Probabilistic Voltage Instability Assessment of Smart Grid Based on Cross Entropy Concept

Mohammad Reza Iranpour
Faculty of Electrical and Computer Engineering
University of Kashan, Kashan, Iran
mr.iranpour@ieee.org

Maryam A.Hejazi
Faculty of Electrical and Computer Engineering
University of Kashan, Kashan, Iran
Mhejazi@kashanu.ac.ir

Reza Arghandeh
Department of Computing, Mathematics and Physics
Western Norway University of Applied Sciences, Norway
rajo@hvl.no

Abstract—**Smart grids, which benefit from the deployment of new technologies such as renewable energy sources, electric vehicles, SCADA and PMU systems and etc., are becoming increasingly complex and uncertain, that can affect security and reliability of these systems. Power variation in generation units and loads demand causes variation in buses voltage that can reach to collapse point and cause instability in smart grids. Therefore instability assessment from buses voltage point of view under uncertain behaviors of smart grids is necessary and important for operation studies of these systems. In this paper by use of cross entropy concept, a new method for bus voltage instability assessment, in which stochastic and uncertain behavior of smart grids had been considered, is introduced. The distance between operational voltage and collapse point for each bus by use of Kullback Libeler divergence is evaluated to calculate instability of system voltage under uncertain operation of system. The IEEE 30-bus system has been analyzed by use of Monte Carlo simulation to evaluate the proposed method.**

Index Terms—**Instability, Collapse Voltage, Cross Entropy, Uncertainty, Probabilistic Behavior, Big data, Information Measure, Monte Carlo Simulation.**

I. Introduction

By use of new technologies, smart grid operation is becoming increasingly complex. This complexity creates different source of uncertainties such as renewable energy sources, electric vehicle charging, SCADA systems, PMU and etc., that can affect the smart grid operation, planning and control and makes new challenges for system planners and operators about the need for developing tools and methodologies for stability, security and reliability assessments [1]. One of these challenges, is instability issues related to bus voltage instability and collapse, after a disturbance in smart grids operation because of power variation in generation units and load demand [2]. Stochastic and uncertain behavior of renewable sources leads to variation in power generation. Also interaction between micro grids (such as new issues raised in competitive electricity markets, including the cost of electric power for subscribers or the reliability of power provided to subscribers by various manufacturing companies), for example demand reduction requested by subscribers from a micro grid and rushing to neighboring grids with better prices and reliability index at different hours of the day and presence of uncertain loads such as, electric vehicles, makes the variation in load demand [3, 4]. Expected energy not supplied (EENS), the expected load curtailment and the loss of load probability [5] are example of different reliability indices which are not suitable indices to voltage reliability assessment because they assess reliability of system from real power point of the view [6-10] and are not directly related to system operating conditions such as voltage, frequency and reactive power (which are related to stability of the bus voltage). Although a security based reliability approach by considering network constraints based on the real power balance has been presented in [7] and system contingency states have been classified into normal, alert, emergency and extreme emergency states, but shortage of reactive power which yields to voltage problems was not considered in this study. Exactly conventional reliability indices related to real power, such as EENS, try to maintain the normal voltage around ±5% of the nominal voltage based on the load curtailed algorithms. These indices could not represent distance between normal and abnormal operating condition such as collapse voltage index [8, 9]. Bus voltages that are related to system's real and reactive power balance can be used to assess system operational instability through monitoring bus voltages in [5] operational reliability assessment of power systems based on bus voltage had been introduced.

Also when smart grids are considered, different factors of uncertainties arise on voltage security. There are many potential sources of uncertainties in smart grid that are related to cyber and physical structures, such as temporal variations of natural resources, forecast errors pertaining to supply and demand, measurement and monitoring errors, and parameter estimation errors of physical systems [1]. In [11], the risk of voltage collapse is measured by considering Poisson probability density function (PDF) to model the probability of transmission line outage. In [12], voltage stability in distributed generation is done by considering uncertainty effects of renewable energy. All of these researches considered a specific form or source of uncertainty in power systems. Several forms of uncertainties in power systems have been presented in [13] that aleatory and epistemic are two main forms of it. The aleatory uncertainty represents the inherited random behavior

