

Sensorless Control of Non-Sinusoidal Permanent Magnet Brushless Motor using Selective Torque Harmonic Elimination Method and Sliding Mode Observer

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Abstract— Nowadays, due to excellent advantages of permanent magnet brushless motors (PMBL), they are used in many industrial and variable-speed electrical drive applications. If the fabricated PMBL motors have neither ideal-sinusoidal nor ideal-trapezoidal back-EMF voltages, they are named non-sinusoidal (or non-ideal) PMBL motor. Employing the conventional control strategies of PMSMs and BLDCMs decreases the efficiency and leads to unwanted torque ripple, vibration and acoustic noises. Moreover, in many applications, to reduce the cost and to enhance the reliability, sensorless control techniques are used. This paper proposes a novel sensorless control for a non-sinusoidal PMBL motor with minimum torque ripple. To develop smooth torque, the selected torque harmonic elimination strategy is employed. Furthermore, to estimate the rotor position and speed, a novel full-order sliding mode observer is designed. Proposed observer estimates the position and speed of motor from standstill to final speed. The proposed observer is robust to uncertainty of harmonic contents in phase back-EMF voltage and able to run the motor from standstill with close loop control scheme. The capabilities of torque ripple minimization and sensorless strategies are demonstrated via simulations.

Index Terms— Brushless machine, Motor drive, Sensorless control, Sliding mode observer, Torque ripple.

I. INTRODUCTION

In two past decades and with reducing the price of permanent magnets material, design and manufacturing of permanent magnet brushless (PMBL) motors have developed in various industrial and non-industrial applications. Superior features such as high efficiency, high power or torque density, low maintenance cost, simple structure and ease of control are the reasons for tendency to these motors [1]. The PMBL motors depending on the shape of phase back-EMF voltage are including two main categories of AC brushless (PMSM or BLAC) and DC brushless (BLDC). The induced back-EMF voltages in stator windings of PMSMs are quite sinusoidal, whereas for BLDCMs are trapezoidal waveforms with flat portion over a range of 120 degrees as shown in Fig. 1(a) and Fig. 1(b). The difference of back-EMF voltage waveforms causes that their control methods to be different [2]. To develop the constant instantaneous torque for PMSMs, vector based control methods such as field oriented control (FOC) and

direct torque control (DTC) are usually used. But for BLDCMs, using of vector based methods isn't common and their utilization, leads to torque ripple. Therefore, simple six-step current methods are employed. However, there are some fabricated PMBL motors that their phase back-EMF voltages are neither ideal trapezoidal like BLDCMs nor sinusoidal like PMSMs. It is due to imprecise design or restrictions during fabrication of PMBL motors. These motors are briefly named non-sinusoidal PMBL motor. Fig. 1c shows a typical back-EMF voltage of non-sinusoidal PMBL motor. Employing conventional control methods of PMSMs and BLDCMs such as vector control or six-step current control for non-sinusoidal PMBL motors causes significant instantaneous torque ripple that depends directly on the harmonic contents of phase back-EMF voltage [3,4]. In special applications such as military under-water vehicles, existence of torque ripple leads to mechanical vibration or acoustic noise that is inelible.

On the other hands, closed-loop control of all types of PMBL motors need to electronic or electromechanical sensors for measuring of rotor speed and position. Regarding to non-sinusoidal PMBL motors, to have a constant instantaneous torque, exact instantaneous value of rotor position is essential. It means that exact position sensors must be used that are costly. In addition, using of position/speed sensors leads to hardware complexity and

