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MODELING AND SIMULATION METHODS FOR BRUSHLESS DC MOTOR DRIVES

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ABSTRACT

The purpose of this paper is comprehensive modeling and simulation of Brushless DC (BLDC) motor drives with introducing their capabilities. Firstly, linear state-space modeling is presented. After that, because of attractive aspects of using the Field Oriented Control (FOC) for other types of AC machines lead the researchers to use it for modeling of trapezoidal BLDC drive systems. But, trapezoidal Back-EMF waveform of BLDC motor causes poor accuracy. FOC method along with Multiple Reference Frame (MRF) analysis is another solution to have more accuracy, but with more complicated equations and more time for simulation. Then, the functional simulation model via using Switching Function Concept (SFC) is studied. To obtain both steady state and dynamic performances by using the stator circuit equation and the motion equation, numerical modeling methods is explained. Finally simulation results SFC method are shown.

1. INTRODUCTION

The permanent magnet synchronous motor is being used in computer, aerospace, military, automotive, industrial and household applications. In addition, trapezoidal type of this motor (BLDC), offers several advantages over their sinusoidal counterparts including greater power density, ease of construction and smaller inverter size. On the other hand, prediction of motor performance is necessary for the evaluation characteristics of motor designs and motor modeling. Simulation design is a preferred method in motor designing compared to building motor prototypes that is more costly and needs longer time [1, 2]. Effective modeling and simulation of such systems require a software tool that can handle all these functions in an integrated environment. Available simulation softwares for electronic circuits or dynamic systems can be classified into two main categories: (1) circuit simulation programs such as Spice-based, EMTP, Saber (2) equation solver programs such as Matlab/Simulink, SIMNON. These programs are not designed specifically for power electronic systems so that the users have to develop their own models to fulfill their needs [3, 4]. In this paper, we compare the different modeling and simulation methods of BLDC motor drives and explore their advantages and disadvantages.

2. ANALYSIS OF 3-PHASE BLDC MOTOR

The typical mathematical model of a three-phase BLDC motor is described by the following equations:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \times \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L-M & 0 & 0 \\ 0 & L-M & 0 \\ 0 & 0 & L-M \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \quad (1)$$

Where R is the resistance, L is the stator winding inductance, M is the mutual inductance and v_a, v_b, v_c and e_a, e_b, e_c and i_a, i_b, i_c are phase voltage, back-EMF voltage and phase current of each phase of stator respectively. The electromagnetic torque is expressed as

$$T_e = \frac{1}{\omega_m}(e_a i_a + e_b i_b + e_c i_c) \quad (2)$$

The interaction of T_e with the load torque determines how the motor speed builds up:

$$T_e = T_L + J\frac{d\omega_r}{dt} + B\omega_r \quad (3)$$

Where T_L is load torque, J is inertia, and B is damping. Figure 1 shows the voltage and current parameters in a three-phase BLDC motor. For this motor, in each time only two phases are excited through the conduction operating modes. Figure 2 shows the three quasi-square waveform phase currents with the trapezoidal back-EMF of this type of BLDC motor [2,4]. The shown waveforms are the ideal, but as the shapes of slot, skew and magnet of BLDC motor are varied subject to design purposes, the waveform of real back-EMF is at some degree deviated from the ideal waveform as shown in figure 2. In addition, because of time constant, phase current are not same as quasi-square. So in order to obtain accurate motor characteristics in simulation, simulation model should be near to real waveform. In next section the different modeling methods is proposed.

