decreased methods of stress concentration around holes and openings in plate and shell structures. Science International, vol. 26, no. 1. Gale Academic Onefile. https://go.galegroup.com/ps/anonymous?id=GALE%7CA376229623&sid=googleScholar&v=2.1&it=r&linkaccess=abs&issn=10135316&p=AONE&sw=w. Accessed 30 Sept 2019

Mukhtar F, Al-Gahtani H (2016) Finite element analysis and development of design charts for cylindrical vessel-nozzle junctures under internal pressure. Arab J Sci Eng 41:4195–4206

Nguyen Q, Tong L (2007) Voltage and evolutionary piezoelectric actuator design optimisation for static shape control of smart plate structures. Mater Des 28:387–399

Przybylski J (2008) Stability of an articulated column with two collocated piezoelectric actuators. Eng Struct 30:3739–3750

Rajaiah K, Kumar RR (1982) Optimum hole shapes in circular cylindrical shells under tension and torsion. Mech Res Commun 9:85–93

Rajaiah K, Kumar RR (1985) Stresses around circular, square and optimized holes in circular cylindrical shells under torsion. Int J Solids Struct 21:177–185

Ramsey J Jr (1975) Stress concentration factors for circular, reinforced penetrations in pressurized cylindrical shells, Ph.D, thesis-Virginia Univ

Ryu C-H, Lee Y-S, Choi M-H, Kim Y-W (2004) A study on stress analysis of orthotropic composite cylindrical shells with a circular or an elliptical cutout. J Mech Sci Technol 18:808–813

Sensharma PK, Haftka RT (1996) Limits of stress reduction in a plate with a hole using piezoelectric actuators. J Intell Mater Syst Struct 7:363–371

Sensharma PK, Palantera MJ, Haftka RT (1993) Stress reduction in an isotropic plate with a hole by applied induced strains. J Intell Mater Syst Struct 4:509–518

Shah D, Joshi S, Chan W (1994) Stress concentration reduction in a plate with a hole using piezoceramic layers. Smart Mater Struct 3:302

Uchino K, Debus J-C (2012) Applications of ATILA FEM software to smart materials: case studies in designing devices. Elsevier, Amsterdam

Van Dyke P (1965) Stresses about a circular hole in a cylindrical shell. AIAA J 3:1733–1742

Wu N, Wang Q (2011) An experimental study on the repair of a notched beam subjected to dynamic loading with piezoelectric patches. Smart Mater Struct 20:115023

Xue M-D, Du Q-H, Hwang K-C, Xiang Z-H (2010a) An analytical method for cylindrical shells with nozzles due to internal pressure and external loads—part I: theoretical foundation. J Press Vessel Technol 132:031206

Xue M-D, Du Q-H, Hwang K-C, Xiang Z-H (2010b) An analytical method for cylindrical shells with nozzles due to internal pressure and external loads—part II: design method. J Press Vessel Technol 132:031207