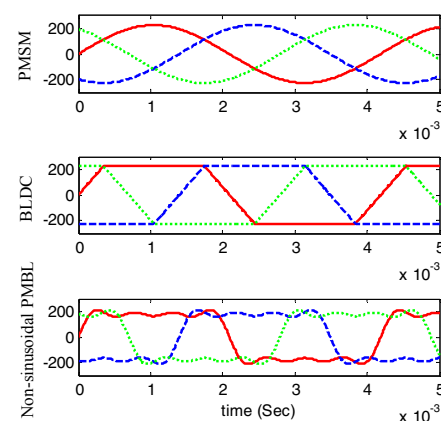


Fig. 1. The back-EMF voltage waveforms of different types of permanent magnet brushless motors (PMBL)

decreases the overall reliability of system. Moreover, using of such sensors may not have economic justification in low power drives. Consequently, using of adequate position/speed estimators can reduce the cost and enhances the reliability. This paper proposes a novel closed-loop control of speed with minimum torque ripple for a given non-sinusoidal PMBL motors. As follows, we briefly examine previously presented control methods for PMSMs and BLDCMs and introduce the superior control method. Also in conjunction to developed control method, a novel rotor position/speed estimation based on sliding mode observer is developed.

II. DYNAMIC MODEL OF NON- SINUSOIDAL PMBL MOTOR

The non-sinusoidal PMBL motor, unlike PMSMs has not sinusoidal flux distribution in the air gap and using two-axis dq reference frame leads to lots of error due to existence of higher harmonics components [5]. There are two main approaches for dynamic modeling of these motors; (1) modeling in multiple dq reference frames (MRF) [6] (2) modeling in stationery three axis abc reference frame. In the modeling based on multiple reference frames, according to harmonic contents of back-EMF voltage, multiple dq reference frames are considered with speeds equal to available harmonics, and motor quantities include voltages, currents, fluxes are transferred to these multiple dq reference frames [7]. Another modeling method of non-sinusoidal PMBL motor is the manner that is used for modeling of BLDC motor. Fig. 2 shows the electrical equivalent circuit of dynamic model for non-sinusoidal PMBL motor. The voltage equations of three-phase non-sinusoidal PMBL motor to the motor star point are expressed as follows:

$$\begin{bmatrix} v_{an} \\ v_{bn} \\ v_{cn} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L_s - M & 0 & 0 \\ 0 & L_s - M & 0 \\ 0 & 0 & L_s - M \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \quad (1)$$

that v_{an}, v_{bn}, v_{cn} are stator terminal voltages to star point, i_a, i_b, i_c are phase currents of motor, e_a, e_b, e_c are phase back-EMF voltages and L_s and M are self-inductance and mutual inductance of stator phases. Electromagnetic torque is developed as follows:

$$T_e = \frac{e_a i_a + e_b i_b + e_c i_c}{\omega_m} \quad (2)$$

in which ω_m is mechanical speed of rotor shaft and obeys the following rotational motion equation:

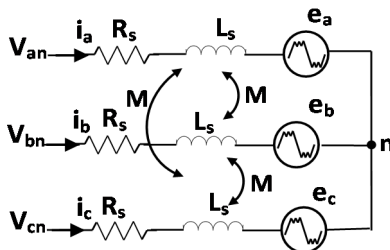


Fig. 2. Electrical equivalent circuit of dynamic model of three-phase non- sinusoidal PMBL motor in abc reference frame

$$T_e = T_L + J \frac{d\omega_m}{dt} + B_f \omega_m \quad (3)$$

that T_L is load torque and B_f and J are load friction coefficient and moment of inertia referred to the rotor shaft respectively.

III. CONTROL OF NON-SINUSOIDAL PMBL MOTOR WITH MINIMUM TORQUE RIPPLE

Due to source of torque ripple in PMBL motors that can cause from current commutation, tooth grooves, or non-ideal waveform of back-EMF voltage, various methods have been suggested [3]. The category of methods based on reference current shaping is the most common and adequate control method for torque ripple elimination. It can be carried out by analyzing the components of phase back-EMF voltage in dq rotating reference frames. On this way, based on the components of back-EMF voltage (e_d and e_q) the reference value of different harmonics of current are made. Afterwards, current regulation for each harmonic is carried out in corresponding dq reference frame, and finally their outputs are combined together to make the output voltages. In [8], this manner has been done for a PMSM motor with non-sinusoidal back-EMF voltage with fifth harmonic order. The PI current regulators have been used for controlling the d and q components of current in fundamental and fifth times speed dq rotating reference frames.