978-0-7381-2409-4/20/$31.00 ©2020 IEEE

of power systems [14], that in power system reliability assessment, has been quantified using the sampling approach in the Monte Carlo simulation techniques [15]. In this paper a new method is introduced that can be useful for smart grid voltage instability assessment under different forms of uncertainty. As mentioned earlier, renewable sources, interaction between micro grids and exists of uncertain loads such as electric vehicles, which create aleatory uncertainty sources and uncertainty originated from cyber systems, when voltage is measured and monitored, create epistemic uncertainty sources (which can be arises from cyber-attacks) and affect load demand and bus voltage in smart grid.

Complexity increases in smart grids creates uncertainties in these systems and cause growth in information (because devices, services, and processes in smart grid, contain a vast amount of information). Information and Communication Technology (ICT) solutions collect the big amount of data, but models and tools for creating knowledge from these data is required. Data mining methods should be used to predict the correct performance in smart grids. So a hybrid of data and probabilistic-based approaches seem to be most effective as a comprehensive voltage instability assessment to consider big amount of data and uncertainties forms and sources effects on smart grids operation simultaneously. There are different researches on voltage stability assessment which in continue have been reviewed.

Voltage stability assessment divided into two sub-sections:

1- Static assessment and 2- Dynamic assessment.

In dynamic voltage stability assessment most methods are based on dynamic model of systems and data gained from PMUs [16]. Methods for static voltage stability assessment are based on the Jacobin matrix, power flow, optimal power flow, steady state stability, modal analysis, PV and QV curves [17, 18]. In voltage collapse analysis [19], different scenarios such as power flow methods are employed to determine load margins, margin to thermal limits and voltage violations. Probabilistic voltage stability assessment considering renewable sources with the help of PV and QV curves had been presented in [4]. Voltage stability Index (VSI) has been presented in [10], to evaluate bus voltage stability, by just considering load level and reactive power as the voltage constraints in reliability assessment. Probabilistic methods, VSI index and load factor have been used to voltage instability assessment based on the contingency analysis in [20]. For composite system reliability evaluation in [21], VSI index has implemented to select the buses for load curtailment. Indexes for identifying weak and sensitive bus to the voltage collapse, based on the catastrophe theory were used in [22, 23]. Existing approaches in probabilistic stability analysis of power systems have been reviewed in detail in [24]. Lack of data to estimate long-term system profiles and identification of influential uncertainties in power systems are examples of challenges in instability assessment of modern power system which introduced in [24].

In this work, probabilistic risk of voltage collapse by considering uncertainty in the amount of loads and by use of an information measure tool has been assessed. In fact, by defining voltage collapse PDF and weakest bus PDF, stochastic behavior of system voltage under different uncertainties sources can be modeled. Then distance between these two PDF has been calculated by use of Cross Entropy (Kullback Libeler divergence) concept as an index for measuring and mining information about voltage instability of system. Also probability approaching of this distance to 0 has been introduced as a new voltage instability index that is calculated by use of a modified Monte Carlo method. This approach has been used in [25] for critical infrastructure reliability assessment, by calculating distance between the safe performance measures of the system (that calculated based on the entropy concept) and zero level of entropy which shows the safe performance of the system. Cross Entropy (or Kullback Libeler divergence), is a fundamental idea of modern information theory [26, 27]. Initial applications of this concept in power systems can be found in [28-30].

The main contributions of this paper may be summarized as follows: 1) Performing power flow in any state and determine the collapse point and weakest bus voltage. 2) Calculating distance between weakest voltage bus and collapse point based on Kullback Leibler divergence by considering the uncertain behavior of system. 3) Calculating probability of approaching distance between weakest voltage bus and collapse point to 0. Also different advantages of this new proposed method include: (1) Taking into account operational condition of system under different forms of uncertainties includes aleatory and epistemic uncertainties, (2) and (3) good accuracy and sensitivity in comparison to similar indices, (4) short convergence time which can be used in dynamic probabilistic voltage assessment, (5) weakest buses identification in different states. But most important novelty of this method is (6) ability of it to assess voltage instability by use of the information theory which can be used in the artificial intelligence or machine learning techniques to instability assessment of smart grids.