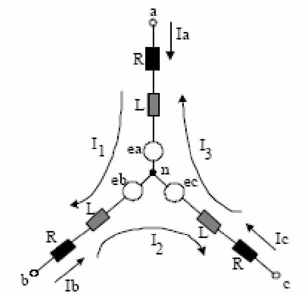

Figure 1. *Equivalent circuit of the three- phase BLDC motor*

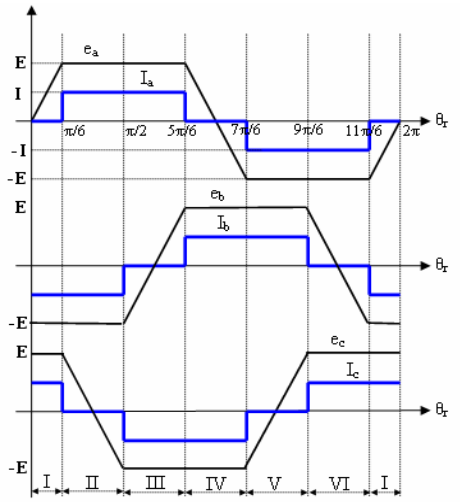

Figure 2. *Back-EMF and phase current waveforms of BLDC motor*

3. MODELING OF 3-PHASE BLDC MOTOR DRIVE

The major of various modeling and simulation method for 3-phase BLDC motor drive have been proposed that are as steady-state representation, field oriented based techniques, finite element method, switching function concept which they have different capabilities.

3.1. State Space Representation

The instantaneous induced emfs can be written as

$$\begin{cases} e_a = f_a(\theta_r)\lambda_p\omega_m \\ e_b = f_b(\theta_r)\lambda_p\omega_m \\ e_c = f_c(\theta_r)\lambda_p\omega_m \end{cases} \tag{4}$$

Where, $f_a(\theta_r), f_b(\theta_r), f_c(\theta_r)$ functions have the same shape as e_a, e_b, e_c with a maximum magnitude ± 1. The electromagnetic torque then is

$$T_e = \lambda_p(f_a(\theta_r)\cdot i_a + f_b(\theta_r)\cdot i_b + f_c(\theta_r)\cdot i_c) \tag{5}$$

It is significant to observe that the phase-voltage equation is identical to the armature-voltage equation of dc machine [5]. That is one of the reasons for naming this machine the PM brushless dc machine. Combining all the relevant equations, the system in state-space form is

$$\dot{x} = Ax + Bu \tag{6}$$

$$x = [i_a \quad i_b \quad i_c \quad \omega_m \quad \theta_r]^t \tag{7}$$

$$A = \begin{bmatrix} -\frac{R}{L_1} & 0 & 0 & -\frac{\lambda_p}{L_1}f_a(\theta_r) & 0 \\ 0 & -\frac{R}{L_1} & 0 & -\frac{\lambda_p}{L_1}f_b(\theta_r) & 0 \\ 0 & 0 & -\frac{R}{L_1} & -\frac{\lambda_p}{L_1}f_c(\theta_r) & 0 \\ \frac{\lambda_p}{J}f_c(\theta_r) & \frac{\lambda_p}{J}f_c(\theta_r) & \frac{\lambda_p}{J}f_c(\theta_r) & -\frac{B}{J} & 0 \\ 0 & 0 & 0 & \frac{P}{2} & 0 \end{bmatrix} \tag{8}$$

$$B = \begin{bmatrix} \frac{1}{L_1} & 0 & 0 & 0 \\ 0 & \frac{1}{L_1} & 0 & 0 \\ 0 & 0 & \frac{1}{L_1} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

$(L_1 = L - M)$

$$u = [v_a \quad v_b \quad v_c \quad T_l]^t \tag{10}$$

Above state space representation is a complete linear modeling which, linear analysis and control methods can be applied to it [5, 6].

3.2. Field Oriented Based Modeling Techniques

Trapezoidal type of BLDC motor has more advantage rather than its sinusoidal type such as simplicity in control and greater torque. However, with sinusoidal supply BLDC motor, current regulators in x-y axes are much appreciated and it works properly. Moreover, control in x-y axes or field oriented reference frame is very simple and it is possible to use powerful and classic control techniques such as robust control, sensorless techniques and field weakening methods. Therefore, it is appreciated to extend field oriented methods for trapezoidal BLDC motor [7, 8, 9].