The vector control of non- sinusoidal PMSMs has been improved by modifying the q component of reference current in [9]. In other words, the reference component i_{qs} is modified by back-EMF voltage component e_q that changes due to higher order harmonics using air-gap power relationship in dq reference frame. A park-like transformation has been used in [7] to reduce the torque ripple and control of a non-sinusoidal BLDC motor. Another method was proposed in [10] that used an extension of Park's transformation to model the non-sinusoidal PMSM by means of the so-called pseudo-dq axes reference frame. In [11], an alternative approach called "Pseudo-Vector Control" (PVC) to reduce torque ripple in BLDCM. Two major issues of mentioned methods are; high computational complexity and dependency on the motor parameters in which most of them are open loop and don't include the variations of motor parameters into control law. Therefore as follows, the selective torque harmonic elimination method is used for non-sinusoidal PMBL motor.

A. Selective torque harmonic elimination control of non-sinusoidal PMBL motor

This method has been applied for the BLDC motors and also is called harmonic current injection [12]. It is briefly expressed for non-sinusoidal PMBL motor with phase back-EMF voltage shown in Fig. 1c that contains the harmonics of order $n=1,3,5$ and 7 with the amplitude percentage of 100%, 33%, 20% and 14% respectively. Suppose that the back-EMF voltage of phase 'a' can be rewritten as:

$$e_a(t) = E_1 \sin \alpha t + E_3 \sin 3\alpha t + E_5 \sin 5\alpha t + E_7 \sin 7\alpha t \quad (4)$$

To gain the maximum power/torque, it is desired that phase current 'a' to be in phase with phase back-EMF voltage as:

$$i_a(t) = I_1 \sin \alpha t + I_3 \sin 5\alpha t + I_7 \sin 7\alpha t \quad (5)$$

Because in the configuration assumed for given PMBL motor, the neutral connection is not used so that current harmonics of order multiple of three cannot exist. Moreover, higher order harmonic currents than seven are not imposed because they only cause more stator copper losses. The instantaneous air-gap power of phase 'a' includes an average component and even higher order harmonics until 14th order as:

$$P_a(t) = e_a i_a = P_0 + P_2 \sin 2\omega t + P_4 \sin 4\omega t + P_6 \sin 6\omega t + \dots + P_{14} \sin 14\omega t \quad (6)$$

Considering the symmetry for phase voltages and currents of different phases, the total air-gap power will contain an average component as well as harmonics of order multiple of six as:

$$P_g(t) = 3(P_0 + P_6 \sin 6\omega t + P_{12} \sin 12\omega t) \quad (7)$$

and the other harmonics are eliminated. Therefore, the instantaneous torque can be written as:

$$T_e(t) = \frac{P_g}{\omega_r} = T_0 + T_6 \sin 6\omega t + T_{12} \sin 12\omega t \quad (8)$$

where:

$$T_0 = \frac{3}{2\omega_r} [E_1 I_1 + E_5 I_5 + E_7 I_7] \quad (9)$$

$$T_6 = \frac{3}{2\omega_r} [I_1 (E_7 - E_5) - I_5 E_1 + I_7 E_1] \quad (10)$$

$$T_{12} = \frac{3}{2\omega_r} [-I_5 E_7 - I_7 E_5] \quad (11)$$

Since the torque is proportional to the multiply of the back-EMF and the feed current, it is possible to determine an appropriate combination of e and i that reduces the torque ripple to a minimum value for the given average torque T_0 . Therefore, only harmonic order multiples 5 and 7 (I_5, I_7) are added to fundamental harmonic in which the most important torque harmonics T_6 and T_{12} are cancelled out. So, for the given average torque of T_0 and E_1, E_5 and E_7 , by solving of following algebraic equation:

$$\begin{bmatrix} E_1 & E_5 & E_7 \\ E_7 - E_5 & -E_1 & E_1 \\ 0 & E_7 & E_5 \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_5 \\ I_7 \end{bmatrix} = \frac{2\omega_r}{3} \begin{bmatrix} T_0 \\ 0 \\ 0 \end{bmatrix} \quad (12)$$

and for given values $E_1=100\%$, $E_5=20\%$, $E_7=13\%$, and load torque T_b , the feed current harmonics are obtained from:

$$\begin{bmatrix} I_1 \\ I_5 \\ I_7 \end{bmatrix} = \begin{bmatrix} 1.0063 \\ -0.047 \\ 0.0235 \end{bmatrix} \frac{2\omega_r T_b}{3} \quad (13)$$