This paper is organized as follows. In section II, a new index based on Cross Entropy concept has been offered for Voltage Instability assessment. In section III, two numerical test cases are provided to demonstrate the effectiveness of the proposed method in this paper and Section IV concludes this paper.

II. NEW INDEX FOR VOLTAGE INSTABILITY ASSESSMENT BASED ON CROSS ENTROPY METHOD

A. Voltage stability concept

Power system can be represented by an equivalent generator seen from a load bus as shown in Figure.1.

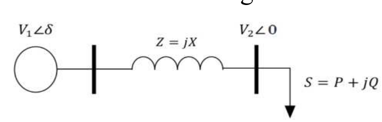

Figure 1. Two Bus System

In the steady-state condition, bus voltage magnitude $V(P, Q)$ is a function of real (P) and reactive power (Q) loads that are presented as [31]: (X is the reactance of the line between bus 1 and 2)

$$P = \frac{V_1 V_2}{x} \sin\delta \quad (1)$$

$$Q = \frac{V_1 V_2}{x}[\cos\delta\,] - \frac{V^2}{x} \quad (2)$$

By elimination of δ from (1) and (2), voltage of 2nd bus can be written as follows:

$$V_2 = \sqrt{\frac{V_2^2}{2} - Q_L x \pm x\sqrt{\frac{V_2^2}{4X^2} - P_L^2 - Q\frac{V_1^2}{x}}} \quad (3)$$

This equation has two answers for bus voltage under a determined power value that one of them is acceptable. With increasing the load bus, voltage drops, until it reaches to (V_c,P_c) (concave point in P-V curve). This point is collapse point of bus and after this point bus voltage is instable and just before voltage collapse is defined to represent collapse voltage V_c. Capability to sustain the control of bus voltage when real and reactive power load increases is the voltage stability of a power system [21]. Equation (3) can be expanded to the whole network. With increasing network load in all *p-q* buses simultaneously, until the power flow does not converge, weakest bus and collapse point of system can be identified.

B. Collapse Of Voltage Probability index for voltage stability based on Cross Entropy concept

B.1. Cross Entropy concept:

There are different methods to describe information about experiments which are used in power system analysis as follow: Some of the Information theories defines the measurements in a random experiment via a random vector $X = (x_1,\ldots, x_n)$ with PDF *f*. Sometimes is need to describe information about the experiments with just a few key numbers, such as the expectation and the covariance matrix of *X*, which provide information about the mean measurements and the variability of the measurements, respectively. Also other informational measure comes from coding and communications theory, which in it Shannon entropy characterizes the average number of bits needed to transmit a message *X* over a (binary) communication channel [32]. Parameter vector $\boldsymbol{\theta}$ is key parameter in point estimation theory which PDF *f* is depended on it. Main question in estimation theory is how well $\boldsymbol{\theta}$ can be estimated via an outcome of X. In other words, how much information about $\boldsymbol{\theta}$ is contained in the “data” *X*. Various measures for this type of information are associated with the maximum likelihood, and the (Fisher) information matrix. Finally, the amount of information in a random experiment can often be quantified via a distance concept, such as the Kullback-Leibler “distance” (divergence), also called the cross-entropy [33]. Let $g(x)$ and $h(x)$ be two densities function on X. The Kullback-Leibler (cross-entropy) between *g* and *h* is defined as:

$$\begin{aligned} D(g,h) = E_g[ln\frac{g(x)}{h(x)}] &= \int g(x)\,ln\frac{g(x)}{h(x)}\,dx \\ &= \int g(x)\,ln\,g(x) \\ &- \int g(x)\,ln\,h(x)dx \end{aligned} \quad (4)$$

And $D(g,h) = 0$ if and only if $g(x) = h(x)$.

In fact similarity level of two density function is evaluated with Cross Entropy concept. Whatever, the samples produced and the mean of the two distribution functions is more similar, distance or Cross Entropy between them is less. When behaviors of two distribution functions are quite similar, the distance between them is zero and they are the same.