Figure 3 depicts a cross section of the 3-phase BLDC motor. The stator winding a-a', b-b' and c-c' are shown as lumped winding for simplicity. Mechanical rotor position and speed are denoted θ_{rm}, ω_{rm}, respectively. 2-axis x-y reference frame is located in rotor and rotates synchronous with rotor. The shown reference frame can be used the control of 1'th harmonic (xy_1) of stator phase current and leads to much oscillations in the developed motor's torque and current. This method is simple implementation but have no precise control. Therefore, it is not suitable for modeling of trapezoidal BLDC motor drive [10, 11]. A good solution to solve above problem is the multiple reference frames (MRF) and average-value model. So harmonic analysis should be applied Hence we extended the rotating reference frames for dominant harmonics such as 5'th, 7'th and 11'th harmonics. Then the proper controllers should be applied to different harmonic separately. Figure 4 shows the block diagram of the proposed control algorithm. MRF modeling and average value model can be used for both transient and dynamic system analysis [12, 13].

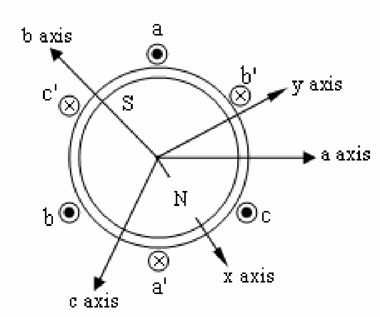

Figure 3. *Cross section of a 3-phase BLDC motor*

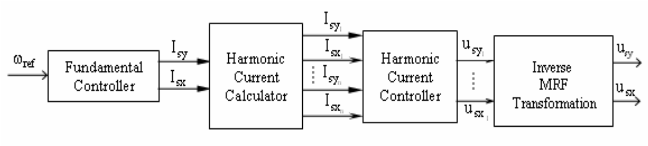

Figure 4. *Block diagram of the harmonic controller*

3.3. Modeling Based on Switching Function Concept

It has shown that the switching function concept is a powerful tool in understanding and optimizing the performance of the static power converters/inverters [14, 15]. Using the switching function concept, the power conversion circuits can be modeled according to their functions, rather than circuit topologies [16]. Therefore, it can achieve simplification of the overall power conversion functions and also allow for the development of analytical concepts that are applicable to families of converters instead of individual ones [14].

Figure 5 shows the overall system configuration of the 3-phase BLDC motor drive. The inverter topology is six-switch voltage-source configuration with constant DC-Link voltage (V_d), which is identical with the induction motor drives and the permanent magnet ac motor drives. The PWM three-phase inverter operation can be divided into six modes according to the current conduction states as shown in figure 2. The three phase currents are controlled to take a form of quasi-square waveform in order to synchronize with the trapezoidal back-EMF to produce the constant torque. In this section, the modeling approach based on switching function is explained. Figure 6 shows the overall block diagram of the developed model for BLDC motor drives. It comprises of six functional blocks [17].

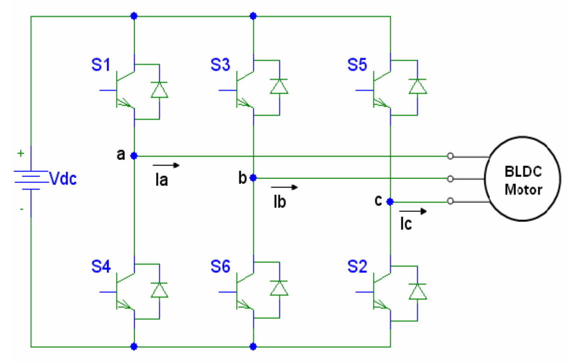

Figure 5. *Configuration of BLDC motor drive system*

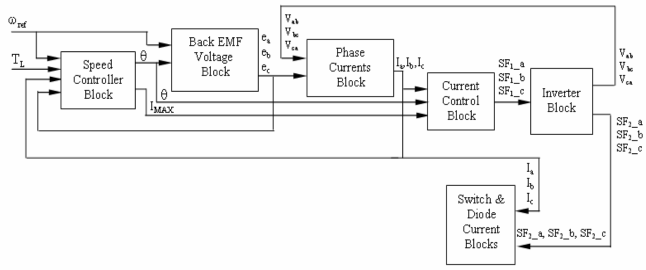

Figure 6. *Switching Function concept based model for the BLDC motor drive*

3.3.1 *Back- EMF Block*

As shown in figure 2, the back-EMF is a function of rotor position (θ_r) having the amplitude of $E = K_e \omega$ (K_e is back-EMF constant). Based on the rotor position, the numerical expression of back-EMF can be obtained. In practical situation, due to manufacturing imperfection, deterioration of permanent magnets, or unbalanced stator windings, the back-EMF waveforms become unbalanced. In this case, the real back-EMF could be modeled and so, back-EMF block can be modified [4, 17].