By imposing three-phase reference currents with the first, fifth and seventh harmonic amplitude as obtained, the torque ripple due to harmonics of phase back-EMF voltage will be exactly cancelled. Fig. 3 shows the block diagram of speed closed-loop system of non-sinusoidal PMBL motor controlled by using selective torque harmonics elimination method. The used motor has rated power and speed of 2.5 kW and 1500 rpm that its equivalent circuit parameters are listed in table II at appendix. Fig. 4 shows the simulation results of motor behavior by this method. The reference speed reaches its rated value within a tenth of a second and the motor actual speed also tracks it as well. The reference current, which is determined by control system, is not sinusoidal and includes harmonics with order multiple fifth and seventh. Three hysteresis current controllers are

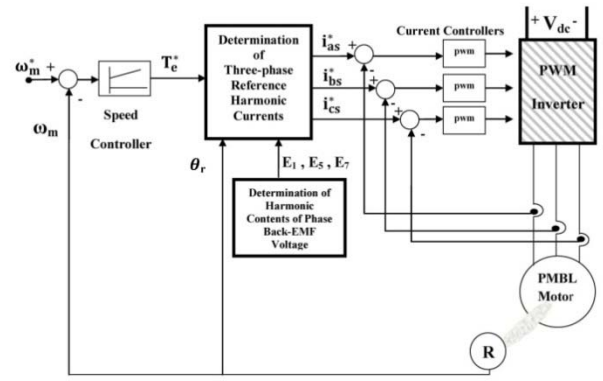


Fig. 3. The block diagram of non-sinusoidal PMBL motor drive by selective torque harmonic elimination method

employed for tracking of reference currents and the results show the current response is satisfactory. Electromagnetic torque reaches to its determined maximum value of 40 N.m during the startup and at final speed, it tends to load torque. The peak-to-peak value of torque ripple is 2.7 N.m or 16% at final speed which shows a significant improvement rather than other suggested methods.

To prove this claim, the motor behavior is also simulated by using three common control methods of PMBL motors including; three-phase quasi-square current control, direct torque control and vector control. The simulation results of these methods are compared with selective torque harmonic elimination method as summarized in Table I. Simulation results confirm that proposed selective torque harmonic elimination method has significant advantages such as the ease of implementation, low calculations and less torque ripple value in compared to other methods.

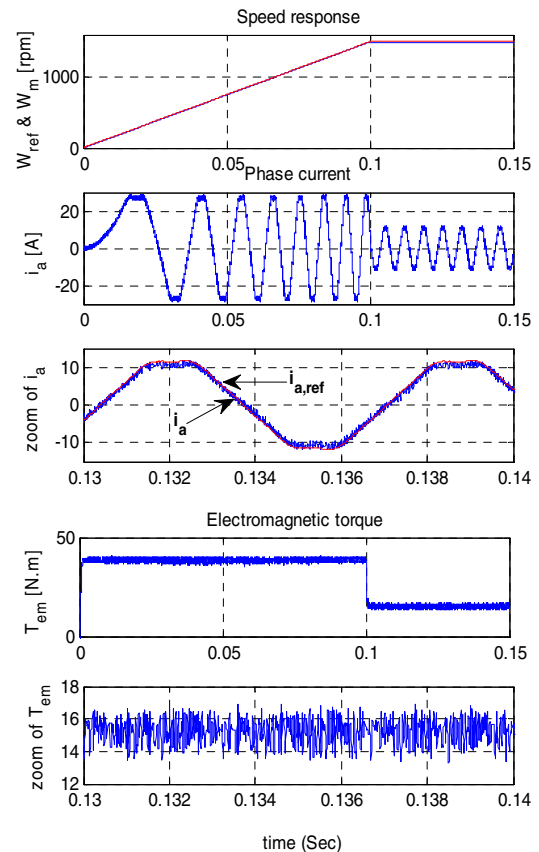


Fig. 4. Simulation results of the non-sinusoidal PMBL motor controlled by the selective torque harmonic elimination method