B.2. Voltage Instability Formulation Based On Cross Entropy Method:

A large blackout is starting by collapse of voltage at one or more buses at the system after a shook. So that network instability can be assesses based on the distance between the actual and collapse point of the buses voltage in a power network [34]. Based on uncertain and stochastic behavior of smart grids in different states, for voltage collapse and operational voltage, two PDFs can be considered. Suppose $V_c(x)$ be voltage collapse PDF and mean of this PDF is the collapse point of system and $V_o(x)$ be operational voltage PDF for weakest bus and measured voltage of this bus after power flow in each state is mean of this PDF. Cross entropy concept in each state, can be used to measure distance between these two PDFs. If this distance is zero, means that, collapse of voltage will occur in the system.

$$\begin{aligned} D(V_c(x), V_o(x)) = E_{V_c(x)}[ln\frac{V_c(x)}{V_o(x)}] \\ = \int V_c(x)\,ln\frac{V_c(x)}{V_o(x)}\,dx \\ = \int V_c(x)\,ln\,V_c(x) \\ - \int V_c(x)\,ln\,V_o(x))dx \end{aligned} \quad (5)$$

This distance, provides very useful information for system operator to make a correct decision in a contingency state and for system planner to make an accurate planning decision to future programs. Therefore, we can introduce a new index for voltage instability assessment entailed by ***"Collapse of Voltage Probability (COVP)"***. Based on this description, *COVP* of the system can be defined as follow:

The *COVP* is the probability of the distance between $V_c(x)$ and $V_o(x)(|V_c(x), V_o(x)|)$ approaches to zero. In this definition, the concept of reaching to zero is introduced based on limitation concept as follows:

$$\lim_{V_o \to V_c}(COVP) = L \Leftrightarrow \forall\varepsilon > 0, \exists\delta > 0 : \left|D(V_o, V_c)\right| < \delta \Rightarrow \left|COVP - L\right| < \varepsilon \quad (6)$$

Because of stochastic behavior of the system, different methods based on probability concepts can be useful to calculate this probability, such as Monte Carlo simulation (MCS)-based tools [35] as follows:

$$COVP = \frac{1}{N}\sum_{k=1}^{N} I\{D(V_o(x), V_c(x)) \le \varepsilon\} \quad (7)$$

In this equation, *I* is identity matrix, $\varepsilon > 0$ is a very small value and vector *X* is a Bernoulli random variable that defined as:

$$X_i \begin{cases} for: V = V_{o,p.u} \rightarrow X_i = Ber(V_o) = \begin{cases} 1 & if: V = V_{o,p.u} \\ 0 & if: V \neq V_{o,p.u} \end{cases} \\ for: V = V_{c,p.u} \rightarrow X_i = Ber(V_c) = \begin{cases} 1 & if: V = V_{c,p.u} \\ 0 & if: V \neq V_{c,p.u} \end{cases} \end{cases} \quad (8)$$

COVP can be calculated in each bus or in whole of the system. For calculating the COVP in whole of the system, weakest bus of system should be identify and then probability of the distance between weakest bus and collapse point (in this bus) to approach 0 should be calculated. Weakest bus in a power network is recognized as the first bus that reaches to its collapse point when the load of the system is increasing during a contingency state.

B.2.1. Weakest bus identification:

Identifying precise of the bus which is caused to collapse of system in a contingency state is a complex procedure. This bus has been determined based on the voltage variations from normal to abnormal state [5] and also is considered as a bus with 1) heavy load and weak links to other buses of system and 2) lowest voltage and vulnerable behavior to further load increase when considering different forms of uncertainties [36]. For identifying weakest bus, first collapse point of system should be determined which this collapse point can be calculated by increasing the load in p-q buses simultaneously until power flow in the system does not converge. Math power tool can be used to perform this scenario. Bus *w* in the bus set Ω_L is the weakest bus for contingency state *i*, if the distance (Cross Entropy) between $V_{w,i}$ and V_c be the minimum, defined as:

$$\begin{aligned} &for\ i = 1:1:L, \forall V_i \in \Omega_L \\ &V_i = V_{w,i} \Leftrightarrow D(V_{w,i}, V_c) = Min\ D(V_i, V_c) \end{aligned} \tag{9}$$

In (6), L is the number of load bus in bus set Ω_L.

