

Effect of Lorentz Force on Non-Axisymmetric Thermo-Mechanical Behavior of Functionally Graded Hollow Cylinder

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Abstract. This paper presents the effect of the non-axisymmetric magnetic and steady state thermal load on thermo magneto mechanic functionally graded (FG) hollow cylinder. An analytical solution for stresses and Lorentz force were determined using the separation of variables and complex Fourier series and the power law functions for material, except Poisson's ratio, through the thickness of the cylinder. The results show that the magnetic field has a significant effect on stresses and displacements along the cylinder and with the specific magnetic field vector the stresses and displacements along the cylinder can be controlled and optimized. The aim of this work was to understand the effect of Lorentz force on a functionally graded hollow cylinder subjected to mechanical, thermal and magnetic load. The magnetic field decrease circumferential displacements and stress and increase the radial displacements and stress due to thermo mechanical loads.

Introduction

Functionally graded materials (FGMs) have attracted widespread attention in recent years. A FGM is usually a combination of two material phases that varying continuously along certain directions. This continuous transition allows the certain of multiple properties without any mechanically weak interface. This makes FGM suitable for many specific applications such as sensors or actuators. Most scientists work on symmetry conditions but in some cases, the non-axisymmetric conditions must be considered. The problem of Functionally graded hollow spheres under non-axisymmetric thermo-mechanical loads was investigated by Poultangari, et al. [1]. They assumed analytical solution in the form of Legendre series. The general solution for mechanical and thermal stresses in a functionally graded hollow cylinder due to non-axisymmetric steady-state loads was presented in the form of complex Fourier series by Jabbari, et al.[2]. Transient thermoelastic problem of functionally graded thick strip due to non uniform heat supply were researched by Ootao and Tanigawa [3]. They investigate the influence of temperature and material non homogeneity on displacement and stress distributions. Dai and Fu showed the analytical method on magnetothermoelastic interactions in hollow structures of functionally graded material subjected to mechanical loads [4]. The exact solution of steady-state two-dimensional axisymmetric mechanical and thermal stresses for a short hollow cylinder made of functionally graded material is developed by Jabbari, et al.[5]. The alternative state space formulations for magnetoelectric thermoelasticity with transverse isotropy and the application to bending analysis of non homogeneous plates were studied by Chen and Lee[6]. In this paper at first, by using the separation of variables and complex Fourier series for Navier equation, the magneto thermo mechanical behavior of functionally graded hollow cylinder due to non-axisymmetric magnetic and thermal load was solved. Then the effect of Lorentz force on displacement and stress perturbation in cylinder was investigated.

Formulation of the Problem

Consider a thick FG hollow cylinder subjected to a uniform magnetic field vector $\vec{H}(0,0,H_z)$ and temperature gradient (T). The components of displacement and stress in the cylindrical coordinate (r, θ, z) are respectively expressed as:

$$\sigma_{rr} = q_{11} \frac{\partial U(r, \theta)}{\partial r} + q_{12} \left(\frac{U(r, \theta)}{r} + \frac{1}{r} \frac{\partial V(r, \theta)}{\partial \theta} \right) - p_{11} T(r, \theta). \quad (1)$$

$$\sigma_{\theta\theta} = q_{12} \frac{\partial U(r, \theta)}{\partial r} + q_{11} \left(\frac{U(r, \theta)}{r} + \frac{1}{r} \frac{\partial V(r, \theta)}{\partial \theta} \right) - p_{11} T(r, \theta). \quad (2)$$

$$\sigma_{r\theta} = q_{22} \left(\frac{1}{r} \frac{\partial U(r, \theta)}{\partial \theta} - \frac{V(r, \theta)}{r} + \frac{\partial V(r, \theta)}{\partial r} \right). \quad (3)$$

The coefficients q and p are elastic and thermal expansion, respectively and vary the along radial direction. U and V are displacements along radial and tangential direction of cylinder, respectively. Substituting Eqs. (1) to (3) into two equilibrium equations in cylindrical coordinates, and consider the Lorentz force and material properties as power-law (r^β) function across the thickness of the cylinder, yields two Navier equations as:

$$\frac{\partial^2 U(r, \theta)}{\partial r^2} + \frac{d_1}{r^2} \frac{\partial^2 U(r, \theta)}{\partial \theta^2} + \frac{d_2}{r} \frac{\partial U(r, \theta)}{\partial r} + \frac{d_3}{r^2} U(r, \theta) + \frac{d_4}{r} \frac{\partial^2 V(r, \theta)}{\partial r \partial \theta} + \frac{d_5}{r^2} \frac{\partial V(r, \theta)}{\partial \theta} - 2d_6 r^{\beta-1} T(r, \theta) - d_6 r^\beta \frac{\partial T(r, \theta)}{\partial r} = 0. \quad (4)$$

$$\frac{\partial^2 V(r, \theta)}{\partial r^2} + \frac{d_7}{r^2} \frac{\partial^2 V(r, \theta)}{\partial \theta^2} + \frac{d_8}{r} \frac{\partial V(r, \theta)}{\partial r} + \frac{d_9}{r^2} V(r, \theta) + \frac{d_{10}}{r} \frac{\partial^2 V(r, \theta)}{\partial r \partial \theta} + \frac{d_{11}}{r^2} \frac{\partial U(r, \theta)}{\partial \theta} - d_{12} r^{\beta-1} \frac{\partial T(r, \theta)}{\partial \theta} = 0. \quad (5)$$