TABLE I. COMPARISON OF THE RELATIVE PEAK-TO-PEAK TORQUE RIPPLE OF NON-SINUSOIDAL PMBL MOTOR BY USING VARIOUS CONTROL METHODS AT FINAL SPEED

Control method	Torque ripple
Direct torque control	80%
Quasi-square current control with three-phase current feeding	45%
Vector control	33%
Selective torque harmonic elimination control	16%

IV. SENSORLESS CONTROL OF NON-SINUSOIDAL PMBL MOTOR

The position estimation methods of PMBL motors can be divided into two categories: open-loop and closed-loop methods [13,14]. The closed-loop techniques are based on observer schemes that use an internal correction mechanism. These methods are mainly based on model reference adaptive systems (MRAS) include Luenberger observer, disturbance and sliding mode observer, and Kalman filter.

Among closed-loop observers, sliding mode observers (SMO) have satisfactory dynamic response and good robustness to the dynamic model parameters changes and linear and non-linear un-modeled dynamics. Major presented sliding mode observers estimate the phase back-EMF voltage components in the stationary (e_α , e_β) or rotating (e_d , e_q) two-axis reference frames using measured stator currents and voltages. Then, the rotor position and speed are calculated from mathematical relations. For instance, in [15], for a typical BLDC motor, the voltages e_α and e_β are estimated using a second-order SMO, and rotor position is obtained from:

$$\theta_r = \frac{\pi}{2} - \tan^{-1} \left(\frac{e_\beta}{e_\alpha} \right) \quad (14)$$

and for calculation the rotor speed, it has suggested to derivative from the rotor position. Derivation may lead to significant computational error due to switching noises. Similar attempts have been presented in [16] that back-EMF components e_d and e_q in rotating dq reference frame have been estimated by the same way. To avoid of errors and problems due to derivation, some researches have been suggested using of phase-locked loop (PLL) to calculate of motor speed from estimated position that actually increases the order of dynamic equations of the observer [14].

A. Design of Full-Order Sliding Mode Observer for Non-Sinusoidal PMBL motor

Here, a novel full-order sliding mode observer is presented to estimate of the rotor position and speed of non-sinusoidal PMBL motor. The designed observer is especially for nonlinear uncertain systems. The observer inputs are stator voltage components in $\alpha\beta$ stationary reference frame ($v_{s\alpha}$, $v_{s\beta}$). The system state variables are; electrical angular position, rotor speed, and two stator current components in $\alpha\beta$ stationary reference frame as follows:

$$x^T = [\theta_r, \omega_r, i_{s\alpha}, i_{s\beta}] \quad (15)$$

The dynamic equations of PMBL motor expressed by (1) can be rewritten in $\alpha\beta$ reference frame as:

$$\frac{d\theta_r}{dt} = \omega_r \quad (16)$$

$$\frac{d\omega_m}{dt} = \frac{1}{J} \left(3 \left(\frac{e_\alpha i_\alpha + e_\beta i_\beta}{\omega_m} \right) - T_L - B\omega_m \right) \quad (17)$$

$$\frac{di_{s\alpha}}{dt} = -\frac{R_s}{L_s} i_{s\alpha} + \frac{1}{L_s} v_{s\alpha} - \frac{1}{L_s} e_\alpha \quad (18)$$

$$\frac{di_{s\beta}}{dt} = -\frac{R_s}{L_s} i_{s\beta} + \frac{1}{L_s} v_{s\beta} - \frac{1}{L_s} e_\beta \quad (19)$$