In the following, first, for assessment of newly introduced method, load level in voltage buses have been increased or decreased linearly with a deterministic scenario and then in the second scenario stochastic behavior for load buses has been considered. Based on these descriptions a general trend for Voltage instability assessment by applying Cross Entropy concept is expressed as follows:

Step1: Input system data

Step2: Apply a particular load level for load buses based on one of the two scenarios that are defined.

Step 3: Perform AC power flow analysis for load level *l* in contingency state *i*.

Step 4: Record the initial voltage at each bus.

Step 5: Identify collapse point of system by use of math power tool.

Step6: Identify the weakest bus based on equation (9).

Step7: Calculate *COVP* based on equation (6,7).

Step 8: If all the load levels are evaluated, save the results and stop the program.

III. CASE STUDY

In this section for new introduced method assessment, as mentioned earlier, two scenarios have been considered. In the first scenario load level in voltage buses have been increased linearly with a deterministic scenario. For assessment of new introduced method in this scenario, 6 bus system [37] and IEEE 30-bus system [38] are analyzed. Based on general trend introduced in this paper for voltage instability assessment, first, collapse point of the system should be identified and after performing AC power flow analysis for a specified load level, based on the equation (6), weakest bus is identified, and then *COVP* will be calculated. Simulation results for normal state is shown in TABLE I and simulation results for different load level in load bus, is shown in TABLE II. As shown in these tables, CPU time which represent the convergence time of new introduced method (computations were performed using an Intel Core i7, 2.20-GHz processor), is very short and acceptable for real time studies.

TABLE I. Six bus simulation results in normal state

Bus number	Voltage magnitude (P.U)	Weakest voltage magnitude based on Eq.6	COVP	CPU time (second)
Bus 4	0.989	0.9788	0.1618	0.0037
Bus 5	0.985	0.9649		
Bus 6	1.004	0.9806		

TABLE II. Six bus simulation results in different load level

Load (P.U)	Bus number	Voltage magnitude (P.U)	COVP	CPU time (second)
1.18	Bus 4	0.976	0.1659	0.001817
	Bus 5	0.969		
	Bus 6	0.991		
1.36	Bus 4	0.963	0.2073	0.001901
	Bus 5	0.950		
	Bus 6	0.987		
1.54	Bus 4	0.948	0.2660	0.001734
	Bus 5	0.931		
	Bus 6	0.964		
1.72	Bus 4	0.932	0.3377	0.001723
	Bus 5	0.910		
	Bus 6	0.949		
1.9	Bus 4	0.915	0.4416	0.001732
	Bus 5	0.887		
	Bus 6	0.933		

In [5], expected bus voltage drop (*EBVD*) index is introduced in a same study, which in it *EBVD* index for each bus is calculated. For a correct comparison between the new introduced method in this paper and [5], a same situation should be considered. So for calculating the *COVP* for each bus in algorithm of this paper, step 6 in general trend will be ignored and *COVP* index for each bus will be calculated. The results of these simulations, for 4 different buses in IEEE 30-bus system have been shown in TABLE.III.

TABLE III. EBVD & COVP index simulation results comparison in IEEE 30-bus system

Bus number	Voltage magnitude (P.U)	EBVD [5]	COVP
Bus 30	0.990	0.2037	0.2752
Bus 26	0.996	0.1966	0.1991
Bus 29	1.002	0.1936	0.1963
Bus 19	1.019	0.1831	0.1857