Where, the new defined coefficients d are constants along the radial direction.

Solving the Problem

The two-dimensional steady state heat conduction equation for a functionally graded material hollow cylinder solved by Jabbari et al. [2] that used in the Navier equations. To solve the Eqs. (4) and (5) the temperature gradient T and the displacement components U and V are considered in the complex Fourier series as:

$$T(r, \theta) = \sum_{n=-\infty}^{\infty} T_n(r) e^{in\theta}. \quad (6)$$

$$U(r, \theta) = \sum_{n=-\infty}^{\infty} U_n(r) e^{in\theta}. \quad (7)$$

$$V(r, \theta) = \sum_{n=-\infty}^{\infty} V_n(r) e^{in\theta}. \quad (8)$$

Where $T_n(r)$, $U_n(r)$ and $V_n(r)$ are the coefficients of the Fourier series. Substituting Eqs. (6) to (8) into Navier Eqs. (4) and (5) yields the system of Equations that must be solved in general and particular solution by considering the boundary conditions. The boundary conditions may be either the given displacements or stresses, or combinations.

Results and Effect of Lorentz Force

Consider a thick FG hollow cylinder in the uniform magnetic field vector $\vec{H}(0,0,H_z)$ where the inside boundary is traction-free with given temperature distribution $T(a,\theta) = 50\sin 2\theta^\circ C$. and the inner pressure $\sigma_{rr} = 200\cos 2\theta$. The outside boundary is assumed to be fixed with zero temperature. Therefore, the boundary conditions at the outside yield $U(b,\theta) = V(b,\theta) = T(b,\theta) = \sigma_{rr}(b,\theta) = 0$. Figs. 1 to 4 show the radial and circumferential stresses and displacements, with and without considering the Lorentz force. It can be seen that the Lorentz force has a significant effect on increasing the radial stress and displacement and decreasing the circumferential stress and displacement along the thickness of cylinder. So with the specific magnetic field vector the stresses and displacements along the cylinder can be controlled.