The mechanical speed ω_m should be replaced with equivalent electrical speed ω_r in (17). Above state space equations can be expressed in general form as following:

$$\dot{x} = Ax + Bu + \Phi(x,u) + Df(y,u) \quad (20)$$

$$y = Cx \quad (21)$$

that x , u , and y vectors are; state variables, inputs, and outputs respectively. $\Phi(x,u)$ is the known nonlinear term of system and is assumed to be Lipschitz with respect to x for all u [17]. $f(y,u)$ represents unknown term of system that is bounded by the known function $\rho(y,u)$. The input vector u includes stator voltage components of $v_{s\alpha}$ and $v_{s\beta}$, the output vector y includes $i_{s\alpha}$ and $i_{s\beta}$ stator current components. The matrices A, B, C, and D and functions $\Phi(x,u)$ and $f(y,u)$ in (20)-(21) are defined in the appendix A. Moreover, the following conditions must be met for this observer:

1. rank (CD) = rank (D)
2. All the invariant zeros of the matrix triple A, D, C lie in the left half plane.

For this system, condition of rank (CD) = rank (D) = 2 is satisfied. Also, the system has not any zero; therefore the condition 2 is satisfied. Then, state space equations of system (20)-(21) can be written as follow:

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1u + \Phi_1(x,u) \quad (22)$$

$$\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + B_2u + \Phi_2(x,u) + D_2f(y,u) \quad (23)$$

$$y = C_2x_2 \quad (24)$$

where Φ_1 and Φ_2 are the first two components and the last two components of matrix $\Phi(x,u)$ respectively. B_1 , B_2 are the first two rows and the last two rows of matrix B respectively. Given that $f(y,u)$ are the output uncertainties, then its first two rows which are related to x_1 are zero and D_2 is the last two rows of matrix D . Now, let us to introduce a coordinate transformation $z = Tx$ for designing the sliding mode observer, where T is defined as follow:

$$T = \begin{bmatrix} I_2 & L \\ 0 & I_2 \end{bmatrix}, \quad T^{-1} = \begin{bmatrix} I_2 & -L \\ 0 & I_2 \end{bmatrix} \quad (25)$$

Then, the state equations are obtained in the new coordinate system as follows:

$$\dot{z}_1 = (A_{11} + LA_{21})z_1 + (A_{12} - A_{11}L + L(A_{22} - A_{21}L))z_2 + [I_2 \ L]Bu + [I_2 \ L]\Phi(T^{-1}z,u) \quad (26)$$

$$\dot{z}_2 = A_{21}z_1 + (A_{22} - A_{21}L)z_2 + B_2u + \Phi_2(T^{-1}z,u) + D_2f(y,u) \quad (27)$$

$$y = C_2z_2 \quad (28)$$

So, in new z coordinates, sliding mode observer equations have the following form:

$$\dot{\hat{z}}_1 = (A_{11} + LA_{21})\hat{z}_1 + (A_{12} - A_{11}L + L(A_{22} - A_{21}L))\hat{z}_2 + [I_2 \quad L]Bu + [I_2 \quad L]\Phi(T^{-1}\hat{z}, u) \quad (29)$$

$$\dot{\hat{z}}_2 = A_{21}\hat{z}_1 + (A_{22} - A_{21}L)\hat{z}_2 + B_2u + \Phi_2(T^{-1}\hat{z}, u) + D_2f(y, u) \quad (30)$$

$$y = C_2\hat{z}_2 \quad (31)$$

where,

$$v = \rho \cdot \text{sign}(y - C_2\hat{z}_2) \quad (32)$$

and ρ is a positive scalar. Dynamic equations of the estimation error vector are obtained by combining (26)-(28) with (29)-(32) as follows:

$$\dot{e}_1 = (A_{11} + LA_{21})e_1 + [I_2 \quad L](\Phi(T^{-1}z, u) - \Phi(T^{-1}\hat{z}, u)) \quad (33)$$

$$\dot{e}_y = C_2A_2e_1 + (C_2(A_{22} - A_{21}L)C_2^{-1} + C_2K)e_y + C_2D_2f(y, u) - C_2v + C_2(\Phi_2(T^{-1}z, u) - \Phi_2(T^{-1}\hat{z}, u)) \quad (34)$$