Comparison of these results with [5], indicate good accuracy of the proposed method. As seen in TABLE III, COVP is more sensitive to voltage changes than EBVD. This sensitivity can be seen in Figure. 2 based on the voltage sensitivity analysis for voltage buses which shown in TABLE III. As mentioned before, in the second scenario for assessment of new introduced method in this paper, stochastic behavior of load buses has been considered. Normal distribution probability function with mean μ= *"weakest bus voltage amplitude"* and standard deviation $\sigma = 0.05$ is considered to generate random samples for voltage of weakest bus. After generating 10000 random samples (N=10000) for a 6 bus test system by use of normal distribution probability function, *COVP* has been calculated and the results are shown on Figure.3. Histogram of different bus voltage, collapse voltage and COVP are shown in Figure. 3 COVP is calculated based on the distance between collapse voltage distribution and bus voltage distribution functions. PDF of bus voltage can be created from aleatory or epistemic uncertainties. So that this method can be considered a PDF for bus voltage originated from aleatory or epistemic uncertainties and calculates voltage instability index.

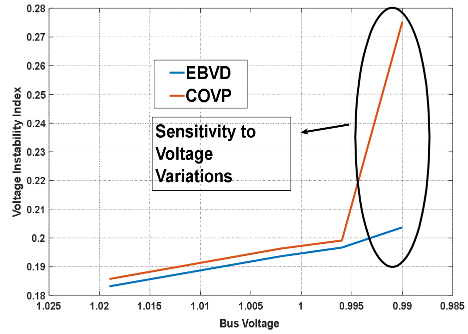

Figure 2. Sensitivity of COVP compared to EBVD

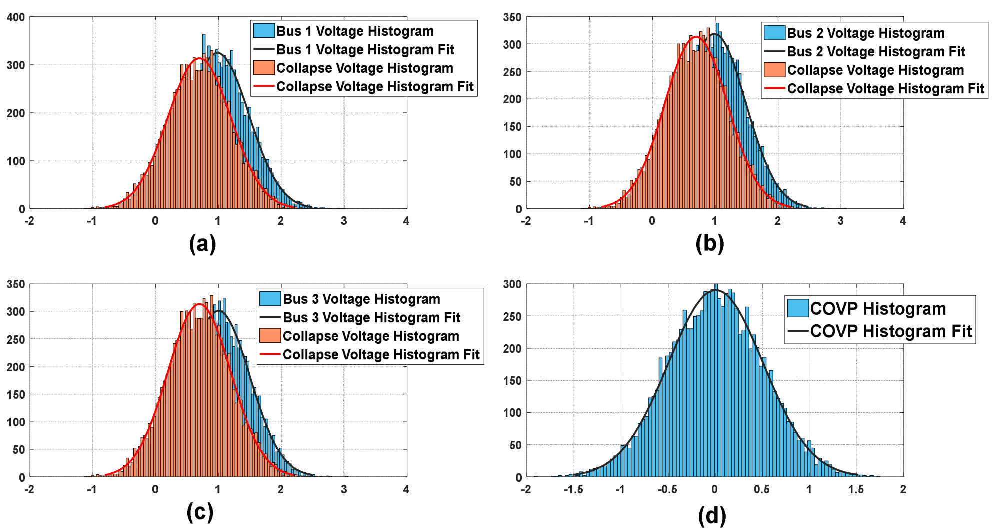

Figure 3. Collapse Voltage, Bus voltage and COVP Histogram.

IV. CONCLUSION AND FUTURE WORKS

This paper proposes a technique to assess instability from the bus voltage stability point of view under uncertain behaviors of smart grids based on distance between operational and collapse point of voltage by use of Cross Entropy concept. The system studies for the 6 bus test system and IEEE 30-bus show that the proposed technique provides a direct and simple instability index for bus voltage instability. In addition, a new method for weakest buses identification has been introduced in this paper by use of Cross Entropy concept which can be used in protection systems and algorithms. Also very short convergence time of method can be useful for real time studies. Aleatory and epistemic are two main forms of uncertainty, related to cyber and physical systems that affected security and reliability of smart grids, which effects of these uncertainties can be assess by use of the new proposed method in this research. In this paper, aleatory uncertainty has been considered and in future works, instability assessment of smart grid voltage under aleatory and epistemic uncertainty will be performed simultaneously for a more accurate study, especially when cyber-attacks are considered.

V. REFERENCES

[1] R. Preece and J. V. Milanović, "Assessing the applicability of uncertainty importance measures for power system studies," IEEE Transactions on Power Systems, vol. 31, no. 3, pp. 2076-2084, 2016.