3.3.2 *Current Control Block*

To control the phase current, a conventional controller such as PI or hysteresis controller can be used in the BLDC drive. The advantages of the hysteresis controller are fast transient response and simple implementation. The current ripple in transient state is a disadvantage of the controller. Compared to the hysteresis controller, the advantage of the PI controller is the zero steady state error. On the other hand, the transient response is slower than the hysteresis controller. Equation 11 shows the transfer function of the PI controller as

$$G_c(s) = k_p + \frac{k_I}{s} \tag{11}$$

The digitized form of the transfer function using the bilinear transformation shown in equation 12 is as follows

$$Y_{a,b,c}(k+1) = Y_{a,b,c}(k) + k_P \cdot [u_{a,b,c}(k+1) - u_{a,b,c}(k)] + \frac{k_I \cdot T_s}{2} \cdot [u_{a,b,c}(k+1) + u_{a,b,c}(k)] \quad (12)$$

Where, u, Y and T_s stand for the input, output and sampling time of the PI controller, respectively. Based on the sign of Y, we can obtain the proper switching functions for each phase of the BLDC motor. To determine the switching functions, output of PI controlled must be modulated via a proper waveform as shown in figure 7. The value of SF_1_a changes to -1 and +1 alternatively. The representations of switching functions (SF1_a,b,c) for hysteresis current control method in [15] are given.

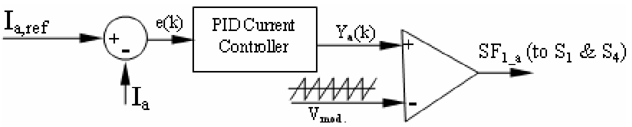

Figure 7. *Generation of the switching pattern for S_1 and S_4*

3.3.3 *Phase Current Blocks*

As shown in figure 2, the only two phases are excited through the conduction operating modes. Therefore, the three phase currents are considered in terms of the line-to-line voltages. From figure 1 and equation 1, the following voltage and current equations can be obtained as:

$$\begin{cases} v_{ao} - v_{bo} = 2R \; i_1 + 2(L-M) \cdot \frac{di_1}{dt} + (e_{ao} - e_{bo}) \\ v_{bo} - v_{co} = 2R \, i_2 + 2(L-M) \cdot \frac{di_2}{dt} + (e_{bo} - e_{co}) \\ v_{co} - v_{ao} = 2R \, i_3 + 2(L-M) \cdot \frac{di_3}{dt} + (e_{co} - e_{ao}) \end{cases} \quad (13)$$

Using switching function SF_1_a,b,c which are obtained from hysteresis current control block, the Vao, Vbo, and Vco can be calculated as

$$\begin{cases} V_{ao} = \frac{V_d}{2} \cdot SF_{1\_a} \\ V_{bo} = \frac{V_d}{2} \cdot SF_{1\_b} \\ V_{co} = \frac{V_d}{2} \cdot SF_{1\_c} \end{cases} \quad (14)$$

Then, from solving equation 13, i_1, i_2 and i_3 can be calculated and so, phase currents (i_a, i_b, i_c) will be obtained [4,15].

3.3.4 *Inverter and Switch & Diode Current Blocks*

From the calculated phase currents, the detailed pure switch and diode currents are derived using switching function SF_2. Each phase has two switching functions of SF_2, such as SF_{2_S1} and SF_{2_S4} for switches S_1 and S_4, respectively with the following definition as

$$\begin{cases} SF_{2\_S_1} = SF_{1\_a} > 0 \\ SF_{2\_S_4} = SF_{1\_a} < 0 \end{cases} \quad (15)$$