According to (33), the matrix L should be chosen so that the term $A_{11} + LA_{21}$ is stable. Also, the gain matrix K can be considered as follows:

$$K = -(A_{22} - A_{21}L)C_2^{-1} + C_2^{-1}A_3 \quad (35)$$

where A_3 is a symmetric positive definite matrix to ensure the following matrix is symmetric negative definite.

$$C_2(A_{22} - A_{21}L)C_2^{-1} + C_2K \quad (36)$$

As a result, the linearized nominal system matrix of the estimation error dynamic system is stable and the error asymptotically tends to zero. The linearized system matrix is described by:

$$\begin{bmatrix} A_{11} + LA_{21} & 0 \\ C_2A_{21} & C_2(A_{22} - A_{21}L)C_2^{-1} + C_2K \end{bmatrix} \quad (37)$$

for this system, the sliding surface is considered as:

$$S = \{(e_1, e_y) | e_y = 0\} \quad (38)$$

In this case, according to the mentioned conditions for selection of matrices L and K , the best response is obtained for $L = -\epsilon ps I_2$ and $A_3 = 2000 I_2$ due to the given uncertainty $f(y, u)$, the value of ρ is chosen to 20. The state estimation error dynamic equations represent sliding dynamics when it is limited to sliding surface S . It is only necessary to ensure stability of e_1 so that asymptotic stability of these equations is proved relative to the sliding surface. For this purpose, Lyapunov function can be considered as $V = e_1^T P e_1$.

V. SIMULATION OF SENSORLESS CONTROL SYSTEM OF NON-SINUSOIDAL PMBL MOTOR

The block diagram of the system is shown in Fig. 5. The estimated position and speed of rotor are compared to the real position and speed in Fig. 6. The maximum amount of position error is 10 degrees which is mainly caused by low pass filters of voltages and currents as observer inputs. However, this amount of error doesn't affect the speed tracking. Also, speed tracking error is less than 8 rpm or 0.6% of reference speed. Fig. 7 shows developed electromagnetic torque that the torque ripple is about 4.5 N.m at final speed. The increase of torque ripple in compare

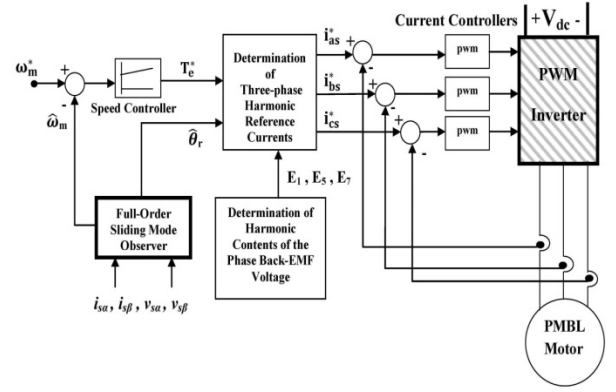


Fig. 5. Block diagram of SMO sensorless control system by selective torque harmonic elimination method for non-sinusoidal PMBL motor

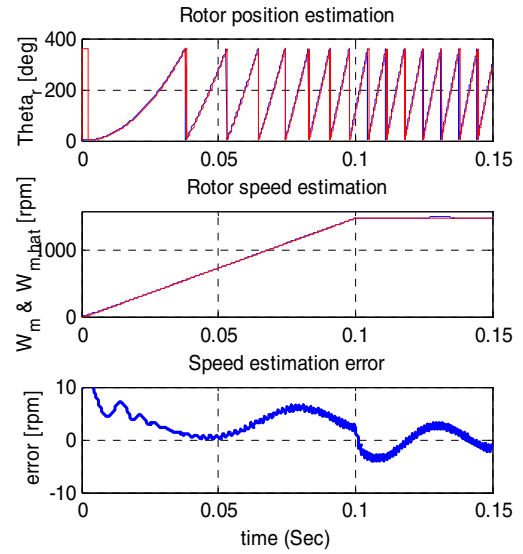


Fig. 6. Estimated position and speed of non-sinusoidal PMBL motor in the sensorless closed-loop control system

to case with sensor is shown in Fig. 4. It is easy to prove that torque ripple caused by mentioned method is less than the event that other control methods are used by the estimated position and speed via sliding mode observer. Fig. 8 shows