[2] A. Invernizzi, B. Buchholz, M. Stubbe, N. Jenkins, B. Dowd, and M. Ceraolo, "Distribution systems and dispersed generation: a new focus for CIGRE," Electra, vol. 213, pp. 17-21, 2004.

[3] A. H. Ranjbar, Reliability analysis of modern hybrid micro-grids. The University of Texas at Dallas, 2013.

[4] A. B. Almeida, E. V. De Lorenci, R. C. Leme, A. C. Z. De Souza, B. I. L. Lopes, and K. Lo, "Probabilistic voltage stability assessment considering renewable sources with the help of the PV and QV curves," IET Renewable Power Generation, vol. 7, no. 5, pp. 521-530, 2013.

[5] W. Qin, J. Song, X. Han, and P. Wang, "Operational reliability assessment of power systems based on bus voltage," IET Generation, Transmission & Distribution, vol. 9, no. 5, pp. 475-482, 2015.

[6] R. Ringlee, P. A. Chmn, and R. Allan, "Bulk power system reliability criteria and indices trends and future needs," IEEE Transactions on Power Systems (Institute of Electrical and Electronics Engineers);(United States), vol. 9, no. 1, 1994.

[7] R. Billinton and E. Khan, "A security based approach to composite power system reliability evaluation," IEEE Transactions on Power Systems, vol. 7, no. 1, pp. 65-72, 1992.

[8] R. Billinton and G. Lian, "Composite power system health analysis using a security constrained adequacy evaluation procedure," IEEE Transactions on Power Systems, vol. 9, no. 2, pp. 936-941, 1994.

[9] R. Billinton and M. Fotuhi-Firuzabad, "Generating system operating health analysis considering stand-by units, interruptible load and postponable outages," IEEE Transactions on Power Systems, vol. 9, no. 3, pp. 1618-1625, 1994.

[10] P. Kessel and H. Glavitsch, "Estimating the voltage stability of a power system," IEEE Transactions on power delivery, vol. 1, no. 3, pp. 346-354, 1986.

[11] N. Aminudin, T. Rahman, N. Razali, M. Marsadek, N. Ramli, and M. Yassin, "Voltage collapse risk index prediction for real time system's security monitoring," in Environment and Electrical Engineering (EEEIC), 2015 IEEE 15th International Conference on, 2015, pp. 415-420: IEEE.

[12] A. Almeida, R. Leme, A. Z. de Souza, B. L. Lopes, and V. O. Albuquerque, "Voltage stability reliability," 2011.

[13] F. O. Hoffman and J. S. Hammonds, "Propagation of uncertainty in risk assessments: the need to distinguish between uncertainty due to lack of knowledge and uncertainty due to variability," Risk analysis, vol. 14, no. 5, pp. 707-712, 1994.

[14] R. Billinton and D. Huang, "Aleatory and epistemic uncertainty considerations in power system reliability evaluation," in Probabilistic Methods Applied to Power Systems, 2008. PMAPS'08. Proceedings of the 10th International Conference on, 2008, pp. 1-8: IEEE.

[15] W. Li, Reliability assessment of electric power systems using Monte Carlo methods. Springer Science & Business Media, 2013.

[16] K. Sun, S. Likhate, V. Vittal, V. S. Kolluri, and S. Mandal, "An online dynamic security assessment scheme using phasor measurements and decision trees," IEEE Transactions on Power Systems, vol. 22, no. 4, pp. 1935-1943, 2007.

[17] F. W. Mohn and A. Z. De Souza, "Tracing PV and QV curves with the help of a CRIC continuation method," IEEE Transactions on Power Systems, vol. 21, no. 3, pp. 1115-1122, 2006.

[18] J. Guckenheimer, "Towards a global theory of singularly perturbed dynamical systems," in Nonlinear Dynamical Systems and Chaos: Springer, 1996, pp. 213-225.

[19] A. Z. De Souza, "Discussion on some voltage collapse indices," Electric Power Systems Research, vol. 53, no. 1, pp. 53-58, 2000.