Based on the equation 14, the switch and diode currents for phase A are calculated as

$$\begin{cases} I_{S_1\_S} = (I_a > 0) \cdot SF_{2\_S_1} \\ I_{S_1\_D} = (I_a < 0) \cdot SF_{2\_S_1} \\ I_{S_4\_S} = (I_a > 0) \cdot SF_{2\_S_4} \\ I_{S_4\_D} = (I_a > 0) \cdot SF_{2\_S_4} \end{cases} \quad (16)$$

Then switch S_1 and S_4 are as

$$\begin{cases} I_{S_1} = I_{S_1\_S} - I_{S_1\_D} \\ I_{S_4} = I_{S_4\_S} - I_{S_4\_D} \end{cases} \quad (17)$$

In addition, the inverter input current, the average and RMS values of current parameters can be obtained [15].

3.3.5 *Speed Controller Block*

According to equation 2, the torque can be calculated from phase back-EMF voltages and currents obtained from proposed blocks. So, the speed can be obtained from solving equation 3. Switching function concept can be implemented in Pspice, but using the circuit elements increase simulation time. Then, the model is simply implemented by using the functional block of Matlab Simulink [4,17].

3.4. Numerical Modeling Techniques

The numerical modeling and simulation of motor drives often requires that the effect of power electronic circuits be considered. This is not just due to the fact that most motor drives are combinations of magnetic components and circuitry, but also because of the need for designers to perform system level simulation. To take geometric complexity, non-linearity, induced eddy currents, mechanical movement and electric circuits with general topologies into account, it is necessary to couple the finite element method (FEM) with electric circuit analysis [19].

Over the past several years, field and circuit coupling has been commonly used in conjunction with the time domain simulation [20]. There are two basic approaches to coupling the FEM with circuit equations [21]. One is direct coupling [22] where the field and circuit equations are coupled directly together and solved simultaneously. The other is indirect coupling which, the FEM is handled as a separate system and communicates with the circuit model by means of coupling coefficients [23]. These coupling coefficients can be lumped inductance matrices extracted from field solutions together with computed winding currents from circuit simulation, or they can be equivalent circuit parameters (impedances, open circuit voltages) across winding terminal ports derived from circuit simulation, and winding currents as well as induced voltages derived from field solutions. For direct coupling, the obvious advantages are reliable convergence and computationally efficiency. Indirect coupling becomes attractive when there is a need to allow field simulator and circuit simulator to work independently, which makes the development of individual simulator more flexible and the implementation of the coupling easier.

4. SIMULATION

Table 1 shows the specifications of the BLDC motor drive used in this paper. Figure 8 shows the generated back-EMF waveforms and rotor position variations according to the operating speed 3500[rpm]. The back-EMF has the amplitude of 22.45[V]. Also, the actual phase currents are successfully obtained by the hysteresis current control algorithm and they are well synchronized with back-EMF waveforms as shown in Figure 9. The detailed operational characteristics of the PWM inverter are shown in figure 10. The PWM inverter modeling is based on switching function concept. From the figure 10, it is noted that the switching function signals are only generated during the 120° conduction periods in order to force the currents to be follow its reference value. The positive (negative) value +1 (-1) expresses the upper (lower) switch or diode is under the conducting state. Figure 11 shows the dynamic responses of the speed controller. With the help of PI controller, the real speed is reaching to the command value in 0.03[sec]. Also, from the generated constant torque reference, maximum current reference value (I_{max}) is calculated, and then it is used in the current control block.

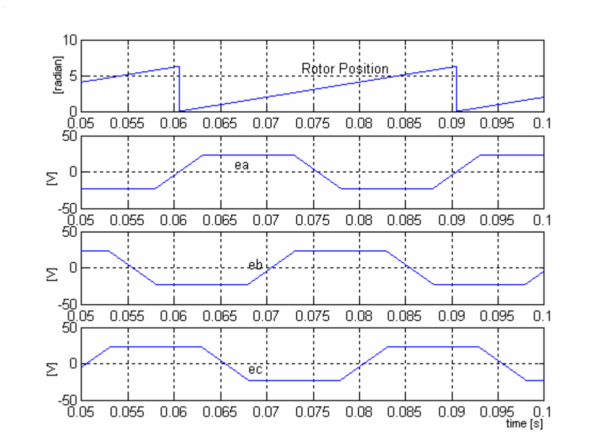

Figure 8. *Rotor position and back-EMF waveforms*

Table 1: *Rated parameter of BLDC motor*

K_t	0.21 [Nm]	R	0.75 [Ω]
K_{e_LL}	0.21 [V/(rad/sec)]	L-M	3.05 [mH]
J	8.2e-5 [kgm^2]	P_{rated}	1[HP]
T_L	0.66 [Nm]	ω_{rated}	3500 [rpm]