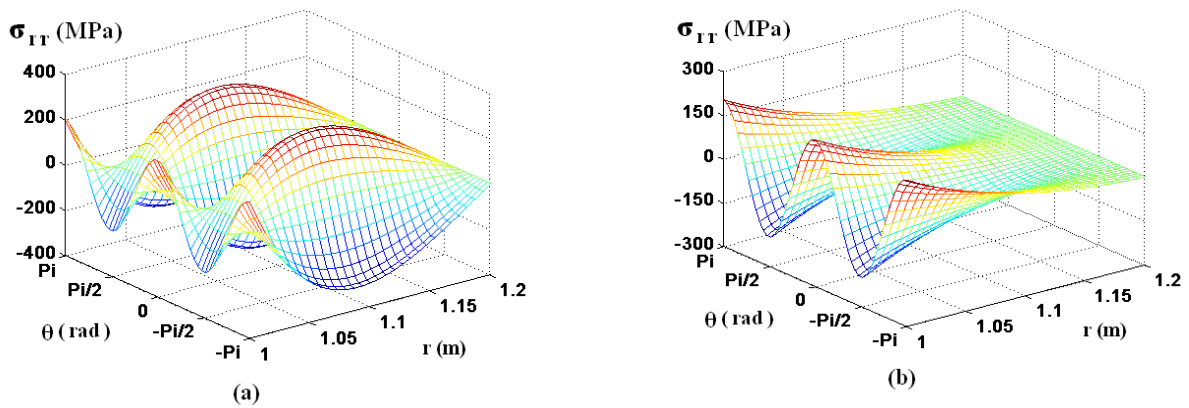


Figure1. Radial stress a) with Lorentz force b) without Lorentz force

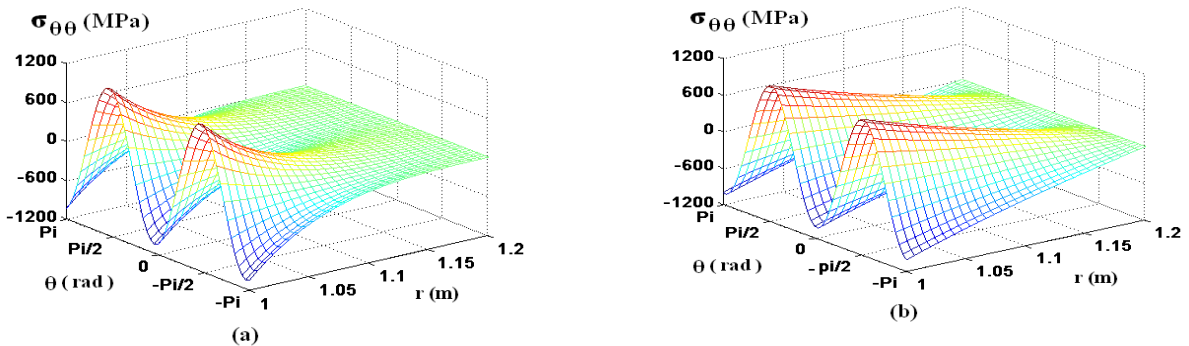


Figure2. Circumferential stress a) with Lorentz force b) without Lorentz force

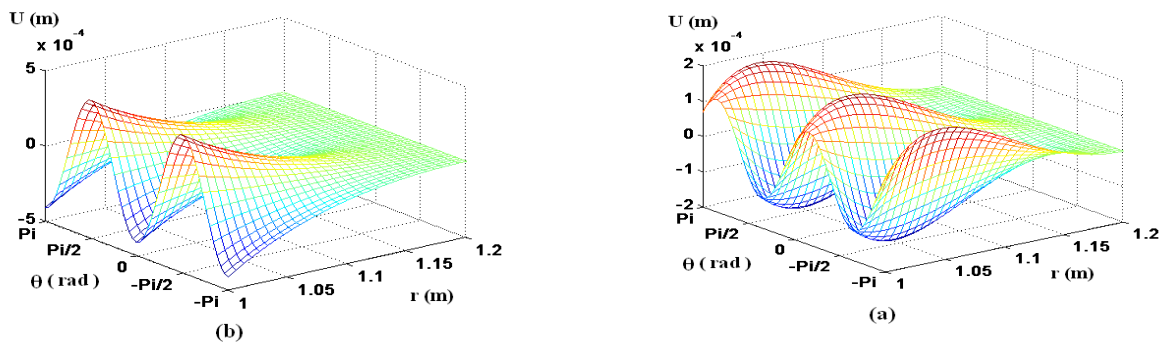


Figure3. Radial displacement a) with Lorentz force b) without Lorentz force

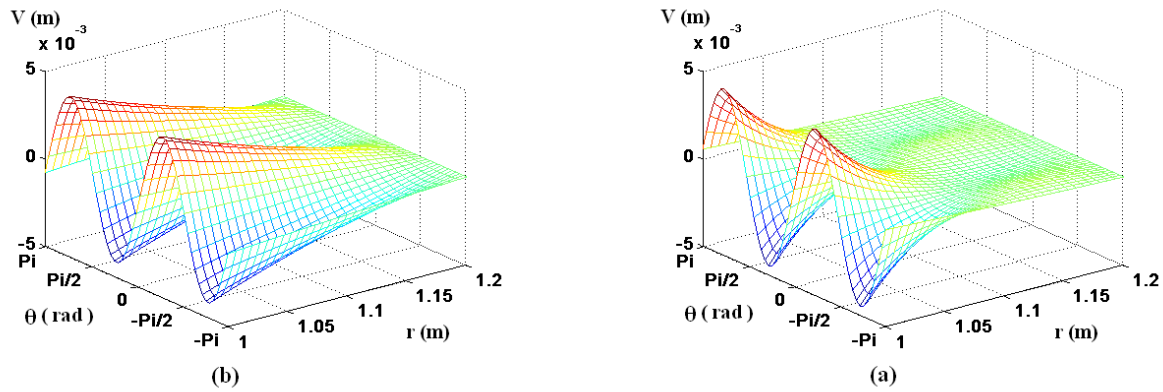


Figure 4. Circumferential displacement a) with Lorentz force b) without Lorentz force

Conclusion

In this paper, the analytical solution for the magneto thermo mechanical behavior of a functionally graded hollow cylinder is presented. The analytical solution for stresses and displacement were determined by using the complex Fourier series method. The mechanical, thermal and magnetic property of materials varied continuously across the radial direction of cylinder according to the power-law functions. The Lorentz force resulted in decrease circumferential displacement and stress and increase the radial displacement and stress due to thermo mechanical loads. In general, the magnetic field has a significant effect on stresses and displacements of a thermo magneto mechanical behavior of a functionally graded hollow cylinder and by determine the specific magnetic field vector the stresses and displacements along the cylinder can be controlled and optimized.

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