[20] S. Aboreshaid and R. Billinton, "Probabilistic evaluation of voltage stability," IEEE transactions on power systems, vol. 14, no. 1, pp. 342-348, 1999.

[21] R. Billinton and S. Aboreshaid, "Voltage stability considerations in composite power system reliability evaluation," IEEE transactions on power systems, vol. 13, no. 2, pp. 655-660, 1998.

[22] P. Raja, M. P. Selvan, and N. Kumaresan, "Enhancement of voltage stability margin in radial distribution system with squirrel cage induction generator based distributed generators," IET Generation, Transmission & Distribution, vol. 7, no. 8, pp. 898-906, 2013.

[23] T. Zabaiou, L.-A. Dessaint, and I. Kamwa, "Preventive control approach for voltage stability improvement using voltage stability constrained optimal power flow based on static line voltage stability indices," IET Generation, Transmission & Distribution, vol. 8, no. 5, pp. 924-934, 2014.

[24] K. N. Hasan, R. Preece, and J. V. Milanović, "Existing approaches and trends in uncertainty modelling and probabilistic stability analysis of power systems with renewable generation," Renewable and Sustainable Energy Reviews, vol. 101, pp. 168-180, 2019.

[25] M. Iranpour, M. A. Hejazi, and M. Shahidehpour, "A Unified Approach for Reliability Assessment of Critical Infrastructures Using Graph Theory and Entropy," IEEE Transactions on Smart Grid, vol. 11, no. 6, pp. 5184-5192, 2020.

[26] T. Homem-de-Mello and R. Y. Rubinstein, "Rare event simulation and combinatorial optimization using cross entropy: estimation of rare event probabilities using cross-entropy," in Proceedings of the 34th conference on Winter simulation: exploring new frontiers, 2002, pp. 310-319: Winter Simulation Conference.

[27] P. Blossom, "The Cross-Entropy Method: A Unified Approach to Combinatorial Optimization, Monte-Carlo Simulation, and Machine Learning," ed: JSTOR, 2006.

[28] D. P. Kroese, K.-P. Hui, and S. Nariai, "Network reliability optimization via the cross-entropy method," IEEE Transactions on Reliability, vol. 56, no. 2, pp. 275-287, 2007.

[29] D. Ernst, M. Glavic, G.-B. Stan, S. Mannor, and L. Wehenkel, "The cross-entropy method for power system combinatorial optimization problems," in Power Tech, 2007 IEEE Lausanne, 2007, pp. 1290-1295: IEEE.

[30] F. Belmudes, D. Ernst, and L. Wehenkel, "Cross-entropy based rare-event simulation for the identification of dangerous events in power systems," in Probabilistic Methods Applied to Power Systems, 2008. PMAPS'08. Proceedings of the 10th International Conference on, 2008, pp. 1-7: IEEE.

[31] C. W. Taylor, Power system voltage stability. McGraw-Hill, 1994.

[32] T. M. Cover and J. A. Thomas, Elements of information theory. John Wiley & Sons, 2012.

[33] R. Y. Rubinstein, "Optimization of computer simulation models with rare events," European Journal of Operational Research, vol. 99, no. 1, pp. 89-112, 1997.

[34] Y.-L. Chen, "Weak bus-oriented optimal multi-objective VAR planning," IEEE Transactions on Power Systems, vol. 11, no. 4, pp. 1885-1890, 1996.

[35] R. Allan, R. Billinton, A. Breipohl, and C. Grigg, "Bibliography on the application of probability methods in power system reliability evaluation: 1987-1991," IEEE Transactions on Power Systems, vol. 9, no. 1, pp. 41-49, 1994.

[36] Y.-L. Chen, C.-W. Chang, and C.-C. Liu, "Efficient methods for identifying weak nodes in electrical power networks," IEE Proceedings-Generation, Transmission and Distribution, vol. 142, no. 3, pp. 317-322, 1995.

[37] P. Kundur, N. J. Balu, and M. G. Lauby, Power system stability and control. McGraw-hill New York, 1994.

[38] H. Saadat, Power system analysis. McGraw-Hill, 1999.