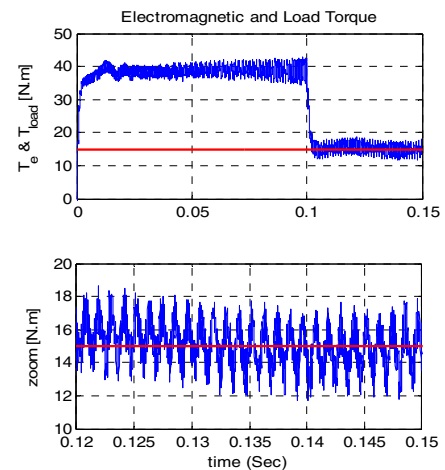


Fig. 7. Developed electromagnetic torque in sensorless closed-loop control system of non-sinusoidal PMBL motor

current response of the driving system. Tracking of phase current are performed well. Also, the reference waveform of stator current component q and its real value are shown. Since the reference current i_{qs} contains the fifth and seventh harmonics it has periodical variations. The real value of i_{qs} has high frequency oscillations due to hysteresis current controller and chattering of SMO.

VI. CONCLUSION

In this paper, a novel sensorless control based on a full-order sliding mode observer in accompany with the torque ripple minimization using selective harmonic elimination have been investigated for close-loop speed control of a non-sinusoidal PMBL motor. Proposed control method has not any dependency to the motor parameters except to harmonic contents of phase back-EMF. Also, contrary to vector controlled based method, it doesn't need any park transformations and voltage decoupling. Torque ripple resulted from proposed method is minimum among various proposed techniques. To reduce the cost and to enhance the reliability, a four-order new sliding mode observer (SMO) has been developed to estimate the instant position and speed of PMBL motor directly. Developed SMO has special features including robustness against uncertainties of the motor parameters such as back-EMF or stator resistance and it can run the motor as closed-loop scheme from standstill without employing open-loop starting that is relevant in other open-loop and some closed-loop estimators.

APPENDIX

TABLE II. THE PARAMETERS OF USED NON-SINUSOIDAL PMBL MOTOR

Quantity	Symbol	Value
Resistance per phase	R_s	0.2 Ω
Self-inductance per phase	L_s	0.8 mH
Mutual inductance	M	0.35 mH
Number of poles	P	12
Constant of back-EMF voltage	K_e	0.15 V/(rad/sec)
Moment of inertia	J	0.015 N.m/s ²
DC link voltage	V_{dc}	300 V
Rated load torque	T_n	15 N.m

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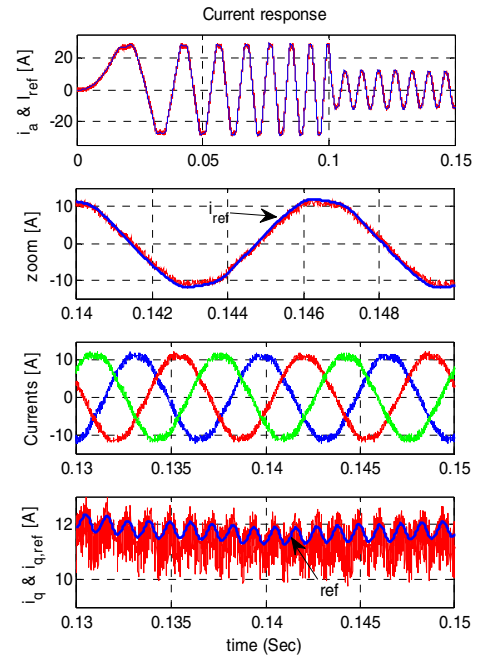


Fig. 8. Non-sinusoidal PMBL motor currents waveforms in the sensorless closed-loop control system

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