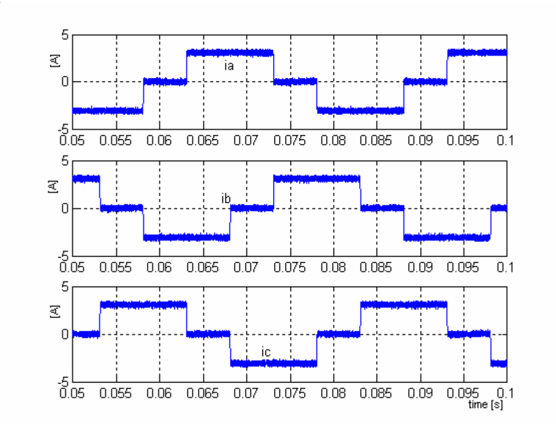

Figure 9. *Phase currents*

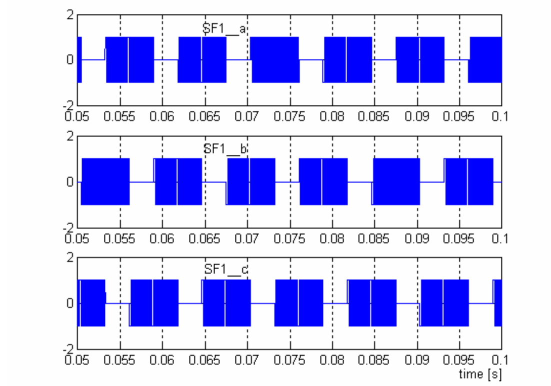

Figure 10. *Switching functions $SF_{1\_a,b,c}$*

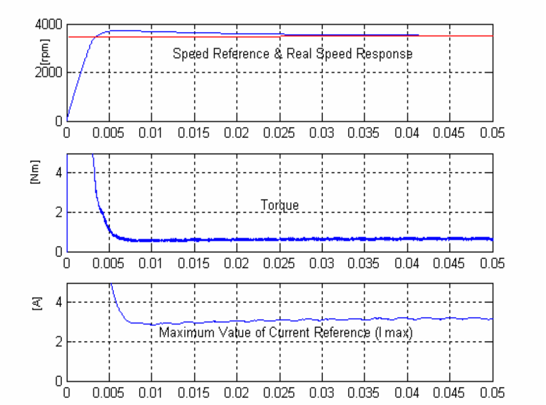

Figure 11. *Dynamic response of speed controller, developed torque and current reference*

5. CONCLUSION

In this paper, we explored the different methods for modeling the BLDC motor drive system. Linear modeling is not an effective method to estimate transient state and motor's nonlinearity. By using field oriented based method and via d-q park transformations, it is possible to use the conventional control techniques, different sensorless schemes, operation in high speed by field weakening techniques. However, because of trapezoidal voltage waveforms and quasi square current waveforms, to enhance the accuracy, it should be used the multiple reference frame that leads complex calculation. Moreover, modeling of inverter needs to extra software such as Saber, Spice. Then we proposed a dynamic simulation model of the BLDC motor drive based on switching function concept. The SFC based developed model has the major following advantages: (1) Simplification of the power conversion circuit (2) Providing an easy-to-design tool for design and examining the performance of the control strategy (3) Avoiding of the convergence problem and simulation run time be reduced by much. Numerical modeling methods have high accuracy and too complex calculations. So they are used often for design the motor. To enhance the accuracy of the modeling of BLDC motor drive such as considering the non-ideal back-EMF voltage waveforms, torque ripple due to cogging effect, it is recommended that the numerical modeling methods such as finite element to be coupled with the proposed switching function based modeling method.

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